Space-Time SIC Multiuser Detection Algorithm and Its Performance in Rayleigh Fading Channel

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Abstract— In this paper, a space-time successive interference cancellation (ST-SIC) scheme equipped with multiple transceiver antennas for direct-sequence code division multiple access (DS-CDMA) communications is presented. The performance of the scheme is analytically examined in a Rayleigh fading channel. It is observed that our analytical results agree well with the empirical results. The validity and usefulness of the ST-SIC scheme are also demonstrated by computer simulations.

1. INTRODUCTION

Two major factors that limit the performance of DS-CDMA systems include the multiple access interference (MAI) and the multipath channel distortion (MCD). Many advanced signal processing techniques, such as the multiuser detection (MUD) algorithm [1], [2] and the space-time processing (STP) algorithm [3], have been studied to combat the MAI and the MCD, respectively.

Verdu [1] proposed the optimal MUD or the maximum likelihood sequence detector, and presented its performance analysis in additive white Gaussian noise (AWGN) environments. Since, however, Verdu's work is computationally too complex to be used in the practical DS-CDMA systems, researches have focused on finding the suboptimal MUD solutions [2], [4] that enable more efficient implementation. One of the suboptimal MUD solutions is the successive interference cancellation (SIC) [4] detector, in which each stage of the SIC detector decides, regenerates, and cancels out one additional DS user from the received signal successively one at a time. The remaining users thus see less MAI in the next stage. Despite its simplicity in hardware implementation, the SIC detection requires one additional bit delay per stage as well as a reordering procedure for the received signals whenever the power profile changes.

The STP technique is often used to suppress the MCD in wireless communications systems involving the exploitation of spatial diversity with multiple transmitter and/or receiver antennas. The STP technique initially focused on the systems employing one transmitter antenna and multiple receiver antennas [3], but recently, much of the work in this areas has focused more on the transmit diversity scheme employing multiple transmitter antennas [5]. Tarokh et al. [6] proposed the space-time trellis coding algorithm, in which the copies of the same symbol are repeatedly transmitted through multiple antennas at different times. The simple space-time block coding scheme was presented by Alamouti [7], and a more generalized concept of the Alamouti's work was developed in [8].

Recently, the space-time multiuser detection (ST-MUD) schemes using multiple transmitter and receiver antennas have been proposed in order to deal with the MAI as well as the MCD problems simultaneously. In [9], the optimal space-time multiuser receiver structure was first presented, followed by the linear ST-MUD methods based on iterative interference cancellation. The maximum-likelihood ST-MUD using orthogonal spreading codes was investigated in [10]. In [11], the performance of the ST-MUD using multiple transmitter and receiver antennas was examined for CDMA communications. Here, however, the decorrelating detection algorithm was used as the MUD scheme, and thus computationally very complex due to the matrix inversion operation.

In this paper, we consider a new and computationally more efficient ST-MUD scheme which employs the SIC scheme as the MUD part. We will utilize the Alamouti space-time block code [7] for two-transmit-antenna configurations. The performance analysis of the proposed scheme in Rayleigh fading channels is also presented for the DS-CDMA system. An analytical expression for the bit error probability is derived.

In Section 2, we analyze the bit error probability of the ST-SIC scheme with multiple transmitter and receiver antennas. For the purpose of the verification of the analytical results, some simulation results in Rayleigh fading channels are demonstrated in Section 3, and concluding remarks are made in Section 4.

2. PERFORMANCE

In this section, we analyze ST-SIC schemes for synchronous CDMA system with multiple transmitter and receiver antennas in Rayleigh fading channels. We consider two cases: 1) two transmitter antennas and one receiver antenna, and 2) two transmitter antennas and two receiver antennas. It is assumed that a space-time block code is employed in systems with two transmitter antennas and the receiver has perfect knowledge of the channel.

2.1. Two transmitter antennas and one receiver antenna

When the transmitter has two antennas, we must first specify how the information bits are transmitted across the two antennas. Here, we adopt the space-time block coding scheme [7]. For user *k*, two information symbols $b_{1,k}$ and $b_{2,k}$ are transmitted over two symbol intervals. At the first time interval, the symbol pair $(b_{1,k}, b_{2,k})$ is transmitted across the two transmitter antennas, and at the second time interval, the symbol pair $(-b_{2,k}, b_{1,k})$ is transmitted [11]. Figure 1 depicts the diagram of the ST-SIC scheme with two transmitter antennas and one receiver antenna.

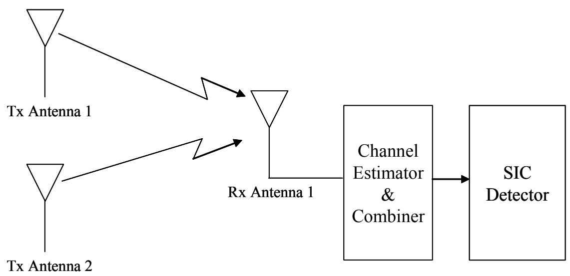

Figure 1. ST-SIC detector scheme with two transmitter antennas and one receive antenna

We begin with a numerical description of this system, where all bits of all users are aligned in time. Assuming there are *K* DS users, the received signal corresponding to these two time intervals are given by

$$\boldsymbol{r}_1 = \sum_{k=1}^{K} |A_k| \left(h_{1,k} b_{1,k} \ \boldsymbol{c}_{1,k} + h_{2,k} b_{2,k} \ \boldsymbol{c}_{2,k} \right) + \boldsymbol{n}_1 \tag{1}$$

$$\boldsymbol{r}_2 = \sum_{k=1}^{K} |A_k| \left(-h_{2,k} b_{1,k} \ \boldsymbol{c}_{1,k} + h_{1,k} b_{2,k} \ \boldsymbol{c}_{2,k} \right) + \boldsymbol{n}_2 \tag{2}$$

where A_k, $b_{i,k}$, and $\boldsymbol{c}_{i,k}$ $(i = 1,2)$ denote the amplitude, information bit, and spreading code vector for user *k*, respectively. The amplitudes are assumed to be Rayleigh distributed with unit mean square value. The signal $h_{1,k}$ ($h_{2,k}$) for the *k*-th user is the complex-Gaussian channel gain between the first (second) transmitter antenna and the receiver antenna. Moreover, $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ are independent received additive white Gaussian noise (AWGN) vectors with $(0, \sigma^2)$ at the two time intervals.

We consider the SIC scheme [4] from (1) and (2) during two time intervals. We assume that the amplitudes of the SIC detector are ordered such as $A_1 > A_2 > \cdots > A_K$. After the MAI component of (*j*-1)st user is cancelled, the decision variables for *j*-th user denote

$$z_{1,j} = |A_j| \left(h_{1,j} b_{1,j} + h_{2,j} b_{2,j} \right) + \upsilon_1 + \sum_{k=j+1}^{K} |A_k| \left(h_{1,k} b_{1,k} + h_{2,k} b_{2,k} \right) \rho_{k,j} \tag{3}$$

$$z_{2,j} = |A_j| \left(-h_{2,j}^* b_{1,j} + h_{1+j}^* b_{2,j} \right) + \upsilon_2 + \sum_{k=j+1}^{K} |A_k| \left(-h_{2,k}^* b_{1,k} + h_{1,k}^* b_{2,k} \right) \rho_{k,j} \tag{4}$$

where $\boldsymbol{\rho}_{k,j} = \boldsymbol{c}_k^T \boldsymbol{c}_j$ is the cross-correlation of *k*-th and *j*-th user, $\boldsymbol{\upsilon}_i = \boldsymbol{c}_i^T \boldsymbol{n}_i$ (*i*=1, 2), and * denotes the complex conjugate. Then, (3) and (4) can be written in a matrix form as

$$\boldsymbol{z}_j = |A_j| \ \boldsymbol{H}_j \boldsymbol{b}_j + \boldsymbol{\upsilon} + \sum_{k=j+1}^{K} |A_k| \rho_{k,j} \ \boldsymbol{H}_k \boldsymbol{b}_k \tag{5}$$

where $\boldsymbol{z}_j = [z_{1,j} \ z_{2,j}]^T$, $\boldsymbol{b}_k = [b_{1,k} \ b_{2,k}]^T$, $\boldsymbol{\upsilon} = [\upsilon_1 \ \upsilon_2]^T$, $\boldsymbol{H}_k = \begin{bmatrix} h_{1,k} & h_{2,k} \\ -h_{2,k} & h_{1,k} \end{bmatrix}$.

In (3), (4), and (5), the first term is the information bit, the second term is the noise component, and the last term is the residual MAI component. The maximum likelihood decision rule for $\boldsymbol{b}_j$ based on $\boldsymbol{z}_j$ in (5) is then given by

$$\hat{\boldsymbol{b}}_j = \text{sign}\left(\text{Re}\left\{ \boldsymbol{H}^T \boldsymbol{z}_j \right\} \right) \tag{6}$$

We must solve $\text{Re}\{\boldsymbol{H}_k^T \boldsymbol{H}_k\}$ before calculate (6).

$$\boldsymbol{H}_k^T \boldsymbol{H}_k = \begin{bmatrix} |h_{1,k}|^2 + |h_{2,k}|^2 & h_{1,k}^* h_{2,k} - h_{2,k}^* h_{1,k} \\ h_{2,k}^* h_{1,k} - h_{1,k}^* h_{2,k} & |h_{1,k}|^2 + |h_{2,k}|^2 \end{bmatrix} \tag{7}$$

$$\text{Re}\{\boldsymbol{H}_k^T \boldsymbol{H}_k\} = \begin{bmatrix} G_k & 0 \\ 0 & G_k \end{bmatrix} \tag{8}$$

where $G_k = |h_{1,k}|^2 + |h_{2,k}|^2$.

From (6), the information bit $\boldsymbol{b}_j$ is given by

$$\hat{\boldsymbol{b}}_{\boldsymbol{j}} = |A_j| G_j \ \boldsymbol{b}_{\boldsymbol{j}} + \mathrm{Re}\{\boldsymbol{H}^{\boldsymbol{T}} \boldsymbol{\upsilon}\} + \sum_{k=j+1}^{K} |A_k| G_k \rho_{k,j} \ \boldsymbol{b}_{\boldsymbol{k}} \tag{9}$$

Total interference component η_j of the ST-SIC scheme for the j-th user based on [4] is given by

$$\eta_j = \frac{1}{N} \sum_{k=j+1}^{K} G_k |A_k|^2 + G_j \sigma^2 + \frac{1}{N} \sum_{k=1}^{j-1} \eta_k \tag{10}$$

where, $N = T / T_c$, T is bit duration, and T_c is chip period. As the result of (10), signal-to-interference ratio $(r_b)_j$ for the j-th user is given by

$$(r_b)_j = \frac{G_j |A_j|^2}{\eta_j} = \frac{G_j |A_j|^2}{\frac{1}{N} \sum_{k=j+1}^{K} G_k |A_k|^2 + G_j \sigma^2 + \frac{1}{N} \sum_{k=1}^{j-1} \eta_k} \tag{11}$$

In (11), the amplitude of desired signal is distributed the Rayleigh fading. Therefore the average signal-to-interference ratio $(\bar{r}_b)_j$ for the j-th user is given by

$$(\bar{r}_b)_j = \frac{G_j E\left[|A_j|^2\right]}{\frac{1}{N} \sum_{k=j+1}^{K} G_k E\left[|A_k|^2\right] + G_j \sigma^2 + \frac{1}{N} \sum_{k=1}^{j-1} \eta_k} = \frac{2 G_j A_k^2}{\frac{2}{N} \sum_{k=j+1}^{K} G_k A_k^2 + G_j \sigma^2 + \frac{1}{N} \sum_{k=1}^{j-1} \eta_k} \tag{12}$$

where $E[\cdot]$ is expected value and $E[|A|_k^2] = 2A_k^2$, i.e., the RMS value of each component is equal to A_k.

2.2. Two transmitter antennas and two receiver antennas

In this section, we consider the ST-SIC detector scheme with two transmitter and receiver antennas as shown in figure 2. We adopt the space-time block coding scheme used in the previous section.

The received signals at antenna 1 during the two symbol intervals are

$$\boldsymbol{r}_1^{(1)} = \sum_{k=1}^{K} |A_k| \left(h_k^{(1,1)} b_{1,k} \ \boldsymbol{c}_{1,\boldsymbol{k}} + h_k^{(2,1)} b_{2,k} \ \boldsymbol{c}_{2,\boldsymbol{k}}\right) + \boldsymbol{n}_1^{(1)} \tag{13}$$

$$\boldsymbol{r}_2^{(1)} = \sum_{k=1}^{K} |A_k| \left(- h_k^{(2,1)} b_{1,k} \ \boldsymbol{c}_{1,\boldsymbol{k}} + h_k^{(1,1)} b_{2,k} \ \boldsymbol{c}_{2,\boldsymbol{k}}\right) + \boldsymbol{n}_2^{(1)} \tag{14}$$

and the received signals at antenna 2 are

$$\boldsymbol{r}_1^{(2)} = \sum_{k=1}^{K} |A_k| \left(h_k^{(1,2)} b_{1,k} \ \boldsymbol{c}_{1,\boldsymbol{k}} + h_k^{(2,2)} b_{2,k} \ \boldsymbol{c}_{2,\boldsymbol{k}}\right) + \boldsymbol{n}_1^{(2)} \tag{15}$$

$$\boldsymbol{r}_2^{(2)} = \sum_{k=1}^{K} |A_k| \left(- h_k^{(2,2)} b_{1,k} \ \boldsymbol{c}_{1,\boldsymbol{k}} + h_k^{(1,2)} b_{2,k} \ \boldsymbol{c}_{2,\boldsymbol{k}}\right) + \boldsymbol{n}_2^{(2)} \tag{16}$$

where in (13) ~ (16), $h_k^{(i,j)}$, $i, j \in \{1, 2\}$ is the complex Gaussian channel gain between transmitter antenna i and receiver antenna j for user k. The noise vector $\boldsymbol{n}_1^{(1)}, \boldsymbol{n}_2^{(1)}$, $\boldsymbol{n}_1^{(2)}$, and $\boldsymbol{n}_2^{(2)}$ are independent and identically distributed with distribution $(0, \sigma^2)$.

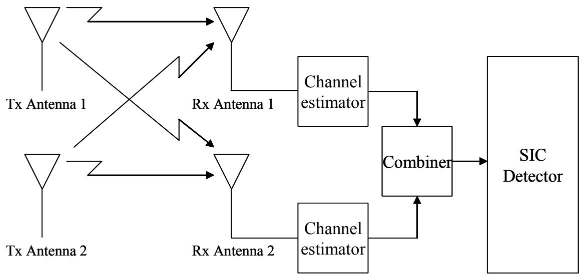

Figure 2. ST-SIC detector scheme with two transmitter antennas and two receiver antennas

As the previous section, we consider the SIC scheme [4] from (13) ~ (16) during two time intervals. After the MAI components of (j-1)-st user is cancelled, the decision variables of j-th user denote

$$z_{1,j}^{(1)} = |A_j| \left(h_j^{(1,1)} b_{1,j} + h_j^{(2,1)} b_{2,j}\right) + \boldsymbol{\upsilon}_1^{(1)} + \sum_{k=j+1}^{K} |A_k| \left(h_k^{(1,1)} b_{1,k} + h_k^{(2,1)} b_{2,k}\right) \rho_{k,j} \tag{17}$$

$$z_{2,j}^{(1)} = |A_j| \left(- \left(h_j^{(2,1)}\right)^* b_{1,j} + \left(h_j^{(1,1)}\right)^* b_{2,j}\right) + \boldsymbol{\upsilon}_2^{(1)} + \sum_{k=j+1}^{K} |A_k| \left(- \left(h_k^{(2,1)}\right)^* b_{1,k} + \left(h_k^{(1,1)}\right)^* b_{2,k}\right) \rho_{k,j} \tag{18}$$

$$z_{1,j}^{(2)} = |A_j| \left(h_j^{(1,2)} b_{1,j} + h_j^{(2,2)} b_{2,j}\right) + \boldsymbol{\upsilon}_1^{(2)} + \sum_{k=j+1}^{K} |A_k| \left(h_k^{(1,2)} b_{1,k} + h_k^{(2,2)} b_{2,k}\right) \rho_{k,j} \tag{19}$$

$$z_{2,j}^{(2)} = |A_j| \left(- \left(h_j^{(2,2)}\right)^* b_{1,j} + \left(h_j^{(1,2)}\right)^* b_{2,j}\right) + \boldsymbol{\upsilon}_2^{(2)} + \sum_{k=j+1}^{K} |A_k| \left(- \left(h_k^{(2,2)}\right)^* b_{1,k} + \left(h_k^{(1,2)}\right)^* b_{2,k}\right) \rho_{k,j} \tag{20}$$

Then, we can rewrite (17) ~ (20) in a matrix form as

$$\boldsymbol{z}_{\boldsymbol{j}} = |A_j| \ \boldsymbol{H}_{\boldsymbol{j}} \boldsymbol{b}_{\boldsymbol{j}} + \boldsymbol{\upsilon} + \sum_{k=j+1}^{K} A_k \rho_{k,j} \ \boldsymbol{H}_{\boldsymbol{k}} \boldsymbol{b}_{\boldsymbol{k}} \tag{21}$$

where $\boldsymbol{z_j} = \left[z_{1,j}^{(1)}\ z_{2,j}^{(1)}\ z_{1,j}^{(2)}\ z_{2,j}^{(2)}\right]^T$, $\boldsymbol{\upsilon} = \left[\upsilon_1^{(1)}\ \upsilon_2^{(1)}\ \upsilon_1^{(2)}\ \upsilon_2^{(2)}\right]^T$, $\boldsymbol{b_k} = \left[b_{1,k}\ b_{2,k}\right]^T$, $\boldsymbol{H_k} = \begin{bmatrix} h_k^{(1,1)} & -\left(h_k^{(2,1)}\right)^* & h_k^{(1,2)} & -\left(h_k^{(2,2)}\right)^* \\ h_k^{(2,1)} & \left(h_k^{(1,1)}\right)^* & h_k^{(2,2)} & \left(h_k^{(1,2)}\right)^* \end{bmatrix}^T$.

It is readily verified in the previous section that

$$\mathrm{Re}\left\{\boldsymbol{H_k^T H_k}\right\} = \begin{bmatrix} G_k & 0 \\ 0 & G_k \end{bmatrix} \tag{22}$$

where $G_k = \left|h_k^{(1,1)}\right|^2 + \left|h_k^{(2,1)}\right|^2 + \left|h_k^{(1,2)}\right|^2 + \left|h_k^{(2,2)}\right|^2$.
From (6), the information bit is given by

$$\boldsymbol{\hat{b}_j} = \left|A_j\right| G_j\ \boldsymbol{b_j} + \mathrm{Re}\left\{\boldsymbol{H^T \upsilon}\right\} + \sum_{k=j+1}^{K} \left|A_k\right| G_k \rho_{k,j}\ \boldsymbol{b_k} \tag{23}$$

Therefore, the signal-to-interference ratio $\left(r_b\right)_j$ and average signal-to-interference ratio $\left(\bar{r}_b\right)_j$ for the j-th user are given by

$$\left(r_b\right)_j = \frac{G_j \left|A_j\right|^2}{\frac{1}{N}\sum_{k=j+1}^{K} G_k \left|A_k\right|^2 + G_j \sigma^2 + \frac{1}{N}\sum_{k=1}^{j-1} \eta_k} \tag{24}$$

$$\left(\bar{r}_b\right)_j = \frac{2 G_j A_k^2}{\frac{2}{N}\sum_{k=j+1}^{K} G_k A_k^2 + G_j \sigma^2 + \frac{1}{N}\sum_{k=1}^{j-1} \eta_k} \tag{25}$$

Comparing (12) with (25), it is different the channel gain between transmitter antenna and receiver antenna according to the number of antennas.

2.3. *Bit error probability in Rayleigh fading channels*

The amplitudes are assumed to be Rayleigh distributed with unit mean square value. In Rayleigh fading channels, the bit error rate (BER) for binary phase shift keying (BPSK) or quadrature PSK (QPSK) can be obtained as [12].

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{r}_b}{1 + \bar{r}_b}}\right) \tag{26}$$

where the $\bar{r}_b$ has been shown in (12) and (25).

3. Simulation Results

In this section, some simulation results are presented in order to illustrate the potential pragmatic possibility of the proposed ST-SIC detector schemes. We assume the synchronous coherent BPSK situation in Rayleigh fading, the equal power of all users when the information bits are transmitted and the receiver perfectly estimates channels between transmitter antenna and receiver antenna.

In Figure 3 and 4, we compare BER among the conventional detector, the SIC detector, and the ST-SIC detector when the number of active users is fixed as 5 and 10, respectively. In figure 4, our simulation results for the BER is fixed 10^{-3} show that the SIC detector, the ST-SIC detector with two transmitter and one receiver (2Tx 1Rx), and the ST-SIC detector with two transmitter and two receiver (2Tx 2Rx) need to about 15dB, 12dB, and 10dB, respectively. Also, the result of theoretical analysis and that of simulation are almost identical. We know that our analytical expression of the ST-SIC schemes is well derived.

It is clear that the performance of the ST-SIC scheme is much better than that of the conventional detector and the SIC detector. The performance of the ST-SIC detector with two transmitter and two receiver antennas is the best in our simulations.

4. Concluding Remarks

In this paper, the ST-SIC schemes equipped with multiple transmitter and receiver antennas have been proposed in order to achieve the reliable performance as well as accommodate more number of active users. A performance analysis of the ST-SIC schemes in Rayleigh fading channels has been made, and the superior characteristics of the ST-SIC scheme have been demonstrated by extensive computer simulations. It is hoped that our work provides an additional design criterion in actual implementation of the 3G and the 4G system.

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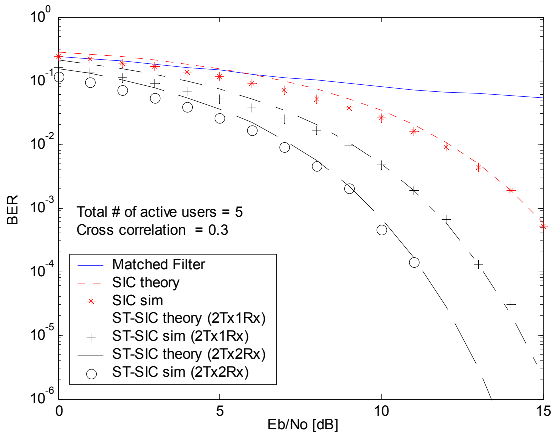

Figure 3. The BER when the number of active users is fixed as 5.

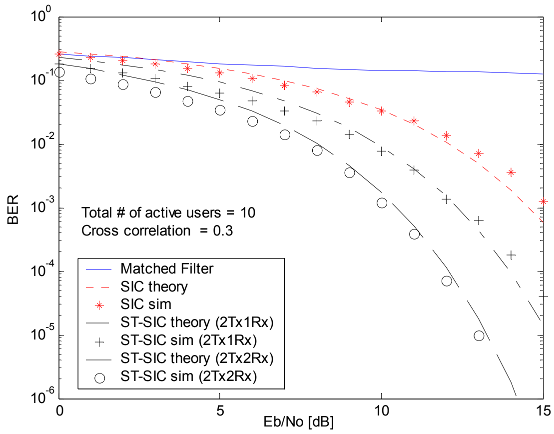

Figure 4. The BER when the number of active users is fixed as 10.