Monte Carlo Simulation

The Monte Carlo method uses a pseudorandom number generator to generate numbers uniformly between zero and one. A simple random number generator uses Lehmer's recursion, i.e.

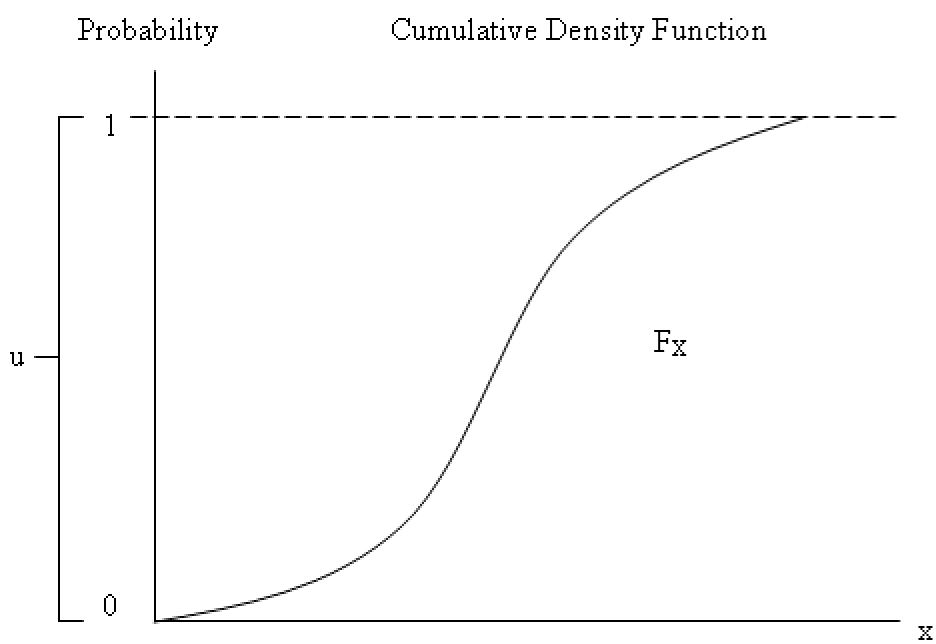
$$z_n = az_{n-1} \bmod m, \; z_o=1$$

Normalizing z_n, one obtains a uniform (0, 1) RN, i.e.

$$u_i=z_i/m$$

The period and quality of the random numbers depends upon the values chosen for "a" and "m". Choosing, a=7^5 and m=2^31-1 results in a sequence of random numbers that passes most of the diehard tests used to measure the quality of a set of random numbers. However, many far superior algorithms are currently in use.

The uniform random number can be manipulated to simulate the characteristics of any probability density function. For power system reliability analysis the exponential and the Weibull distributions are well suited. Simulated exponential and Weibull random variables can be obtained from uniform (0,1) RNs by making use of the fact that the cumulative density function (CDF) is uniform between zero and one as illustrated in the following figure.



For an exponential probability density function (PDF) the CDF is

$F_T(t) = 1 – e^{-\lambda t}$

Making use of the fact that the CDF is a uniform (0, 1) RN, a uniform (0,1) RN can be transformed into an exponential RN as follows

$F_T(t) = 1 – e^{-\lambda t} = u$

$e^{-\lambda t} = 1- u$

$e^{-\lambda t} = u$

$-\lambda t = \ln(u)$

$t = -\ln(u)/\lambda$

A Weibull random variable can be simulated similarly, i.e.

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = u$$

$$1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = u$$

$$1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = u$$

$$e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = 1 - u$$

$$e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = u$$

$$\left(\frac{t}{\alpha}\right)^{\beta} = -\ln(u)$$

$$\frac{t}{\alpha} = \sqrt[\beta]{-\ln(u)}$$

$$t = \alpha \sqrt[\beta]{-\ln(u)}$$

A two state model can be used to sequentially simulate component availability. As the following figure indicates there is one random variable for failures and one random variable for repair times.

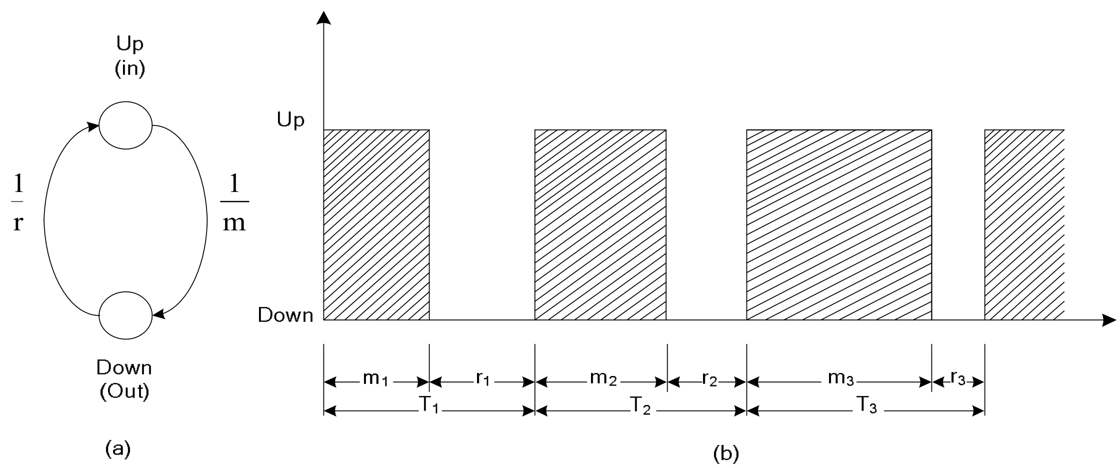
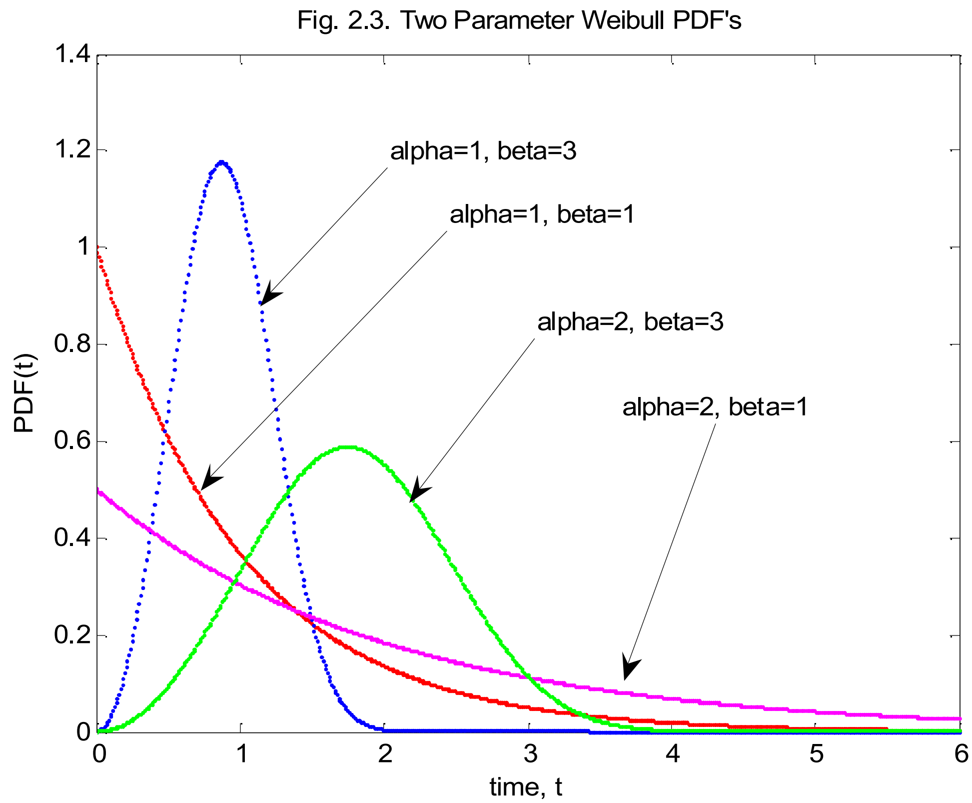


Fig. 2.10. Two-state model in terms of (a) transition diagram and (b) durations

The exponential distribution can be used to simulate random time to failure for both sustained and momentary interruptions. An exponential distribution is characterized as having a constant failure rate, λ. Over the course of a single year, the failure rates of power system components are nearly constant. The use of exponential failure density functions is common practice in power system reliability modeling.

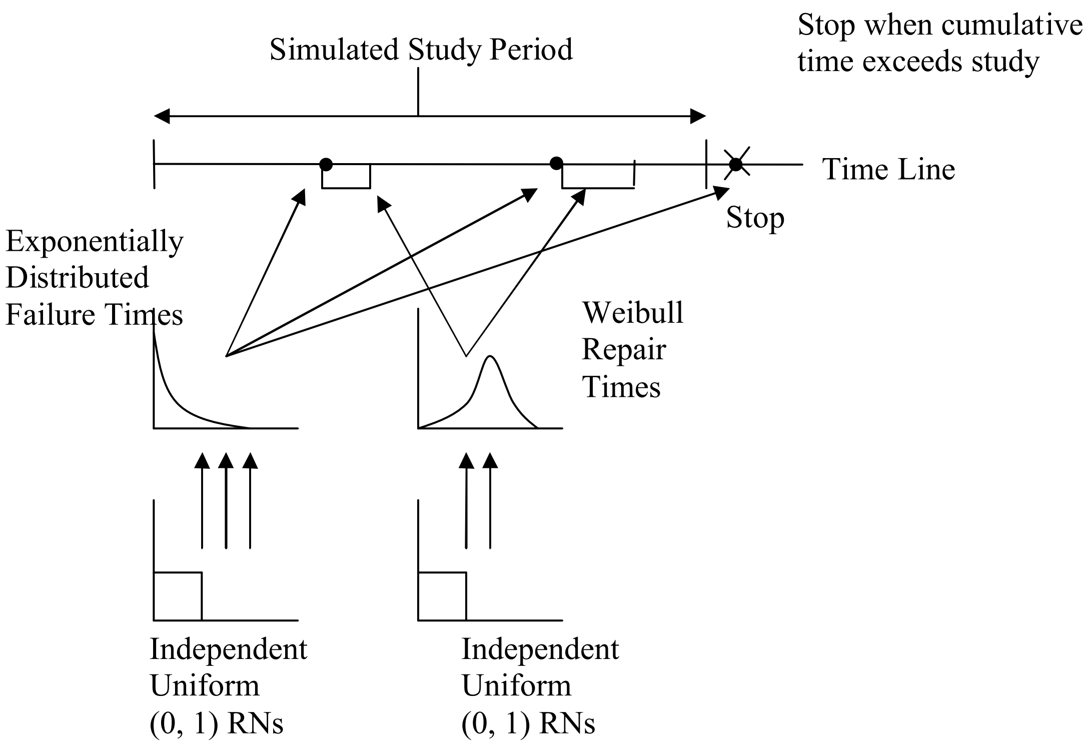
The Weibull distribution is also commonly used to model reliability due to its flexibility. By varying the shape parameter, β, and scale parameter, α, many probability density functions can be approximated as illustrated in the following figure.



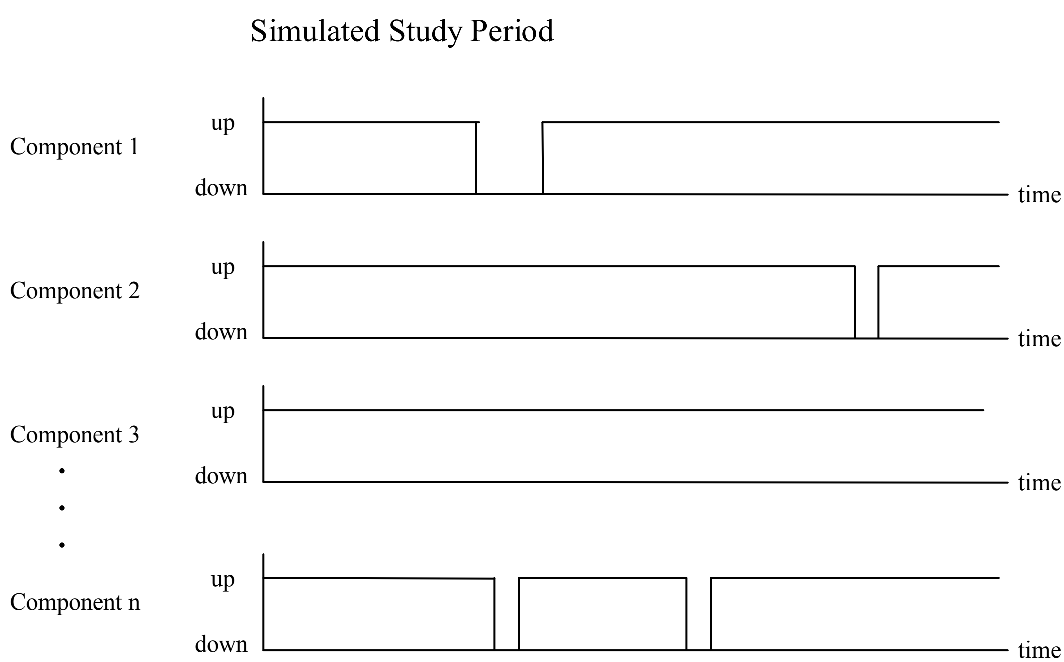
The Weibull distribution will be used to model the time spent in the down state.

The parameters for the failure and repair time distributions can be estimated from the outage history. The failure rate, λ, can be estimated on a component by component basis as the average failure rate over a 5-7 year period, or by analyzing the population of all similar components. The percent of momentary outages can also be determined. Then a uniform (0, 1) RN can be used to determine if a failure is a momentary or sustained outage. The parameters of the Weibull distribution used to model repair times can be determined using least-squares linear regression.

Using an exponential distribution for time to failure, a uniform distribution to determine if the failure is momentary or sustained and a Weibull distribution for the time spent in the down state it is possible to sequentially simulate the study period in question. To simulate the study period failure and repair times are repeatedly generated from the RN sequence until the cumulative time exceeds the study period as shown below.

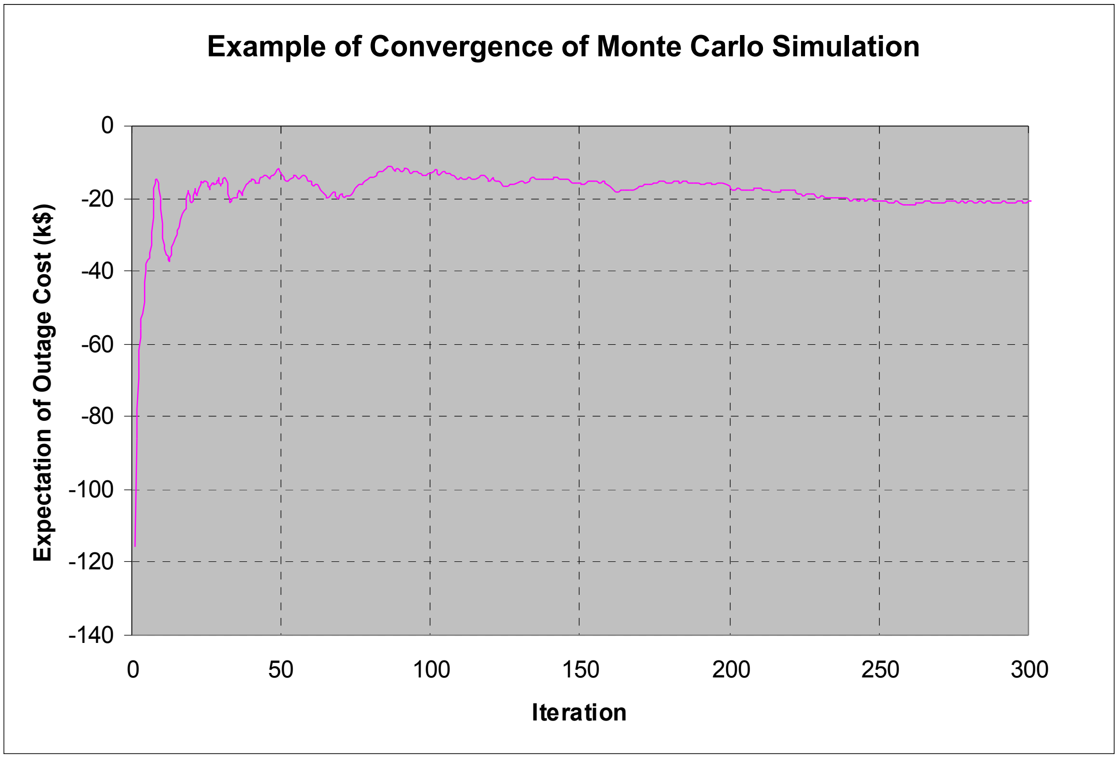


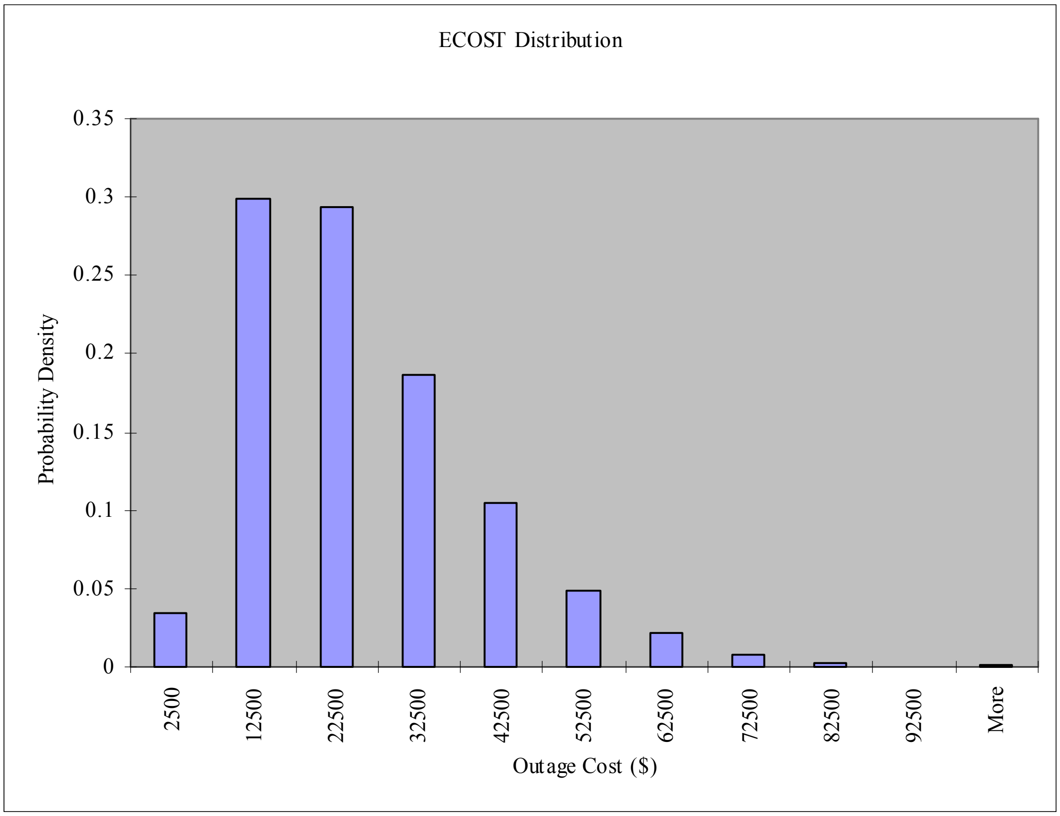
This is done for each component in the system.



At each time step, the status of each component is read and a failure effect analysis is conducted to determine which, if any, customers are out of service. If going from one time step to the next a load point goes from in service to out of service the frequency of failure is updated. While a load point is out of service its unavailability is accumulated.

The study period is repeatedly simulated until indices converge to a steady state value within an acceptable confidence interval.





The Monte Carlo method does provide some very useful benefits. It allows risk to be quantified and results are valid regardless of the probability distributions used. This is true for both transient and steady-state studies. However, this method is very computationally expensive. The study period being analyzed may need to simulated thousands of times to converge to an acceptable confidence interval. Since this method is not enumerative it may overlook rare but important contingencies. Because of the computational burden involved, Monte Carlo simulation is usually reserved for situations where statistical results other than expected values are needed.

Hybrid simulation is a good alternative to sequential Monte Carlo simulation. It produces comparable results in much less time. A Hybrid simulation will repeatedly draw randomly generated reliability parameters for each component from its corresponding distributions and run analytic simulations eliminating the time parameter from the solution process.

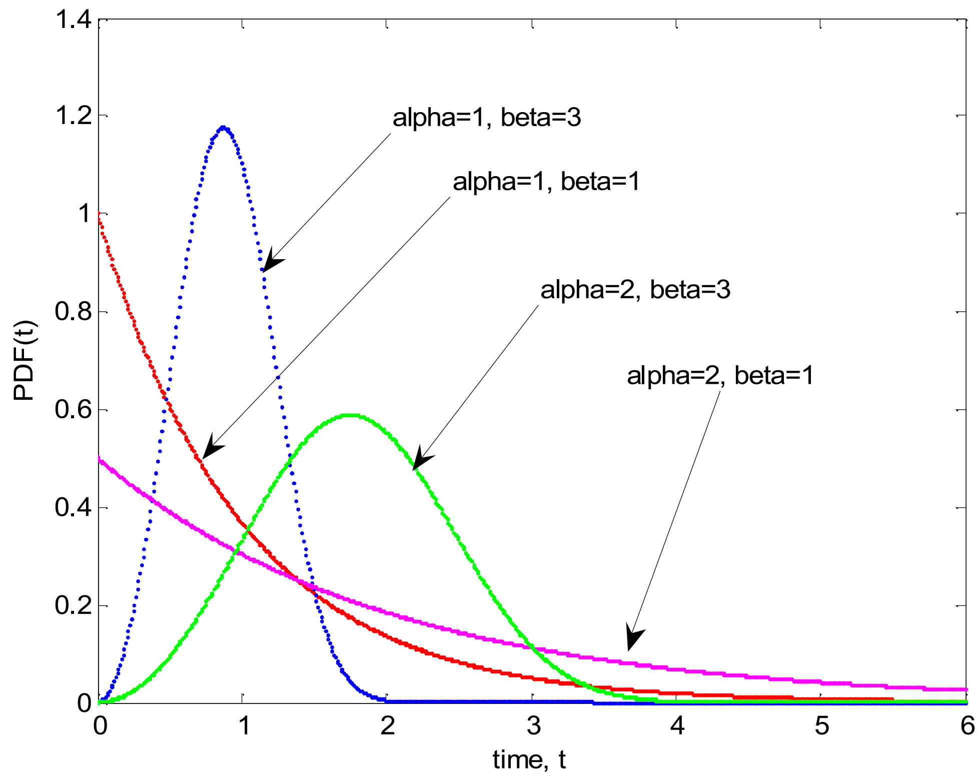
Weibull Analysis

A common method used to predict component failure rates is to model them with a Weibull distribution whose parameters change to represent the component at various stages in its life. The Weibull distribution is also well suited for modeling switching and repair times. A simple and commonly used form of the Weibull PDF, is the two parameter form which is defined by

$$f_T(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \quad t \geq 0, \alpha > 0, \beta > 0, \tag{2.4}$$

where β is the shape parameter and α is the scale parameter. By varying the shape parameter the Weibull distribution can approximate the form of many other probability distributions, for example, for a value of β = 1 the Weibull distribution takes the form of an exponential distribution. Most of the components in a power system have a PDF of failure that can be described very well using a Weibull distribution. Some Weibull distributions with various shape and scale parameters are shown in Fig. 2.3.

Fig. 2.3. Two Parameter Weibull PDF's



The reliability, unreliability and hazard function of the Weibull model are

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \quad t \geq 0, \tag{2.5}$$

$$Q(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \quad t \geq 0 \tag{2.6}$$

and

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}, \quad t \geq 0, \tag{2.7}$$

respectively.

Weibull Parameter Estimation

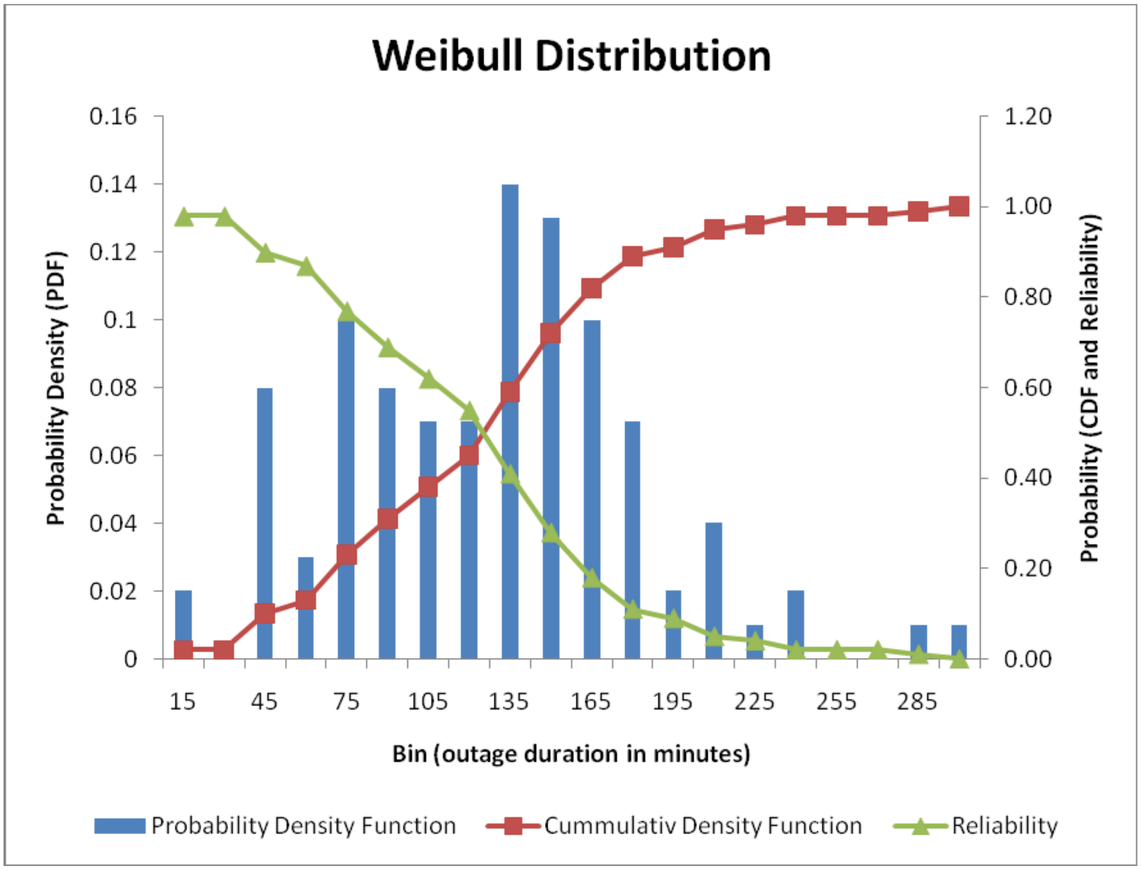
To estimate the parameters of a Weibull distribution construct a histogram of the failure density function from historical outage data.

Integrate the failure density function to obtain the cumulative density function or unreliability.

$$F(t) = \int_{-\infty}^{t} f(x)dx$$

Subtract the unreliability from one to obtain the reliability.

$$R(t) = 1 - F(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$



Take the logarithm of the negative logarithm of the reliability and plot versus the logarithm of the histogram bin midpoints to transform the reliability function into the equation of a line

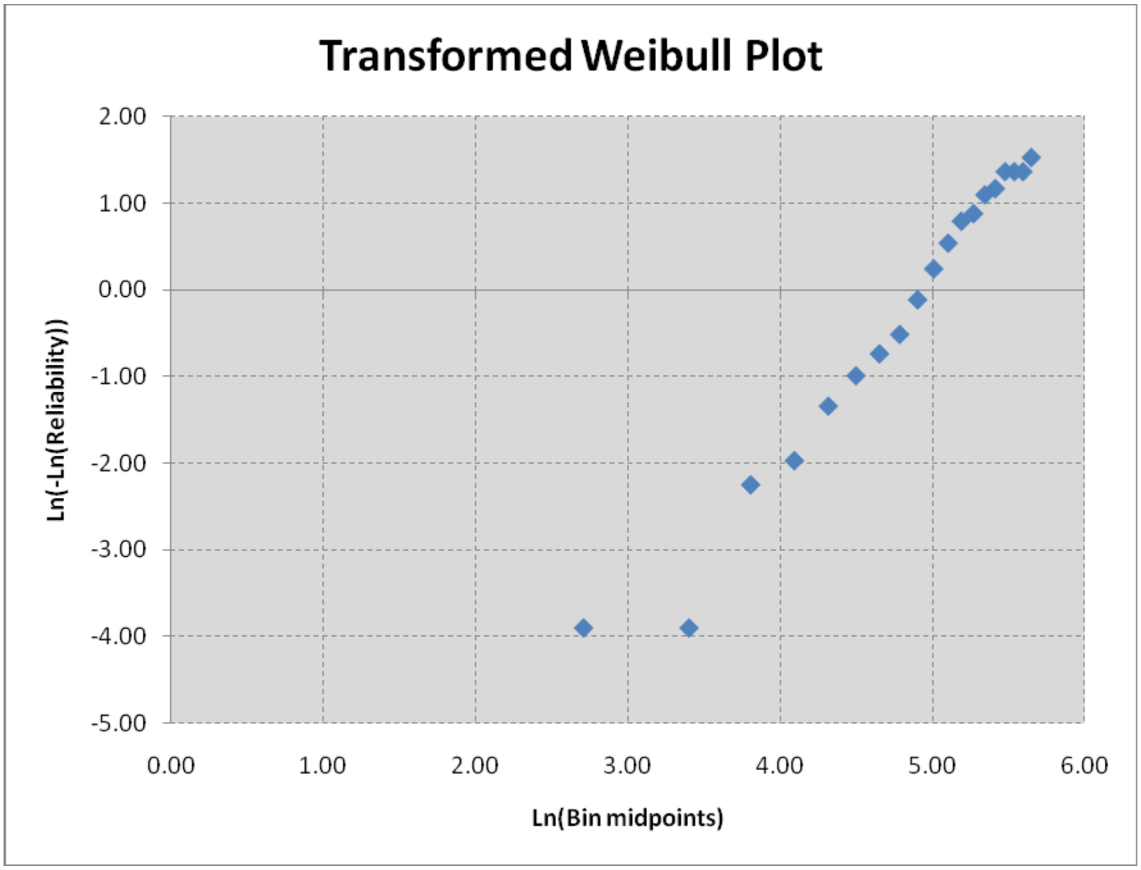
$$-\ln[R(t)] = -\ln\left[e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\right]$$

$$-\ln[R(t)] = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$\ln\{-\ln[R(t)]\} = \ln\left(\frac{t}{\alpha}\right)^{\beta}$$

$$\ln\{-\ln[R(t)]\} = \beta[\ln(t)] - \beta\ln(\alpha)$$

$$\mathbf{y = mx + b}$$



Use least-squares linear regression to estimate the parameters of the Weibull model, find the parameters for the line that minimize the sum of square errors from each point to that line.

$$\varepsilon^2 = \sum (mx + b - y)^2$$

$$\varepsilon^2 = \sum (m^2x^2 + 2mxb - 2mxy + b^2 - 2by + y^2)$$

The minimum square error occurs when

$$\frac{\partial \varepsilon^2}{\partial m} = \sum (2mx^2 + 2xb - 2xy) = 0$$

$$\frac{\partial \varepsilon^2}{\partial b} = \sum (2mx + 2b - 2y) = 0$$

or in matrix form

$$2\begin{bmatrix} \sum x^2 & \sum x \\ \sum x & \sum 1 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} = 2\begin{bmatrix} \sum xy \\ \sum y \end{bmatrix}$$

Solving via Cramer's rule yields

$$m = \frac{\Delta_1}{\Delta} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

and

$$b = \frac{\Delta_2}{\Delta} = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - \left(\sum x\right)^2}$$

or

$$b = \bar{y} - m\bar{x}$$

The slope of the regression line is the estimate of the shape parameter, i.e.

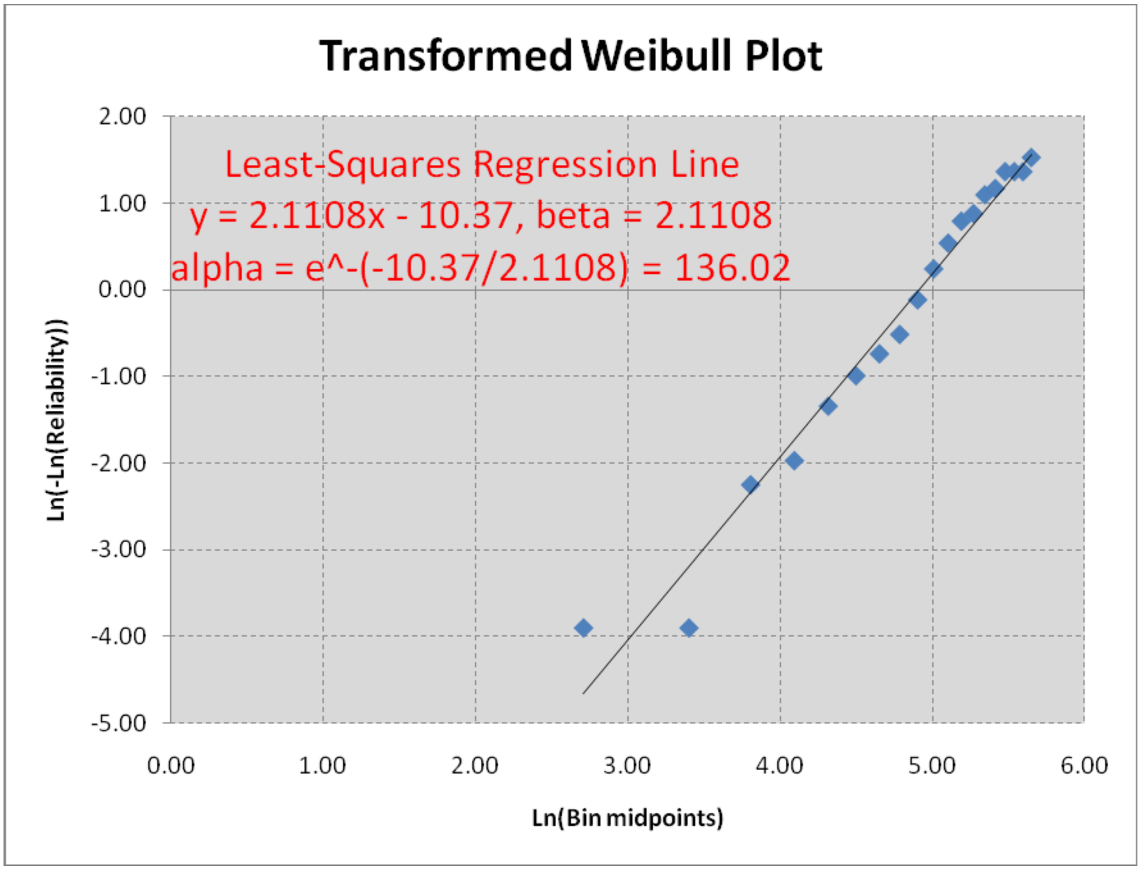
$$\beta = m$$

The scale parameter can be computed from the intercept and the slope via the expression

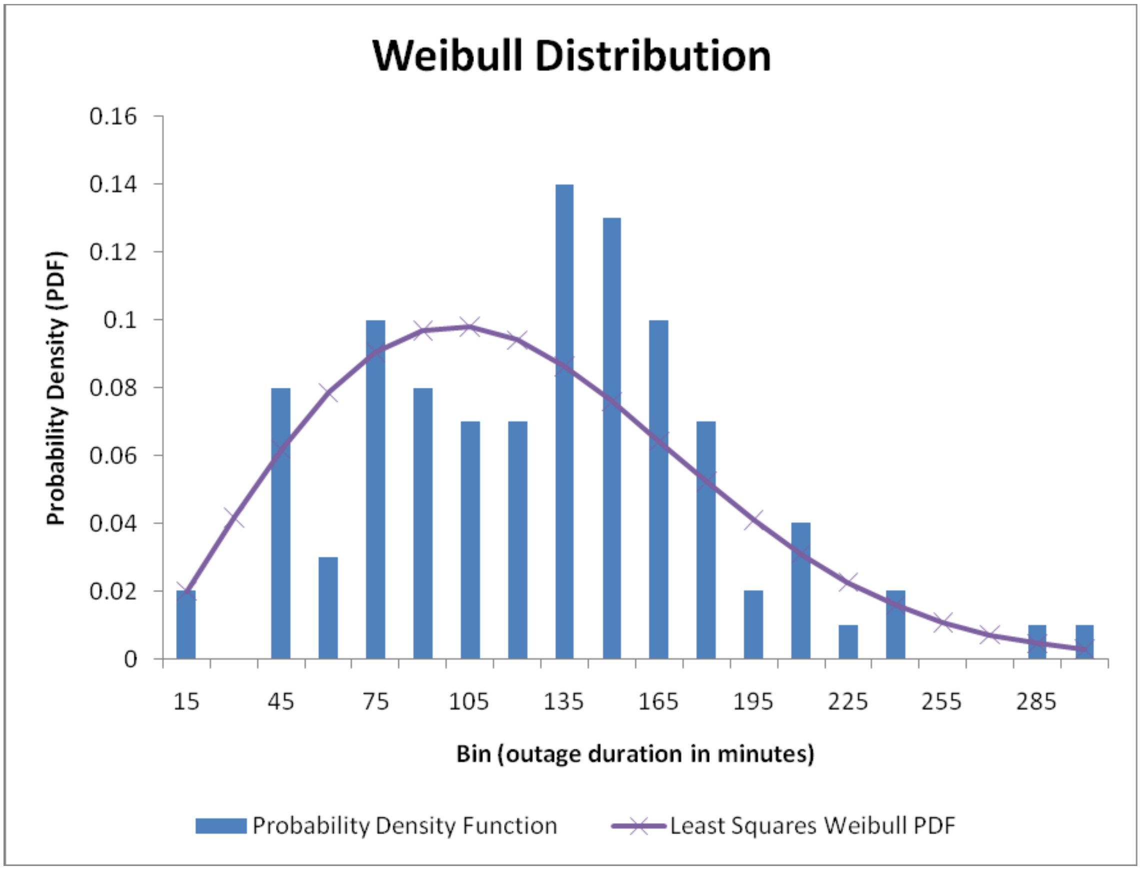
$$b = -\beta \ln(\alpha)$$

which yields

$$\alpha = e^{-\frac{b}{\beta}} = e^{-\frac{b}{m}}$$



The Weibull distribution of best fit is shown in the following figure.


Weibull Distribution
Probability Density (PDF)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
15
45
75
105
135
165
195
225
255
285
Bin (outage duration in minutes)
Probability Density Function
Least Squares Weibull PDF