Introduction to Antenna Array-to-Antenna Array Communication

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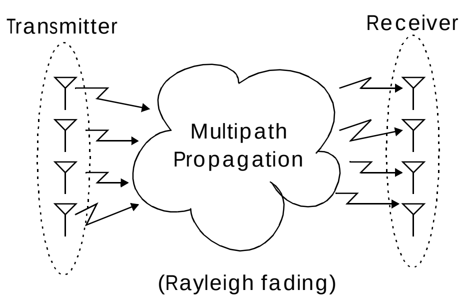
Contents

Dual Antenna-Array Communication Link

An antenna-array to antenna-array link in a multipath fading environment.



Formulation

The discrete-time I/O relationship:

$$\boldsymbol{r}_\tau^{(l)} = \sqrt{E_s}\sum_{k=1}^{n} h_\tau^{(l \leftarrow k)} \boldsymbol{s}_\tau^{(k)} + \boldsymbol{v}_\tau^{(l)}$$

(k), (l): antenna index

τ: discrete time index

or simply

$$\boldsymbol{r}_\tau = \boldsymbol{H}_\tau \sqrt{E_s}\boldsymbol{s}_\tau + \boldsymbol{v}_\tau$$

received signal — transmitted signal — AWGN

$$\begin{bmatrix} r^1 \\ \ldots \\ r^m \end{bmatrix}_\tau = H_\tau \sqrt{E_s} \begin{bmatrix} x^1 \\ \ldots \\ x^n \end{bmatrix}_\tau + v_\tau .$$

Formulation

Simplification assumptions:

- $h_\tau^{(l \leftarrow k)}$ are complex Gaussian (Rayleigh fading)
- correlation among $h_\tau^{(l \leftarrow k)}$ in space: assuming non-existent
- narrowband (no frequency selective fading), no ISI
- Receiver can measure the channel reasonably well

Capacity of Dual Antenna Array Systems

Dual antenna-array systems over multipath fading channels have enormous channel capacity.

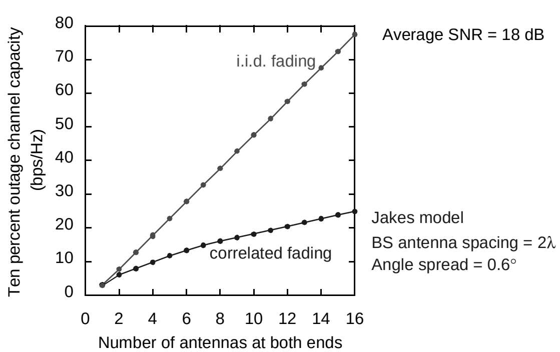
If the channel, H, stays unchanged during a packet transmission, the channel capacity subject to an overall transmit power constraint during this period is

$$C = \max_{\Sigma_s} \log\{\det[I + H\Sigma_s H^\dagger]\} \text{ bits per channel use}$$

where $tr(\Sigma_s)$ is the overall transmit power normalized to noise variance.

Capacity of Dual Antenna Array Systems

Let $m = n$. As $n \to \infty$, if the receiver knows the channel, $\lim_{n \to \infty} \frac{\text{Capacity}}{n} \to \text{nonzero constant}$.

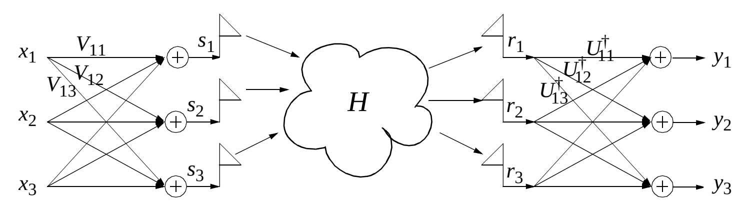


When Are Space-Time Codes NOT Useful?

When the transmitter knows the channel H and performs channel diagonalization.

Singular value decomposition of H: $H = UDV^\dagger$.

- The transmitter can spatially pre-code the to-be-transmitted signal by V: $\boldsymbol{s} = V\boldsymbol{x}$.
- The receiver can spatially equalize the received signal by $U^\dagger$: $\boldsymbol{y} = U^\dagger \boldsymbol{r}$.

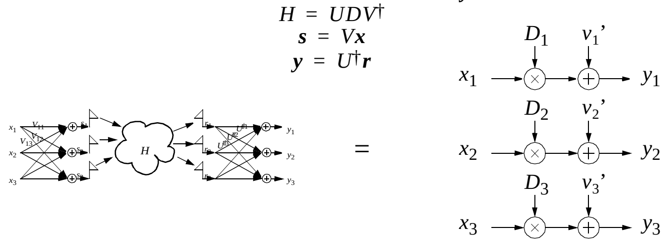


Channel Diagonalization

Channel diagonalization transforms an n-Tx, m-Rx ($m \geq n$) link into n 1-Tx, 1-Rx links whose channel gains are the singular values of H.

$$\mathbf{r} = H\mathbf{s} + \mathbf{v} \longrightarrow \mathbf{y} = D\mathbf{x} + \mathbf{v}'$$

$$H = UDV^{\dagger}$$
$$\mathbf{s} = V\mathbf{x}$$
$$\mathbf{y} = U^{\dagger}\mathbf{r}$$



Channel Diagonalization

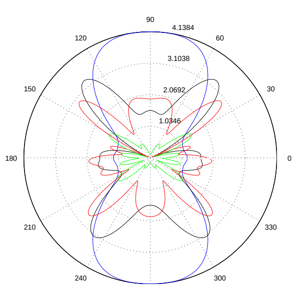
The transmitter knows the channel by

- making use of channel reciprocity in a time-duplex system
- using a feedback link from the receiver

Highest throughput over n 1-Tx, 1-Rx links is achieved using the water-filling solution.

(Impossible to) Visualize Channel Eigenmodes

A side effect of spatial precoding: each spatial eigenmode induces an antenna beam pattern.



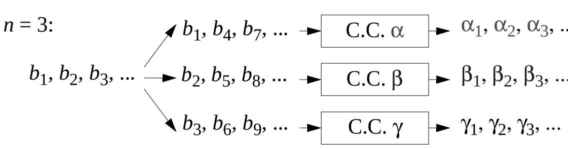
eigenmode beam pattern

- Broadside transmit antenna set has 7 antenna elements
- Angle spread = 15°
- Receiver at 3 o'clock direction

Diagonally Layered Space-Time Codes: Encoding

Under the DLST architecture[1], n constituent codes (CCs) are used.

E.g., $n = 3$:



	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Antenna 1	α_1	β_1	γ_1	α_4	β_4
Antenna 2	0	α_2	β_2	γ_2	α_5
Antenna 3	0	0	α_3	β_3	γ_3

1. Proposed by Foschini.

Expanded-Diagonal DLST

This space-time codeword is represented by a matrix

$$X = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \alpha_4 & \beta_4 & \\ 0 & \alpha_2 & \beta_2 & \gamma_2 & \alpha_5 & \dots \\ 0 & 0 & \alpha_3 & \beta_3 & \gamma_3 & \end{bmatrix}$$

It is also possible to expand the width of a diagonal to multiple symbols.

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
Antenna 1	α_1	α_2	β_1	β_2	γ_1	γ_2
Antenna 2	0	0	α_3	α_4	β_3	β_4
Antenna 3	0	0	0	0	α_5	α_6

Hard Diagonal Decision-Feedback Decoding

Recall that a DLST codeword with $n = 3$:

$$X = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \alpha_4 & \beta_4 & \\ 0 & \alpha_2 & \beta_2 & \gamma_2 & \alpha_5 & \dots \\ 0 & 0 & \alpha_3 & \beta_3 & \gamma_3 & \end{bmatrix}.$$

- The signals transmitted on the leftmost diagonal, α_1, α_2, α_3, are first decoded using the received signals at time 1, 2, and 3.
- Note that intuitively α_1 has the best estimate due to the least number of interfering signals, then α_2, and then α_3.
- The decoded results are used to remove the contribution due to α_1, α_2, and α_3 in the received signal.
- The signals transmitted on the next leftmost diagonal, β_1, β_2, β_3, are then decoded, and so on.

Error Analysis

Consider the probability of decoding a transmitted diagonal (c_1, c_2, ..., c_n) into another possible diagonal (e_1, e_2, ..., e_n).

Define:

- TMEL = weighted number of nonzero $c_k - e_k$;
- TMPD = weighted product of nonzero $|c_k - e_k|^2$;
- TMED = weighted sum of $|c_k - e_k|^2$.

It can be shown that:

- when SNR is high,

$$Prob(c \to e) \approx (TMPD)^{-1}\left(\frac{E_s}{4N_0}\right)^{-TMEL}.$$

- when SNR is low,

$$Prob(c \to e) \approx \frac{1}{1 + (TMED)\frac{E_s}{4N_0}}.$$

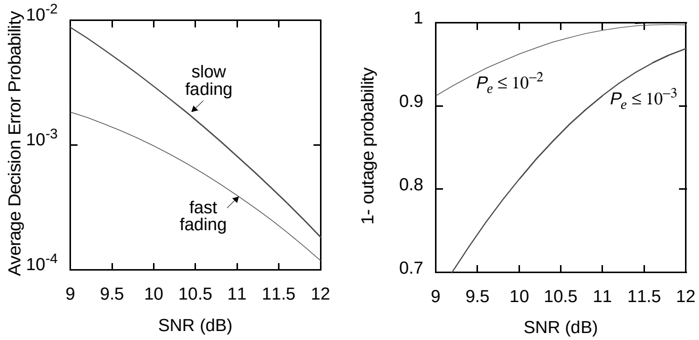
Design Example

Design specification:

- Constituent code must be a rate 1/2, constraint length 10 feedforward convolutional code;
- QPSK constellation;
- Throughput: 8 bits/s/Hz.
- $n = m = 8$.

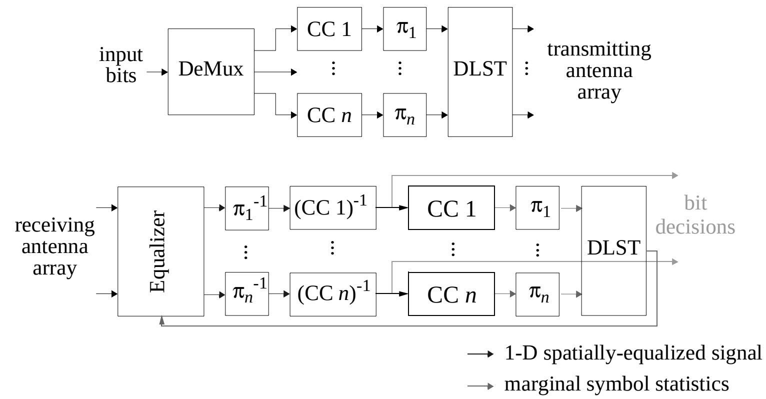
Searching through the space of constituent codes, it is found that the maximum (TMEL, TMPD) is (11, 2^{-19}).

Performance



Iterative Decoding of DLST Codes

- To facilitate iterative decoding, independent interleavers are added at the output of the constituent coders.



Iterative Decoding of DLST Codes (cont.)

- Many options are available for the equalizer and the constituent code decoder
- Linear minimum mean square error equalizer:
  - The equalizer uses the marginal symbol p.m.f.s as the *a priori* symbol probabilities.
  - The kth branch output at time τ is $\tilde{x}_\tau^k = \boldsymbol{w}_k^{\dagger}(\boldsymbol{r}_\tau - \sum_{l \neq k} \boldsymbol{h}_l \hat{x}_\tau^l)$
  - To minimize the mean square error, the soft decision feedback $\hat{x}_\tau^l$ is chosen as the expected value of x_τ^l
  - The equalizer complexity is $O(n^2)$.
- Constituent code decoder:
  - It generates the *a posteriori* symbol expected values and variances.

Error Probability Analysis

It is difficult to analyze the performance of iterative decoding. Instead, we assume maximal likelihood (ML) decoding. Furthermore, we assume slow fading, i.e. $H_\tau = H$.

Consider two DLST codewords $\boldsymbol{C}$ and $\boldsymbol{E}$ differing only at symbols that belong to **interleaved** constituent code 1. For example, $n = 3$,

$$\boldsymbol{B} = \boldsymbol{C} - \boldsymbol{E} = \begin{bmatrix} b_1 & 0 & 0 & b_4 & 0 & 0 & \\ 0 & b_2 & 0 & 0 & b_5 & 0 & \dots \\ 0 & 0 & b_3 & 0 & 0 & b_6 & \end{bmatrix}.$$

Define

- CEL = number of nonzero rows of $\boldsymbol{C} - \boldsymbol{E}$.
- CPD = the product of the norm squared of nonzero rows.

Error Probability Analysis

It can be shown that, at high SNR, the error probability can be approximated as

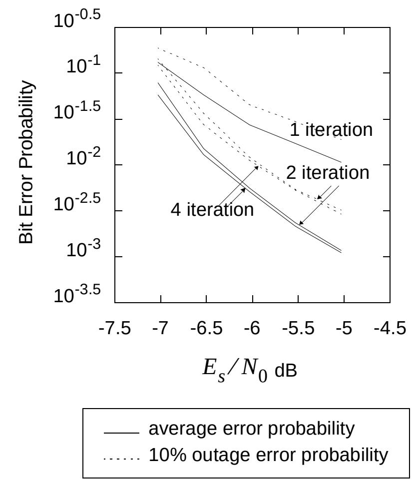
$$Prob(\boldsymbol{C} \rightarrow \boldsymbol{E}) \approx \left(\frac{E_s}{4N_0}\right)^{-m \cdot CEL} (CPD)^{-m} .$$

Design criterion:

Choose the interleaved CC such that

- the minimum of $(E_s/4N_0)^{CEL(\boldsymbol{c}, \boldsymbol{e})} CPD(\boldsymbol{c}, \boldsymbol{e})$ over all $(\boldsymbol{c}, \boldsymbol{e})$ pairs is maximized.
- If the operating SNR is not known:
  - Maximize the minimum $CEL(\boldsymbol{c}, \boldsymbol{e})$ first.
  - Over those $(\boldsymbol{c}, \boldsymbol{e})$ pairs whose CEL equal the minimum CEL, maximizes the minimum of $CPD(\boldsymbol{c}, \boldsymbol{e})$.

Simulation Result



Space-Time Codes with Transmit Diversity Only

Space-time code with dual transmit diversity[1]

Two symbols s_0 and s_1 are transmitted at time 0, and $-s_1{}^*$ and $s_0{}^*$ are transmitted at time 1:

$$r_0 = \begin{bmatrix} h_0 & h_1 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + v_0, r_1 = \begin{bmatrix} h_0 & h_1 \end{bmatrix} \begin{bmatrix} -s_1{}^* \\ s_0{}^* \end{bmatrix} + v_1 \Rightarrow$$

$$\begin{bmatrix} r_0 \\ r_1{}^* \end{bmatrix} = \begin{bmatrix} h_0 & h_1 \\ h_1{}^* & -h_0{}^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} v_0 \\ v_1{}^* \end{bmatrix} .$$

1. Proposed by Alamouti

Space-Time Codes with Transmit Diversity Only

Note that the columns of the matrix H are orthogonal to each other. To detect the transmitted symbol, the receiver simply left-multiplies $\begin{bmatrix} r_0 \\ r_1{}^* \end{bmatrix}$ by $H^{\dagger}$:

$$H^{\dagger} \begin{bmatrix} r_0 \\ r_1{}^* \end{bmatrix} = \begin{bmatrix} |h_0|^2 + |h_1|^2 & 0 \\ 0 & |h_0|^2 + |h_1|^2 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \text{noise} .$$

This effectively transforms a 2-Tx, 1-Rx link with channel gain h_0, h_1 into a 1-Tx, 1-Rx link with channel gain $\sqrt{|h_0|^2 + |h_1|^2}$.

More Than Two Transmit Antennas

Without some redundant transmission, it is not possible to convert an n–Tx, 1–Rx link with gain g_i, $i = 1, 2, \ldots, n$ to a 1–Tx, 1–Rx link with gain $\sqrt{\sum |g_i|^2 / n}$.

For example, with $n = 3$, a rate-$1/2$ code

$$G_c^3 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix}$$

converts the channel to a concatenation of a rate -$1/2$ repetition code with a 1-Tx, 1-Rx channel with a gain $\sqrt{|h_0|^2 + |h_1|^2 + |h_2|^2}$.

More Than Two Transmit Antennas

Define the following vectors:

$\boldsymbol{r} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5^* & r_6^* & r_7^* & r_8^* \end{bmatrix}'$, $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \ldots & x_4 \end{bmatrix}'$, and

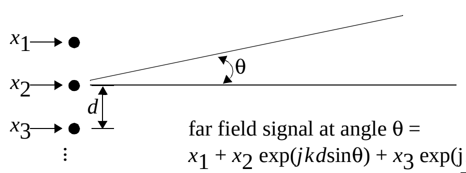
$\boldsymbol{v} = \begin{bmatrix} v_1 & v_2 & v_3 & v_4^* & v_5^* & v_6^* & v_7^* & v_8^* \end{bmatrix}'$.

It can be easily verified that $\boldsymbol{r} = H\boldsymbol{x} + \boldsymbol{v}$, where

$$H = \begin{bmatrix} h_0 & h_1 & h_2 & 0 \\ h_1 & -h_0 & 0 & -h_2 \\ h_2 & 0 & -h_0 & h_1 \\ 0 & h_2 & -h_1 & -h_0 \\ h_0^* & h_1^* & h_2^* & 0 \\ h_1^* & -h_0^* & 0 & -h_2^* \\ h_2^* & 0 & -h_0^* & h_1^* \\ 0 & h_2^* & -h_1^* & -h_0^* \end{bmatrix}.$$

Beam Pattern Perspective

Unlike in traditional beamforming, with the above mentioned space-time block code, the beam pattern is a function of the transmitted data.



far field signal at angle θ =
$x_1 + x_2 \exp(jkd\sin\theta) + x_3 \exp(j2kd\sin\theta) + \ldots =$
$(x_1, x_2, \ldots)\,(1, \exp(jkd\sin\theta), \ldots)^T$

It can be shown that, with n-transmit diversity, the average received power at angle θ is a constant and is not a function of θ.

Summary

- The primary advantage of antenna array-to-antenna array communication is the enormous capacity in a fading environment.
- When the transmitter knows the channel, it can perform channel diagonalization. Traditional one-dimensional codes can be applied; space-time codes are not necessary.
- DLST codes are introduced. DLST codes have good performance and can be decoded using (relatively) low-complexity decoder. The code design criteria and simulation results are also shown.
- If the link has only transmit diversity, there are also technique that can transforms an n-Tx, 1-Rx link into a concatenation of a repetition code and a 1-Tx, 1-Rx link.