The Orbit of Planet Gamma

Part of the IEEE Teacher In-Service Program - www.ieee.org/organizations/eab/precollege

Lesson Focus
Random error and systematic error.

Lesson Synopsis
The Orbit of Planet Gamma is a fun, hands-on classroom activity that leads students to discover the nature of systematic and random errors that are present in all physical measurements. Students are divided into teams of competing “scientists” that are out to capture the prize of recognition as the “most accurate and advanced researchers in the world.” All the teams are given the same data and discover interesting variations in the results produced by the competing teams.

Age Levels
10-14.

Objectives

- Learn basic principles of random error and systematic error.
- Increase understanding of metric system units.
- Solve simple algebraic manipulations involving squares and square roots.
- Learn to present data graphically.

Anticipated Learner Outcomes
As a result of this activity, students should develop an understanding of:

- random error and systematic error
- metric system units
- squares and square roots
- properties of the ellipse
- graphical presentations of data

Planet Gamma: Introduction

The Planet Gamma activity works well both as an introduction for students that have no prior knowledge about random and systematic error, or as reinforcement for students who have been taught the subject. The concepts are applicable to the physical and biological sciences and mathematics teachers will find the activity handy for demonstrating applications involving squares, square roots, and the properties of the ellipse for students who have been taught these concepts. Students are divided into teams of competing “scientists” that are out to capture the prize of recognition as the “most accurate and advanced researchers in the world.” All the teams are given the same data and discover interesting variations in the results produced by the competing teams.

Lesson Activities

The orbit of "Planet Gamma" is examined by teams of students who measure the dimensions of the major axis and the minor axis. Students record the results to the nearest whole millimeter on a data sheet and calculate the eccentricity according to a provided formula. Because three different rulers will be used, students explore systematic error.

Resources/Materials

- Teacher Resource Documents (attached)
- Student Materials (attached)
- Graph Paper Program (available on IEEE Teacher In-Service Program website)
- Optional Excel Program (available on IEEE Teacher In-Service Program website)

Alignment to Curriculum Frameworks

See attached curriculum alignment sheet.

Optional Activity: Excel Spreadsheet

A Microsoft Excel file, gamma.xls, is available on the IEEE Teacher In-Service Program website. The Excel file calculates eccentricities and averages and finds the maximum and minimum values. The teacher can use the Excel spreadsheet to check student's calculations. If computers are available to individual students, then consider having the students use the spreadsheet to perform the required calculations for their team. Some teachers prefer to make an overhead transparency of the data sheet and to write in the data as teams report their results. Individual students can then fill in their data sheets by copying from the image projected overhead. If a computer and projector are available to the teacher, another option is to fill in the spreadsheet as teams report their data and allow the spreadsheet to perform the calculations. Students then copy the results onto their data sheets from the image of the computer screen.

Internet Connections

- Extra-solar Planets Catalog (www.obspm.fr/encycl/catalog.html)
- IEEE Teacher In-Service Program (www.ieee.org/organizations/eab/precollege/tispt)
- IEEE Virtual Museum (www.ieee-virtual-museum.org)
- McREL Compendium of Standards and Benchmarks (www.mcrel.org/standards-benchmarks)
  A compilation of content standards for K-12 curriculum in both searchable and browsable formats.
- National Council of Teachers of Mathematics Principals and Standards for School Mathematics (www.nctm.org/standards)
- National Institute of Standards and Technology (NIST) (www.nist.gov)
  Information about measurements and measurement uncertainty.
- National Science Education Standards (www.nsta.org/standards)

Recommended Reading

- NIST Special Publication 672, Experimentation and Measurement by W. J. Youden (http://physics.nist.gov/Divisions/Div844/facilities/phdet/pdf/expmeas.pdf)

Optional Writing Activity

- Write an essay (or paragraph depending on age) about how random error and systematic error can impact the results of data gathering.

References

Ralph D. Painter, and other volunteers from
Florida's West Coast USA Section of IEEE
URL: http://ewh.ieee.org/r3/floridawc

For Teachers:
Alignment to Curriculum Frameworks

Note: All Lesson Plans in this series are aligned to the National Science Education Standards which were produced by the National Research Council and endorsed by the National Science Teachers Association, and if applicable, also to the International Technology Education Association's Standards for Technological Literacy or the National Council of Teachers of Mathematics' Principals and Standards for School Mathematics.

◆National Science Education Standards Grades 5-8 (ages 10 - 14)

CONTENT STANDARD B: Physical Science

As a result of their activities, all students should develop an understanding of

- Motions and forces

◆National Science Education Standards Grades 9-12 (ages 14 - 18)

CONTENT STANDARD B: Physical Science

As a result of their activities, all students should develop understanding of

- Motions and forces

◆Principals and Standards for School Mathematics (ages 10 - 14)

Measurement Standards

- Understand measurable attributes of objects and the units, systems, and processes of measurement

  - • understand relationships among units and convert from one unit to another within the same system.
  - • understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

- Apply appropriate techniques, tools, and formulas to determine measurements.

  - use common benchmarks to select appropriate methods for estimating measurements.

◆Principals and Standards for School Mathematics (ages 14 - 18)

Measurement Standards

- Understand measurable attributes of objects and the units, systems, and processes of measurement

  - make decisions about units and scales that are appropriate for problem situations involving measurement.

- Apply appropriate techniques, tools, and formulas to determine measurements.

  - analyze precision, accuracy, and approximate error in measurement situations.
  - understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
  - use unit analysis to check measurement computations.

For Teachers:
Teacher Resources

The Orbit of Planet Gamma is a fun, hands-on classroom activity that leads students to discover the nature of systematic and random errors that are present in all physical measurements. Students are divided into teams of competing international "scientists" that are out to capture the prize of recognition as the "most accurate and advanced researchers in the world." All the teams are given the same data and discover interesting variations in the results produced by the competing teams.

Each teacher is provided with all materials required to for a class of up to thirty-five students. The activity works well both as an introduction for students that have no prior knowledge about random and systematic error or as reinforcement for students who have been taught the subject. The concepts are applicable to the physical and biological sciences and mathematics teachers will find the activity handy for demonstrating applications involving squares, square roots, and the properties of the ellipse.

◆ Preparation

The scale drawings of the elliptical orbit of planet gamma given to each student are identical and are included with the lab instructions given to each student. The measuring instruments (paper rulers) provided to each group are, however, not identical. (Note: Paper Rulers may be found on the last page of this document.) Group A rulers are magnified the about 105% of actual size. Group B rulers are reduced to about 95% of full size. Group C rulers are actual size. The rulers are prepared by photocopying the image of a ruler onto paper and adjusting the magnification of the copier to produce the rulers describe above. A ruler 20cm long works well. If your master ruler is longer that 20 cm, then just cut the paper rulers down to 20cm size. Prepare enough rulers for all students.

The differences in the rulers introduce systematic error into the measurements of the minor and major axes. Group A will report measurements for the major and minor axes that are consistently smaller that Groups B and C. Group B will report major and minor axis measurements that are consistently larger than Groups A and C. Group C will tend to report measurements for the major and minor axes that fall between those reported by Groups A and B. However, this scale factor error cancels out in the eccentricity calculation; therefore, the eccentricities calculated by all three groups should be very similar.

The measurement data are likely to show some random error, particularly in the measurements of the long dimension of the ellipse. The long dimension of the ellipse exceeds the length of the paper rulers provided so that the student must mark the paper and re-position the ruler. The need to re-position the ruler results in greater random variation among student measurements for the long dimension. Cutting the rulers in half will generally increase the amount of random variation. Ellipses of other sizes and proportions can be printed with the Graph Paper Printer 4.21 shareware that is included on the disc for the Planet Gamma lab exercise. Also, the ellipse can be copied on to the paper so that the major and minor axes are not aligned with the vertical and horizontal axes of the paper if you wish to make the exercise more challenging.

Do not tell the students beforehand that their rulers are not identical. The point, of course, is to let the students discover the reason for the differences while comparing the results of their group with the other two groups and accounting for the fact that consistent differences among the groups exist in the measurements of the major and minor axes while similar values for the eccentricities are obtained by all groups.

◆ Classroom Procedure

Divide your class into three teams: Team A, Team B, and Team C and appoint a leader of each team to play the role of laboratory director. Each team represents a national research laboratory in competition with the other two teams for recognition as the most accurate and advanced researchers in the world.

Give each student a copy of the lab instructions. The team leader or "lab director" also receives an envelope containing the paper rulers that the teacher has prepared. Read the prologue with your students to be sure that everyone understands the role they will play in this exercise. For your convenience, the prologue is copied below.

◆ Prologue

You are a member of one of three international research laboratories A, B, and C that are competing for global funding, Nobel prizes, and bragging rights as the most accurate and advanced group of researchers in the world. Because of the rivalries among the three laboratories, each laboratory maintains a policy of strict secrecy and never shares preliminary results with the other two laboratories.

Raw data from an international telescope has been provided to all three-research laboratories in the form of scale drawings of the orbit of planet gamma. Each research laboratory is charged with measuring the major axis and the minor axis of the orbit and calculating the eccentricity of the orbit.

The director of your laboratory has decided that each researcher assigned to the planet gamma project will work independently and that the results will be averaged. Averaging the data it is hoped will more or less eliminate random error so that the results from your laboratory will be the most accurate in the world. Unfortunately, one disgruntled member of your laboratory has leaked the averaging technique to the other research laboratories, which decide to also have their researchers work independently and average the results to reduce the effects of random error.

Once the prologue has been read and everyone understands his role, the team leader distributes a paper ruler to each of his team members who independently measure the major and minor axis of the ellipse and calculate the eccentricity.

Besides handing out the paper rulers to his teammates, each team leader collects the measurements of the major and minor axes and the computations of eccentricity from all his team members and transcribes the data onto the data sheet. The director calculates the average values of the major and minor axes and the eccentricity for his team. The team leader acts as spokesperson for his team and reports the averages and the maximum and minimum values to the class when called on by the teacher. After all the teams have announced their results, each student prepares a graph showing the minimum, average and maximum values reported by each team and writes answers to questions 1-8. A blank sheet of graph paper is included with the student materials, but the students must label the scales, etc. A picture of typical graph layout is included at the end of these teacher instructions.

◆ Class Discussion

After each team has graphed the data and has answered the questions, lead the class in a discussion about the differences in the results reported by each team. Most students will notice that Group A results for the major and minor axis measurements are consistently low, Group B are consistently high and Group C results tend to fall in between. If students do not raise the question themselves, point out that the eccentricities calculated by each team are the same and ask how that could be if the major and minor axis measurements vary from one group to another.

Explain to the class that research equipment like telescope observatories cannot be moved to facilitate side by side comparisons. Assuming that each team of researchers is located in a different part of the world and that the “rulers” represent test equipment that cannot be moved from one place to another, ask your students to propose ways to reconcile the differences between the results obtained by the three groups.

Ask the class to decide which team deserves the recognition as the most accurate and advanced researchers in the world and what should be the criteria for this honor.

◆ Follow-up

The table that appears on the next page gives values of the eccentricities of the planets in the solar system. Ask your students whether or not any of the solar system planets could be Planet Gamma.

◆ Solar System Planet Data

	Orbital Eccentricity	Semi-major axis of orbit, AU's	Calculated Semi-minor axis of orbit, AU's
Mercury	0.2056	0.387	0.379
Venus	0.0068	0.723	0.723
Earth	0.0167	1.000	1.000
Mars	0.0934	1.524	1.517
Jupiter	0.0485	5.203	5.196
Saturn	0.0556	9.555	9.540
Uranus	0.0463	19.218	19.198
Neptune	0.0090	30.110	30.108
Pluto	0.2462	39.341	38.130

Sources:

1 Smith, Peter Duffett; Practical Astronomy with Your Calculator, Third Edition. Cambridge University Press, Cambridge, 1988.
2 Abell, George; Exploration of the Universe, Second Edition. Holt, Rinehart and Winston, New York; 1969

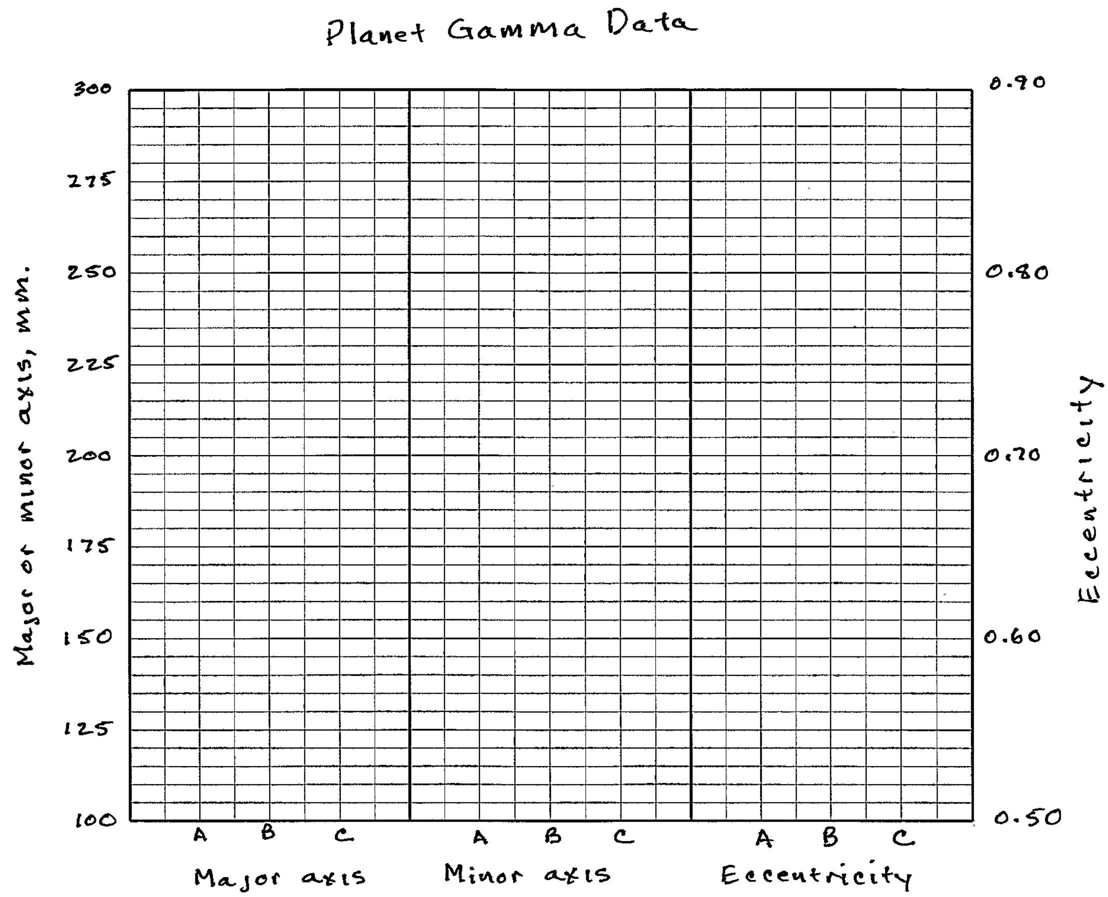
One AU = semi-major axis of earth orbit
One AU = 149,597,893 km
One AU = 92,960,000 miles

If the semi-major axis = a, the semi-minor axis = b and the eccentricity = e, then

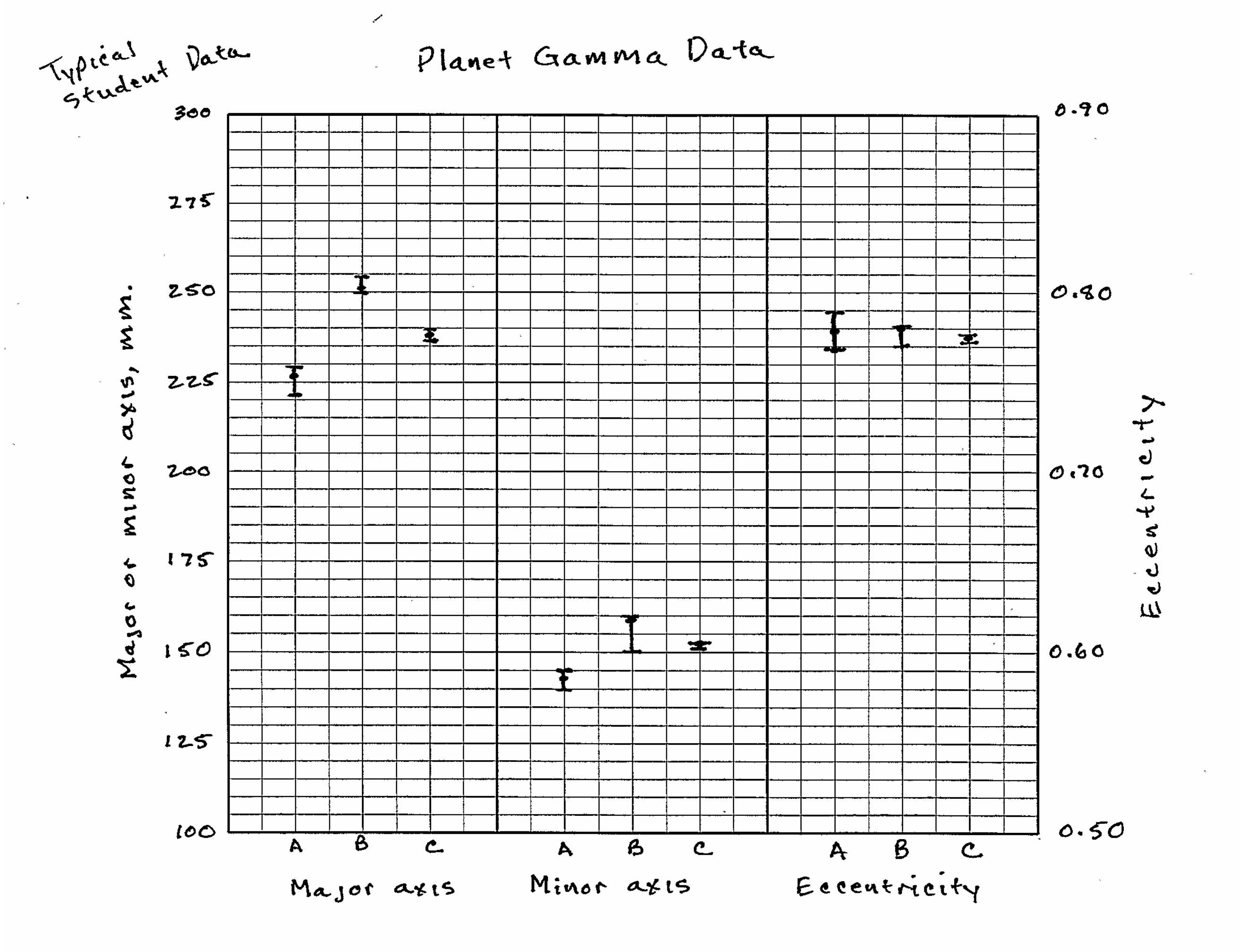
$$b = a\sqrt{1-e^2}$$

Since the eccentricity of Planet Gamma clearly exceeds the eccentricities of all the solar planets, Planet Gamma must be one of the recently discovered extra-solar planets. Challenge your students to search the Internet for orbital data of extra-solar planets to see if any can be found which have eccentricities similar to that of our mythical Planet Gamma. Type “Extra-solar planets” into your favorite search engine and see what comes up. One good place to look is the Extra-solar Planets Catalog at www.obspm.fr/encycl/catalog.html.

◆ Typical Layout of Student Graphs



◆ Typical Layout of Student Graphs



◆ **Typical Student Data**

Record measurements in millimeters rounded to the nearest whole millimeter.

	Group A			Group B			Group C		
	Major Axis	Minor Axis	e	Major Axis	Minor Axis	e	Major Axis	Minor Axis	e
Measurement 1	221	140	0.774	250	160	0.768	238	152	0.769
Measurement 2	228	144	0.775	250	160	0.768	239	152	0.772
Measurement 3	227	145	0.769	251	150	0.802	238	152	0.769
Measurement 4	227	144	0.773	254	160	0.777	238	151	0.773
Measurement 5	227	144	0.773	251	160	0.770	237	151	0.771
Measurement 6	227	140	0.787	250	160	0.768	237	152	0.767
Measurement 7	227	145	0.769	251	150	0.802	238	152	0.769
Measurement 8	227	144	0.773	254	160	0.777	238	151	0.773
Measurement 9	227	144	0.773	251	160	0.770	237	151	0.771
Measurement 10	227	140	0.787	250	160	0.768	237	152	0.767
Average	227	143	0.775	251	158	0.777	238	152	0.770
Minimum	221	140	0.769	250	150	0.768	237	151	0.767
Maximum	228	145	0.787	254	160	0.802	239	152	0.773

◆ Graph Paper Printer 4.21 instructions

Download the program gpaper.exe from the IEEE Teacher In-Service Program website. This program will unzip the file and install Graph Paper Printer 4.21 on your computer. Once Graph Paper Printer 4.21 is installed on your computer, start the program and enter the following setting:

To draw ellipses:

Click on the “Change Variant” icon in the top right hand corner of the screen and choose “Ellipse.” Enter the following parameters. Radial divisions, choose “absent.” Line width, choose “50.” Size, choose any horizontal and vertical size desired. The ellipse provided in the student instructions was printed with a width of 15.30 cm and a height of 24.10 cm.

To print blank graph paper:

Click on the “Change Variant” icon in the top right hand corner of the screen and choose “Custom graduations.” Enter the following parameters and press the “Apply” icon.

	Vertical	Horizontal	
Space between light lines	0.4	0.75	cm
One medium line every	5	2	light line
One heavy line every	8	4	medium line

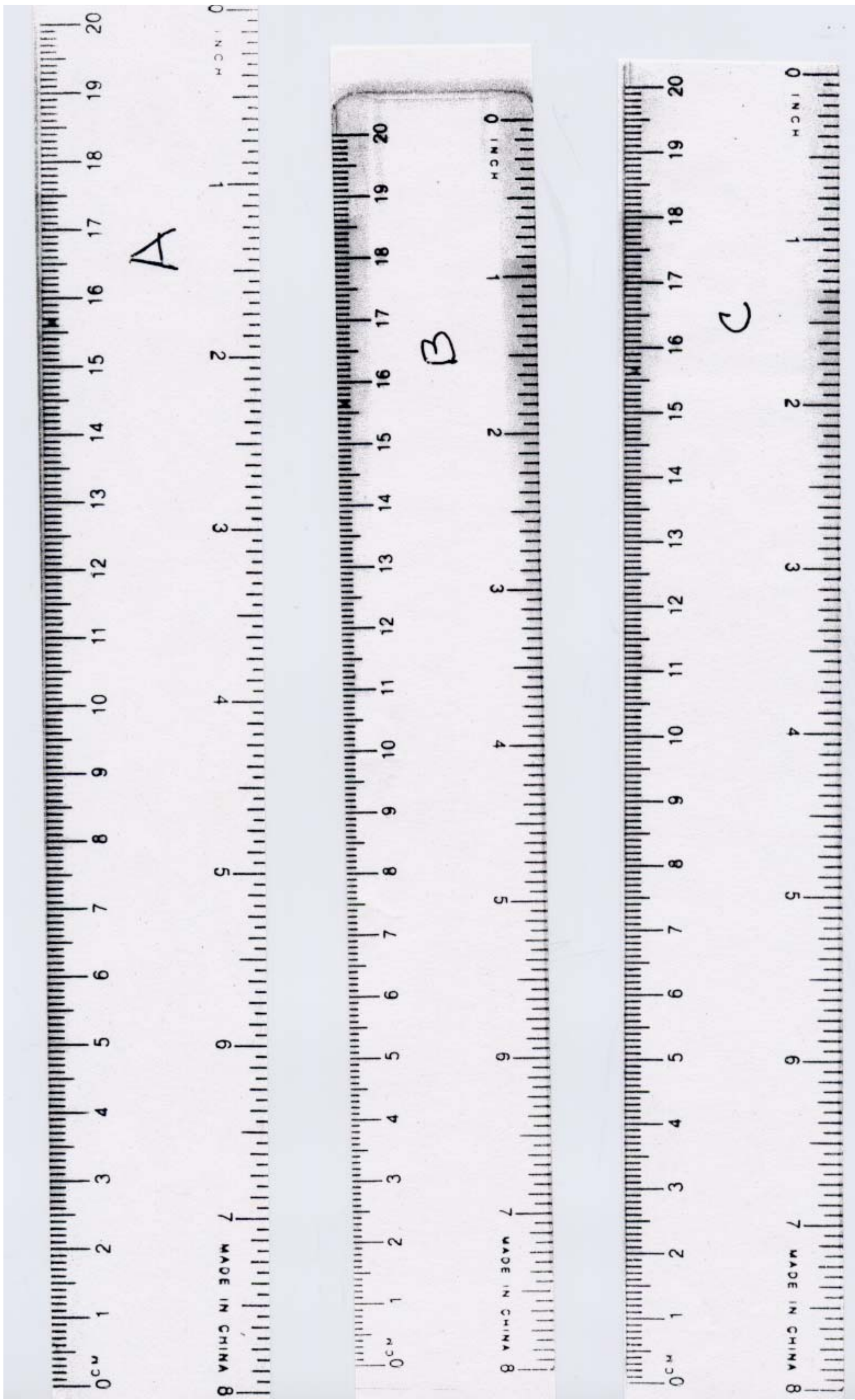
Go to the “lines” section and confirm the following settings:

	Heavy	Medium	Light
Lines 1/100mm	50	20	10

Go to the “size” section near the bottom left hand corner of the screen and enter the width and height shown below and click on the “Apply” icon.

Size (cm)
Width 16.15
Height 18.15

◆ **Paper Rulers**



The Orbit of Planet Gamma

Student Handout:

Planet Gamma Lab Exercise

◆ Prologue

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Raw data from an international telescope has been provided to all three-research laboratories in the form of scale drawings of the orbit of planet gamma. Each research laboratory is charged with measuring the major axis and the minor axis of the orbit and calculating the eccentricity of the orbit.

The director of your laboratory has decided that each researcher assigned to the planet gamma project will work independently and that the results will be averaged. Averaging the data it is hoped will more or less eliminate random error so that the results from your laboratory will be the most accurate in the world. Unfortunately, one disgruntled member of your laboratory has leaked the averaging technique to the other research laboratories, which decide to also have their researchers work independently and average the results to reduce the effects of random error.

◆ Lab instructions

Measure the dimensions of the major axis and the minor axis or the scale drawing of the orbit of planet gamma using the ruler provided. Record the results to the nearest whole millimeter on your data sheet. Calculate the eccentricity according to the formula below and record the calculated eccentricity on the data sheet. Show your calculations including units of measurement.

$$e = \sqrt{(\text{major axis})^2 - (\text{minor axis})^2} \Big/ (\text{major axis})$$

Turn your results into the lab instructor to be tabulated with the results from all researchers.

The lab instructor will assemble the results from all the research laboratories and make them available to all interested parties. Chart the results from all the researchers in "whisker chart" format on the graph paper provided. Analyze the data and answer the following questions on the next page.

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Student Handout:
Planet Gamma Lab Exercise Questions

◆ Does the data from group A show signs of random error? How about the data from groups B and C? Explain your answers.

◆ Which group has the least variation among their measurements of the major axis? Which group has the least variation among their measurements of the minor axis? What does this imply about the skill of the researchers in each group?

◆ Is any group consistently high or low in their measurements of the major axis compared to the other groups? Is any group consistently high or low in their measurements of the minor axis compared to the other groups? What type of error, random or systematic, might account for this? Explain your answer.

◆ Do the calculated values of eccentricity for one group show the same variation when compared to the values for eccentricity calculated by other groups that was noted for the major axis and minor axis data? What might account for this?

◆ Define random error in your own words.

◆ Define systematic error in your own words.

◆ Do systematic errors tend to “average out” over many observations? Do random errors tend to “average out” over many observations?

◆ Which of the three laboratories, A, B or C deserves recognition as the most accurate and advanced researchers in the world? Defend your answer.

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Student Handout:

Data for planet gamma:

Researcher’s Name:

Research group, ___,

Major axis, ____ mm.

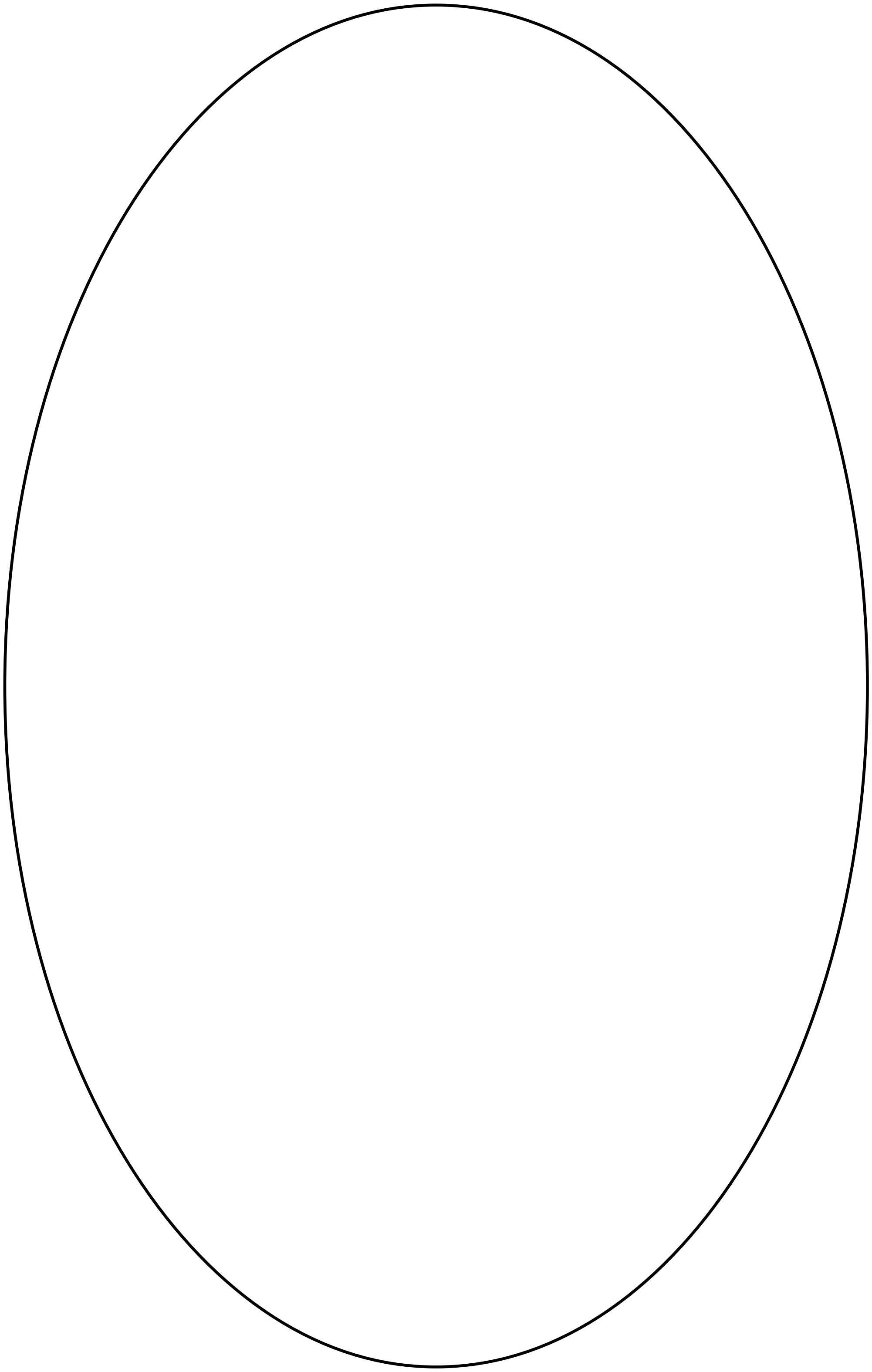
Minor axis, ____ mm.

Eccentricity, ____.

Computations:

The Orbit of Planet Gamma

Student Handout:



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Student Handout:
Graph Paper

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Student Handout: Recording Sheet

Record measurements in millimeters rounded to the nearest whole millimeter.

	Group A			Group B			Group C		
	Major Axis	Minor Axis	e	Major Axis	Minor Axis	e	Major Axis	Minor Axis	e
Measurement 1	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 2	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 3	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 4	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 5	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 6	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 7	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 8	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 9	_____	_____	_____	_____	_____	_____	_____	_____	_____
Measurement 10	_____	_____	_____	_____	_____	_____	_____	_____	_____
Average	_____	_____	_____	_____	_____	_____	_____	_____	_____
Minimum	_____	_____	_____	_____	_____	_____	_____	_____	_____
Maximum	_____	_____	_____	_____	_____	_____	_____	_____	_____