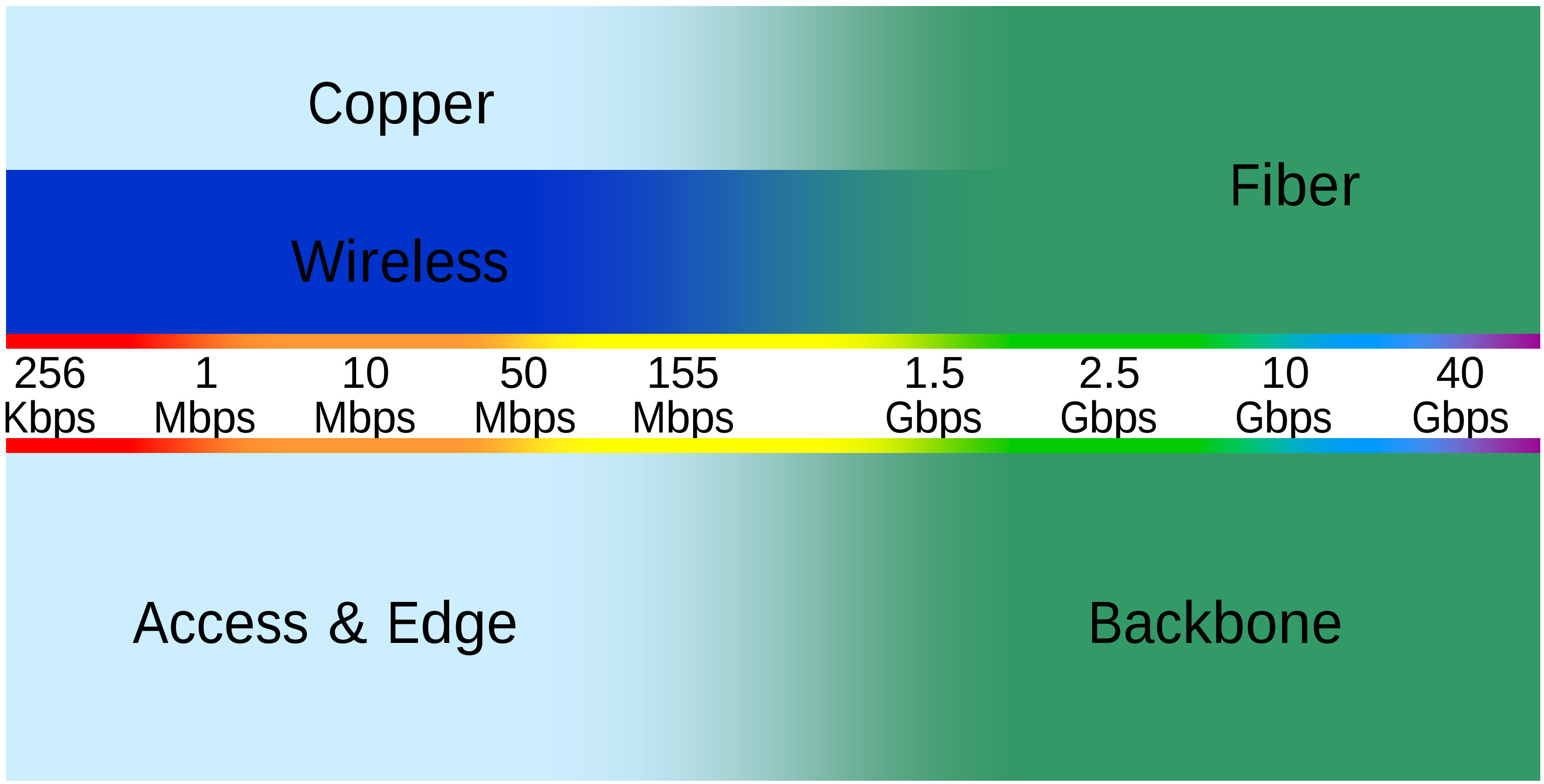
Improving Bandwidth Utilization with Turbo Product Codes

• 2365 NE Hopkins Court • Pullman WA 99163-5601 • tel: 509.334.1000 • fax: 509.334.9000 • www.aha.com •

Agenda

- Communications Systems
- Forward Error Correction
- Turbo Product Codes
- Performance of TPCs
- TPC Products
- Summary and Conclusions
- Hardware demonstration

Communications Medium



Typical System Design Goals

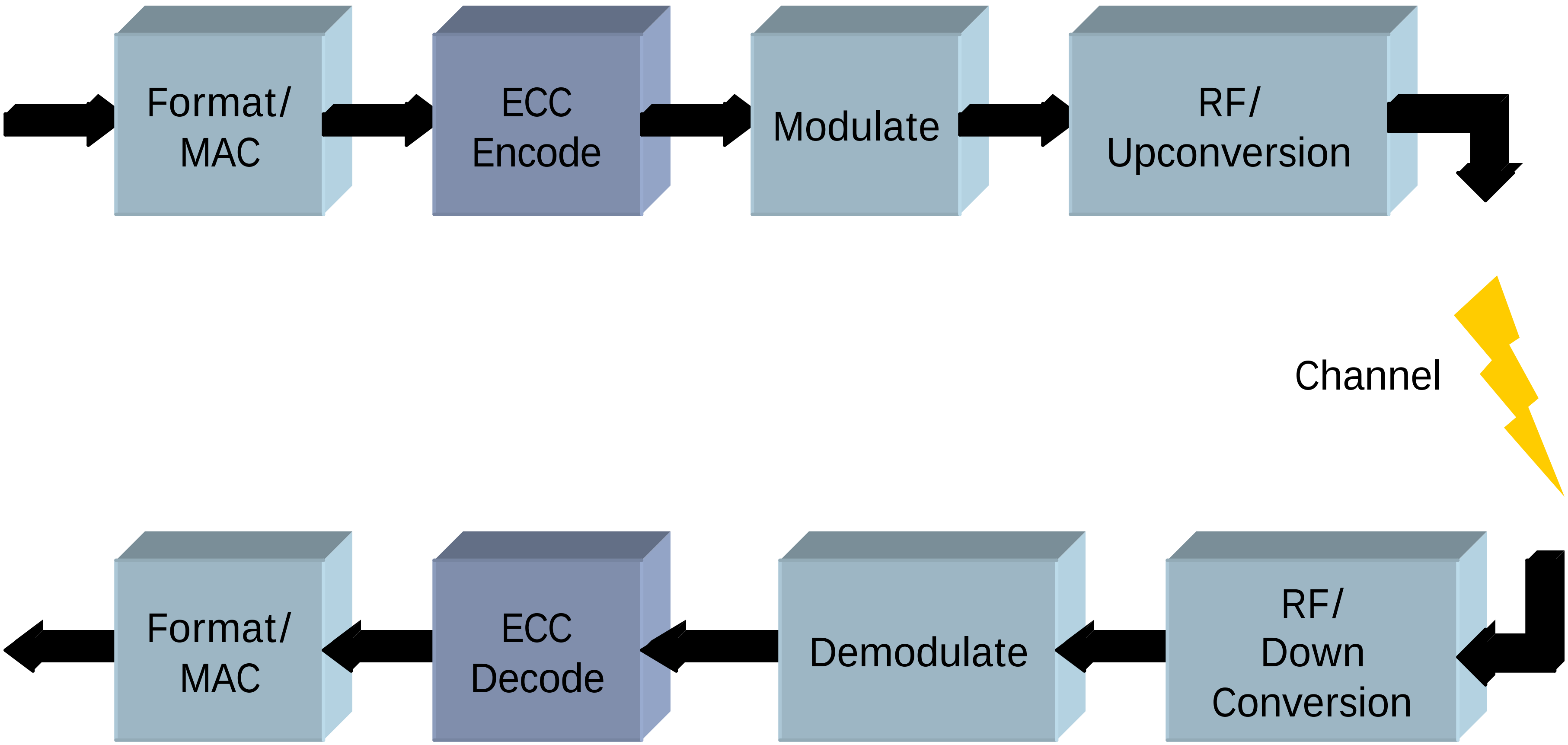
- Maximize data rate
- Maximize data reliability
- Minimize required transmission energy
- Minimize required bandwidth
- Minimize system complexity (cost)

Forward Error Correction can be instrumental in helping meet these goals!

Forward Error Correction

- The addition of redundancy to a message through encoding prior to transmission to enable the receiver (decoder) to correct some number errors.

Forward Error Correction Coding



Why Use Error Correction?

- *Error correction allows the designer to achieve much more efficient bandwidth utilization*
- *Designers can choose between levels of improved data reliability, reduced systems cost or increased range*

More Powerful Error Correction

Advances in Forward Error Correction can offer 3 dB (or more) of added performance over currently used FEC methods.

3 dB of coding gain can:

- reduce the required bandwidth by 50% or
- increase data throughput by a factor of 2 or
- increase range by 40% or
- reduce antenna size by 30% or
- reduce transmitter power by a factor of 2

Bottom line…reduced cost or increased performance (or both)

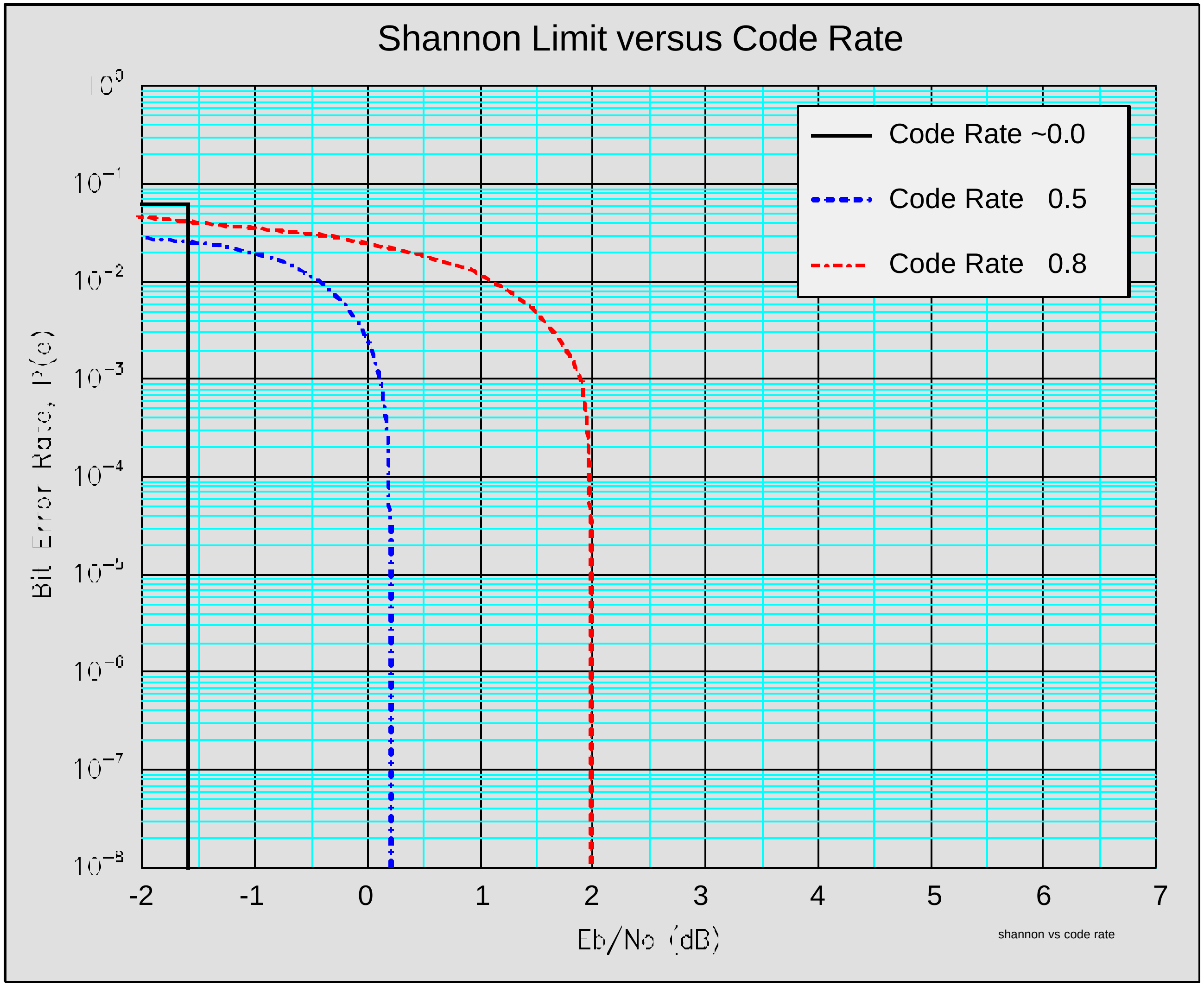
Error Correction Codes

- Block Codes
  - Hamming
  - BCH
  - Reed-Solomon
- Convolutional Codes (Viterbi)
- Trellis Codes
- Concatenated
  - Viterbi/Reed-Solomon
- Turbo Convolutional Codes (TCC)
- Turbo Product Codes (TPC)

Definitions

- Code Rate
  - Ratio of data bits / total bits transmitted
- Shannon Channel Capacity
  - Theoretical bound for channel capacity for a given modulation and code rate

Shannon Capacity vs. Code Rate



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Error Correction Capabilities

- The minimum Hamming distance or “Dmin” of an error correction code characterizes the power of the code
- The error correcting capability, “t”, of a code is defined as the number of guaranteed correctable errors per codeword:

$$t = \left\lfloor \frac{Dmin - 1}{2} \right\rfloor$$

Turbo Product Codes

Turbo Product Code History

- Product codes first described by Elias (1954)
- Iterative decoding of product codes described by Tanner (1981), Lin & Costello (1983) and others
- Hardware implementation had to wait for an efficient SISO decoder algorithm
- AHA's SISO algorithm has made commercially viable TPCs possible
- AHA4501 introduced in November 1998

Turbo Product Codes

- Turbo Product Codes (TPCs) are based on block codes, not convolutional codes
- AHA's TPCs are built using two or three dimensional arrays of extended Hamming codes and parity codes
  - Encoding is done in a single pass
  - Decoding is done iteratively
  - Minimum distance of a 2D product code is square of constituent code; for a 3D code, cubed

Hamming Codes

- Linear and systematic
- Specified as (n,k)
  - n is the encoded number of bits
  - k is the number of information bits
- Example: (7,4)
  - d1 d2 d3 d4 e1 e2 e3
- Easy to both encode and decode

Hamming Codes

- Hamming codes
  - Dmin = 3
- Extended Hamming codes add a parity bit
  - Dmin = 4
- TPCs utilize extended Hamming codes and parity codes to construct Product Codes
  - Dmin >> than constituent codes
    - ✓ 16 for two dimensional (2D) codes
    - ✓ 64 for three dimensional (3D) codes

TPC Constituent Codes

- Constituent Codes can be mixed and matched to achieve desired code characteristics
- Two or three dimensions as desired

Extended Hamming Codes	Parity Codes
(128,120)	(128,127)
(64,57)	(64,63)
(32,26)	(32,31)
(16,11)	(16,15)
(8,4)	(8,7)
-	(4,3)

2D Product Code Example

- (8,4) x (8,4) Code
  - Code is systematic
  - D represents input data
  - E represents ECC bits
- 3D codes follow the same concept, but in three dimensions

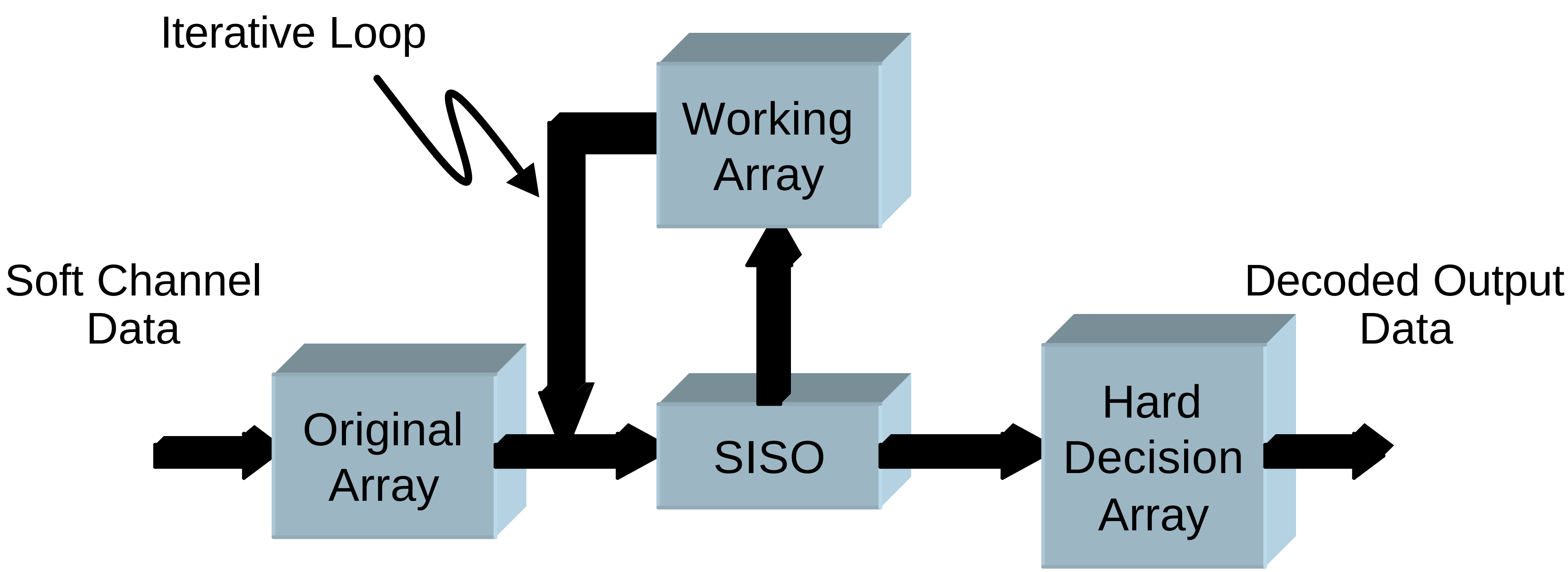
D	D	D	D	E_x	E_x	E_x	E_x
D	D	D	D	E_x	E_x	E_x	E_x
D	D	D	D	E_x	E_x	E_x	E_x
D	D	D	D	E_x	E_x	E_x	E_x
E_y	E_y	E_y	E_y	E_{xy}	E_{xy}	E_{xy}	E_{xy}
E_y	E_y	E_y	E_y	E_{xy}	E_{xy}	E_{xy}	E_{xy}
E_y	E_y	E_y	E_y	E_{xy}	E_{xy}	E_{xy}	E_{xy}
E_y	E_y	E_y	E_y	E_{xy}	E_{xy}	E_{xy}	E_{xy}

Example Codes

- TPCs can provide a wide range of code rates and block sizes
- Code shortening enhances this flexibility
  - Shorten rows, columns and/or bits

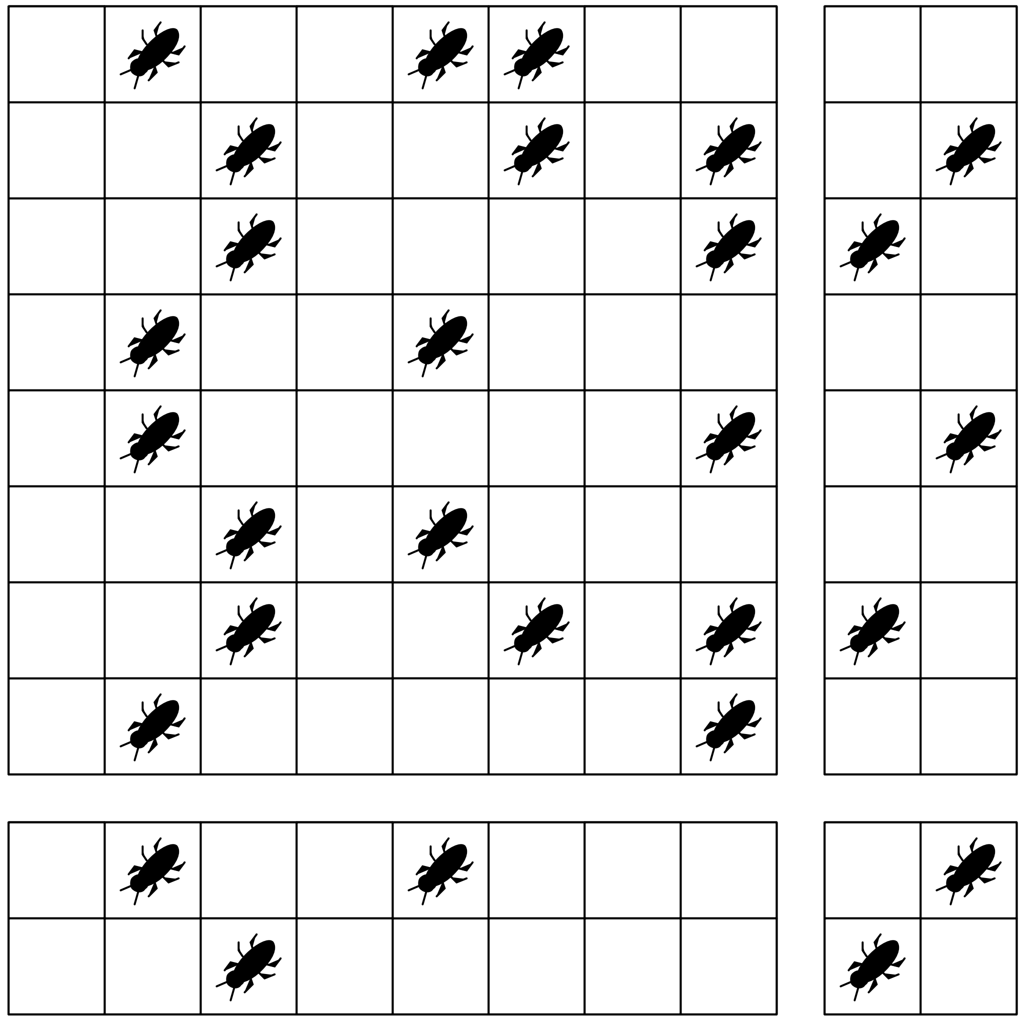
Product Code	Block Size	Code Rate
(128,127) x (128,127)	16,383	0.98
(128,120) x (128,127)	16,383	0.93
(64,57) x (32,26)	2,048	0.72
(32,26) x (16,15) x (8,7)	4,096	0.66
(16,11) x (16,11)	256	0.47
(16,11) x (16,11) x (16,11)	4,096	0.32

TPC Decoding

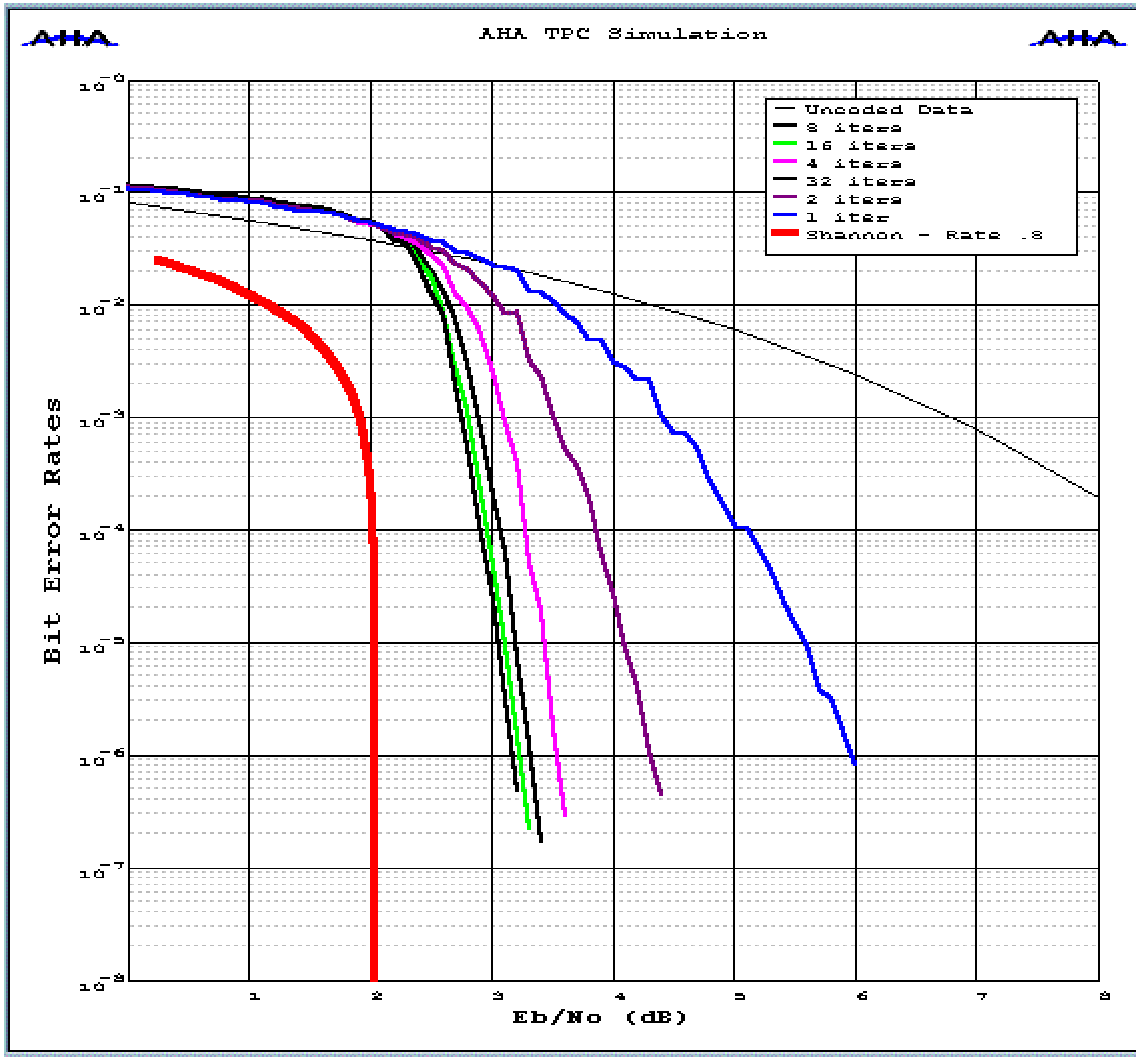


Each Iteration Decodes all Rows, then all Columns

Two-dimensional Iterative



Iterative Decoding



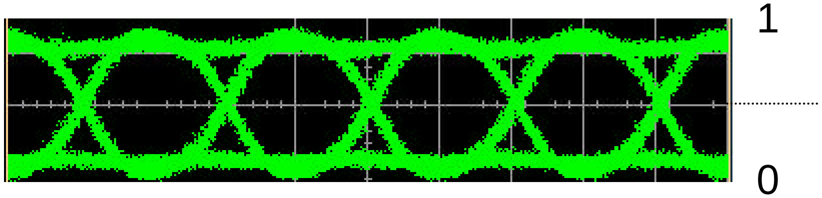
Soft Decision

- Soft decision metrics represent “confidence” that input bit is “1” or “0”
- TPCs can use soft decision information to improve decoding performance

Soft Decision Decoding

- Soft decoder input requires a “soft metric”
  - BPSK/QPSK proportional to I/Q voltage
  - Higher order constellations also supported
- TPCs will work with hard decision or soft decision decoding
- Two bit soft decision decoding typically picks up 2 dB of coding gain over hard decision
- Additional soft bits can add about 1/2 dB more

Hard Decision



Transmit	0 0 1 0	1 1 0	Estimated Error Count
Receive	0 1 1 1	1 1 0	
Closest Code Word 1	0 0 1 0	1 1 0	
Estimated Error 1	0 1 0 1	0 0 0	2
Closest Code Word 2	0 1 1 1	0 1 0	
Estimated Error 2	0 0 0 0	1 0 0	1

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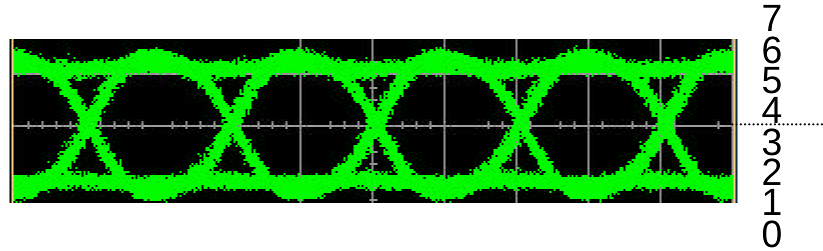
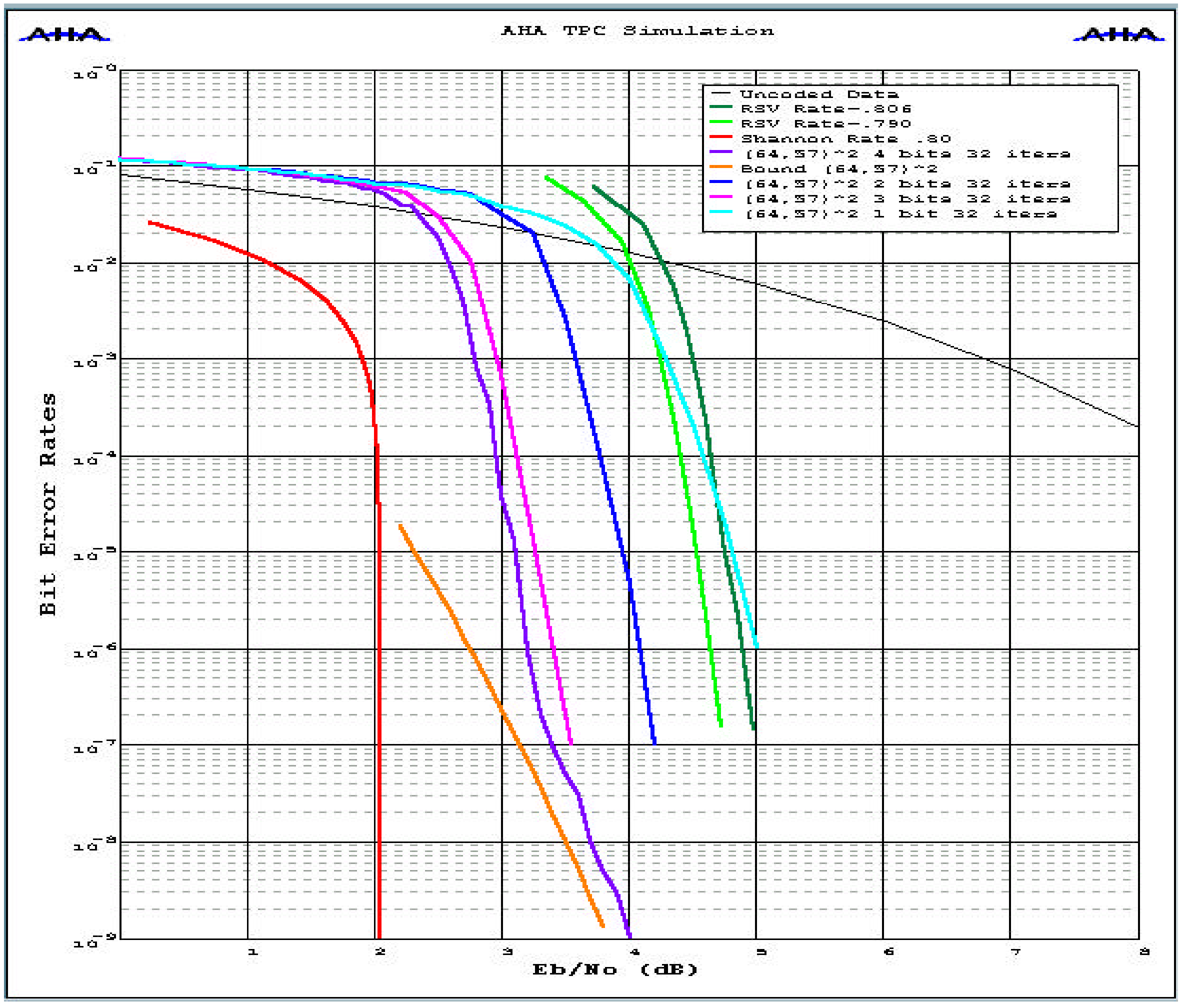
Soft Decision



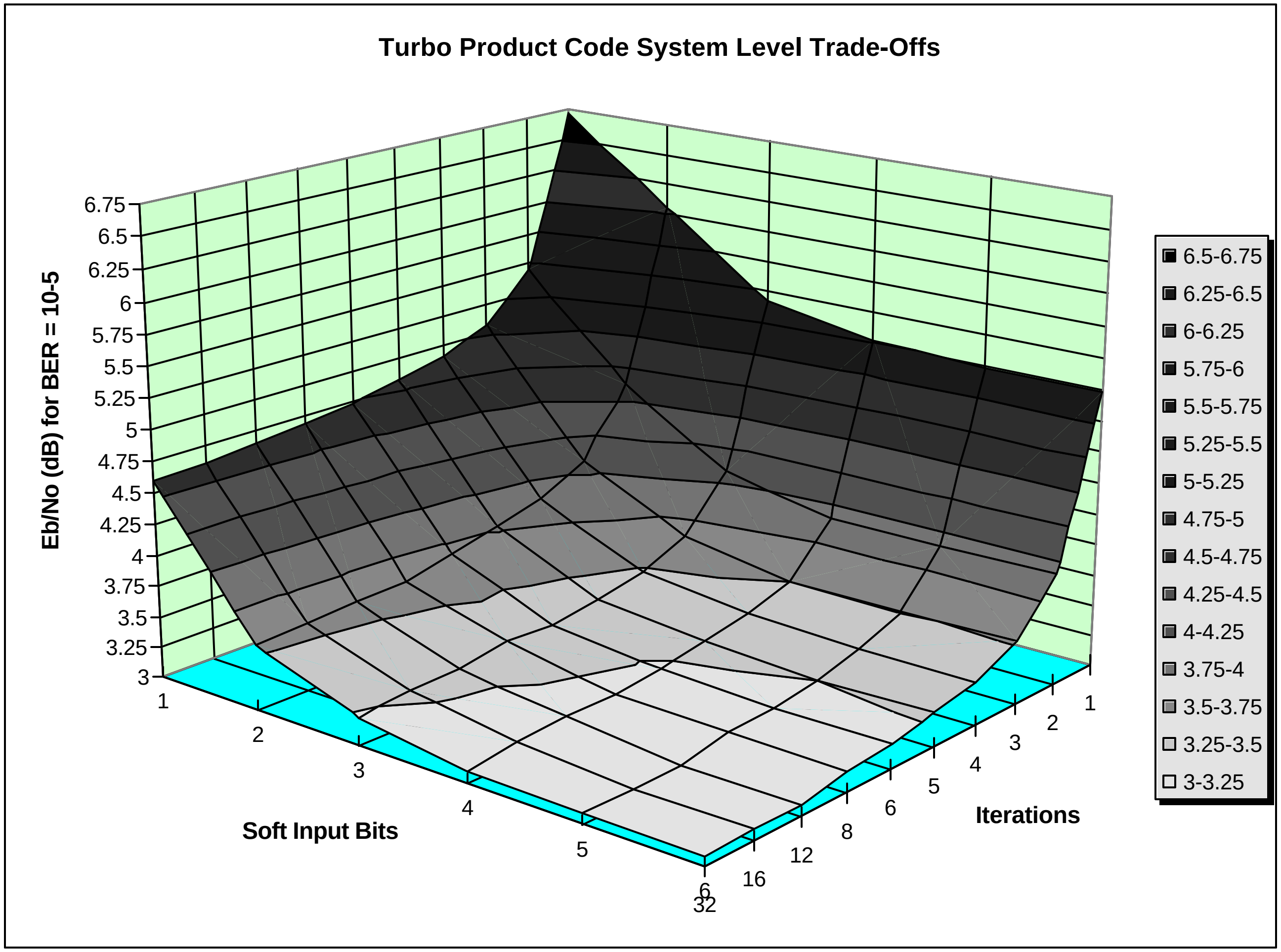
			Estimated Error Weight
Transmit	0 0 1 0	1 1 0	
Receive	0 4 7 4	6 6 1	
Closest Code Word 1	0 0 7 0	7 7 0	
Estimated Error 1	0 4 0 4	1 1 1	11
Closest Code Word 2	0 7 7 7	0 7 0	
Estimated Error 2	0 3 0 3	6 1 1	14

Soft vs. Hard Decision

(64,57)^2 vs. RSV



System Level Tradeoffs



(64,57)x(64,57) Rate 0.793 Code

What's better about TPCs?

- Simple constituent codes
- Multi-dimensional
- Iterative
- Soft-in/Soft-out (SISO)
- Good granularity
- Flexible

Turbo Convolutional Codes (TCCs)

- TCCs are generated from Concatenated Convolutional codes

- Performance of TCCs can be within 1 dB of the Shannon Capacity Limit for low code rates

TPCs vs. TCCs

- Code Rate:
  - TCCs perform best for low code rate applications
  - TPCs perform best for high code rate applications
- Data Rate:
  - TCCs will have difficulty achieving high data rates
  - TPCs can operate to data rates of 10 Gbps in CMOS

TPCs vs. TCCs - continued

- Error Floor
  - TCCs exhibit error floor at BERs below 10^{-5}
  - TPCs error floor (flare) is less pronounced and at lower BER values
- IP/Patents
  - TCCs require license from France Telecom
  - TPCs based on public domain technology
- TPCs have been available in hardware since November 1998

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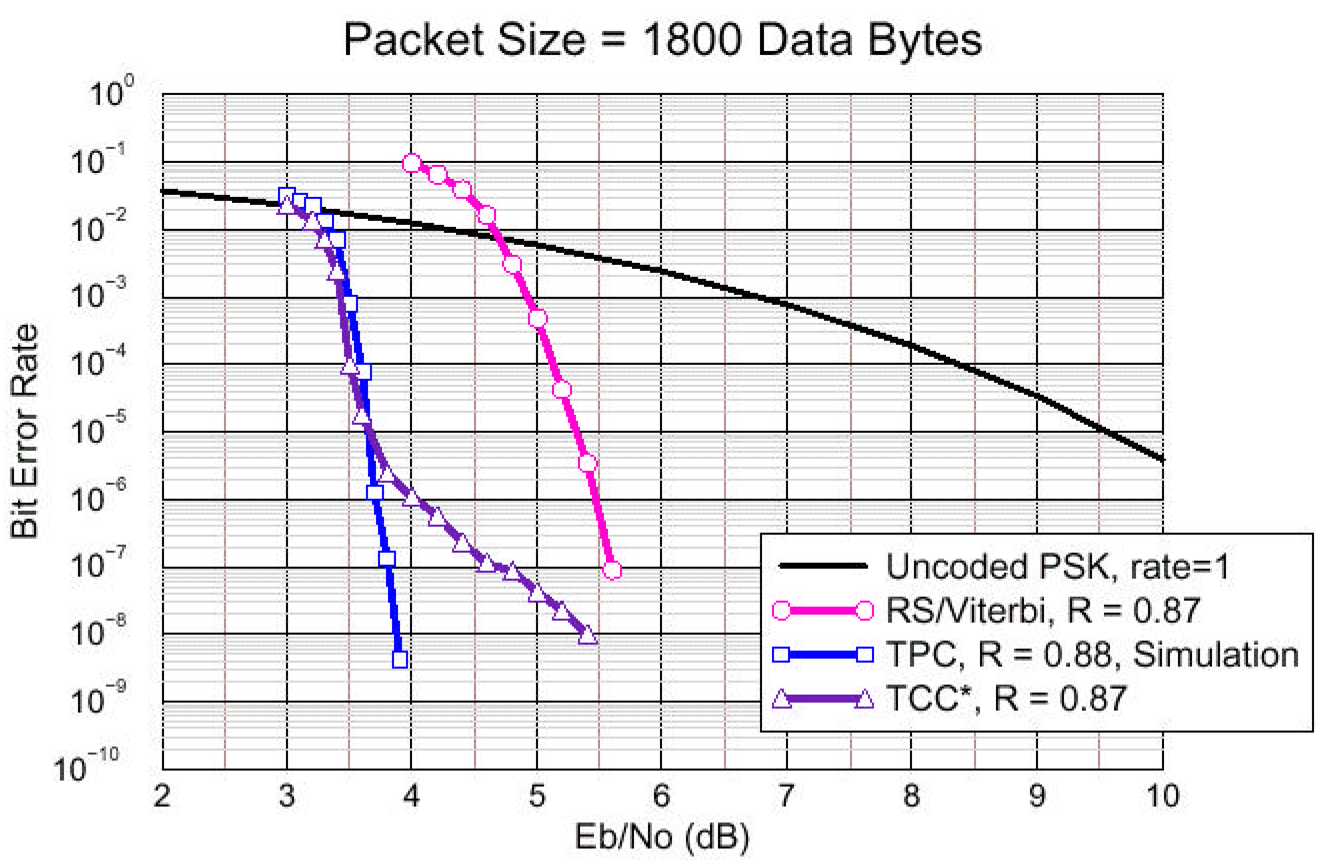
TPC vs. TCC



** PCCC, Simulated with Communications Research Centre (CRC) Turbo Decoding Software*

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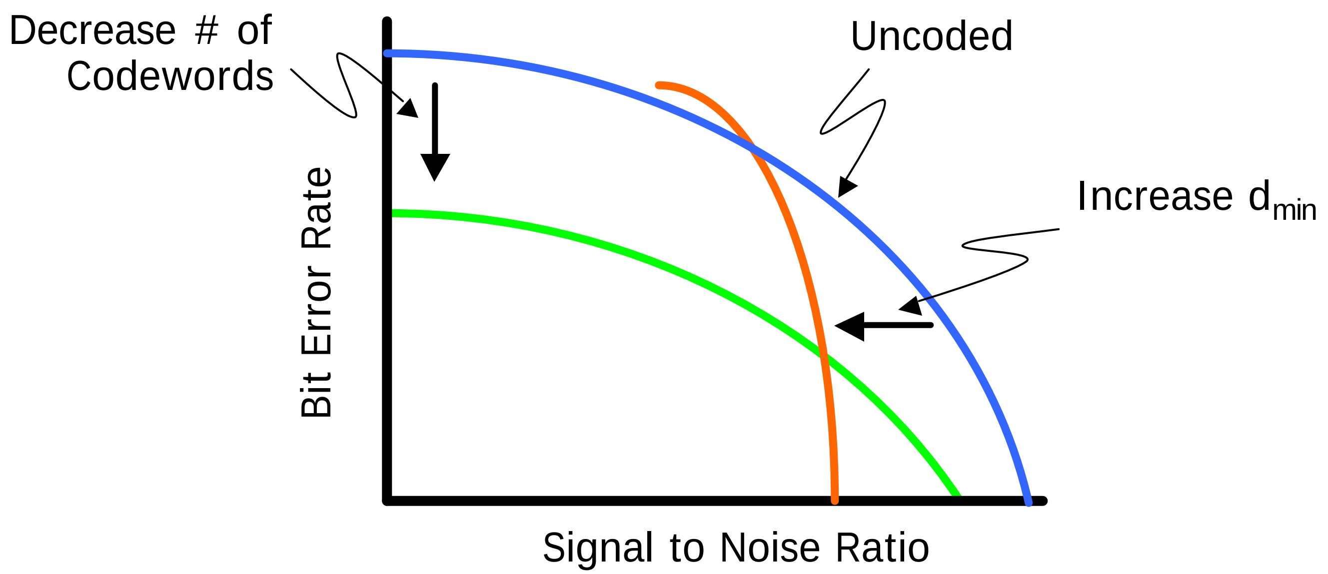
Asymptotic Bounds

- Minimum Distance (dmin) Determines Slope of Asymptotic Bound
- Number of Codewords at dmin Determine Position of Bound



TPC Distance Structure

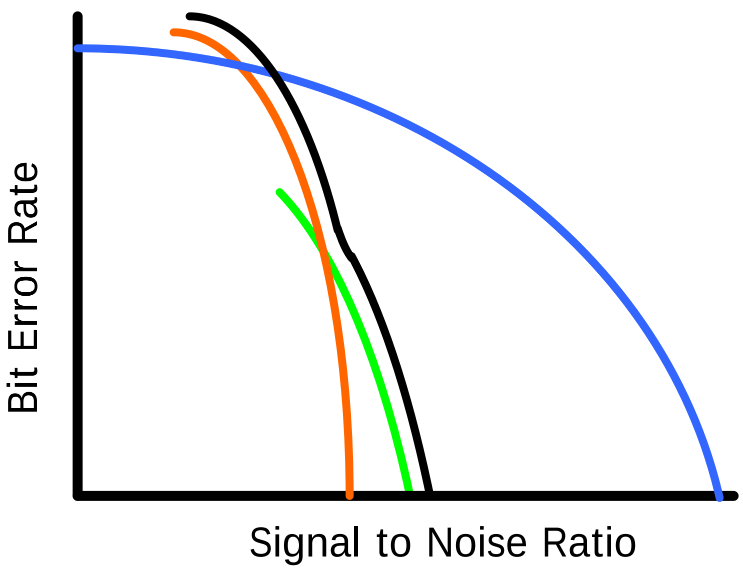
- Distance Properties of Product Code are Equal to the Product of the Constituent Codes

$(32,26)^2$ Product Code

$d_{min} = 16$

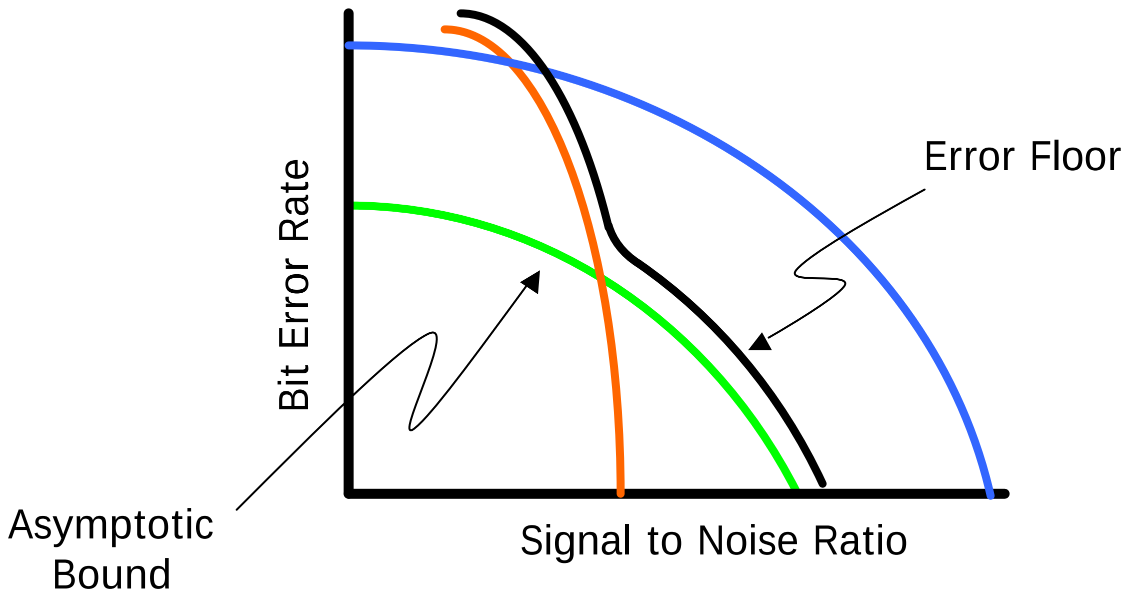
$(16,11)^3$ Product Code

$d_{min} = 64$



TCC Distance Structure

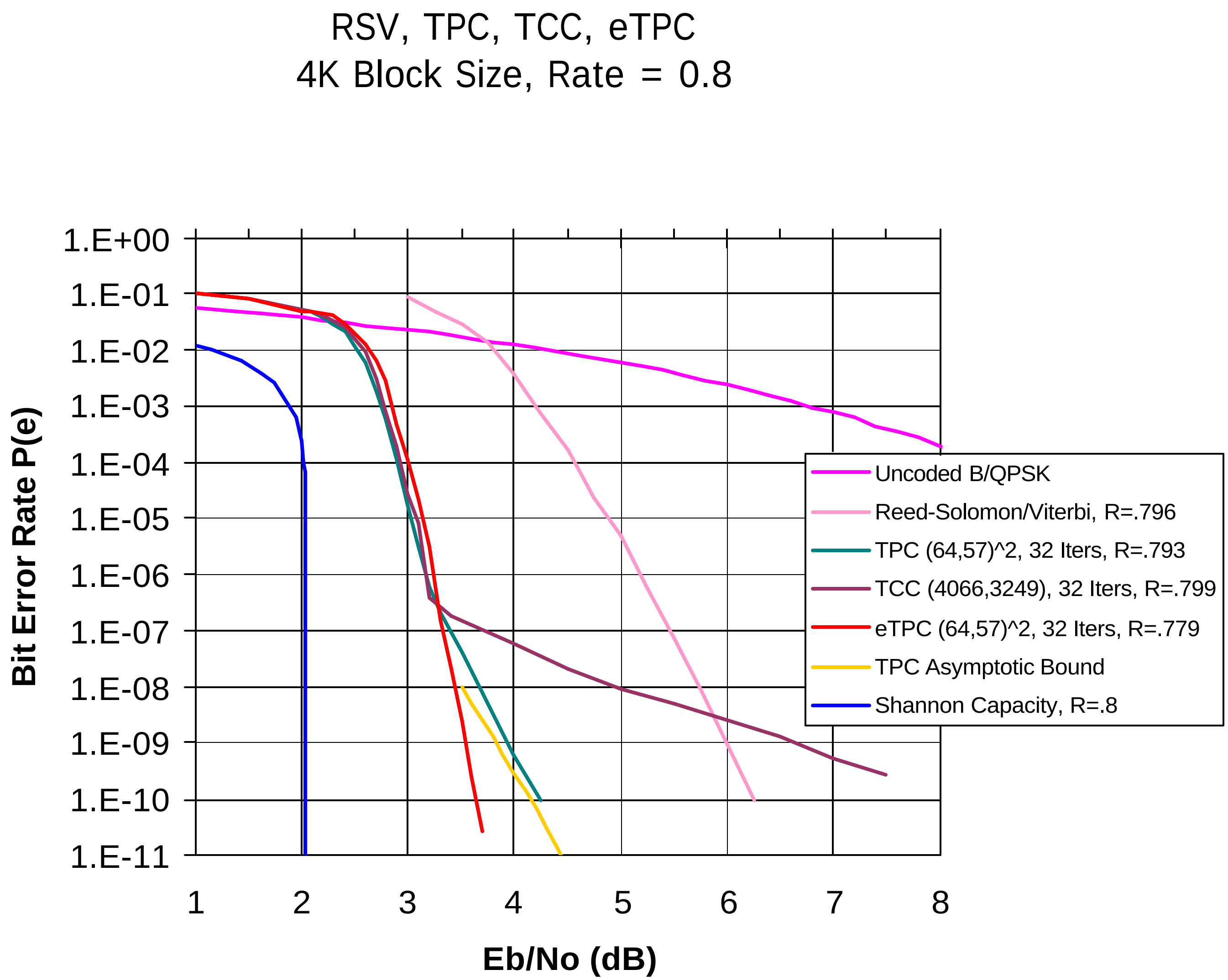
- Decreases Number of Codewords at dmin
- However, dmin is not High (Especially After Puncturing)
- Can Cause Characteristic ‘Error Floor’



Error Floor vs. Flare

- For TPCs, there is minimal error floor because of high Dmin (typ 16 or greater)
- Depending on the TPC code used:
  - Minor flaring starting @ about 10^{-7} to 10^{-12}
  - Flare is predictable
    - Block size
    - Dmin

Performance Comparison



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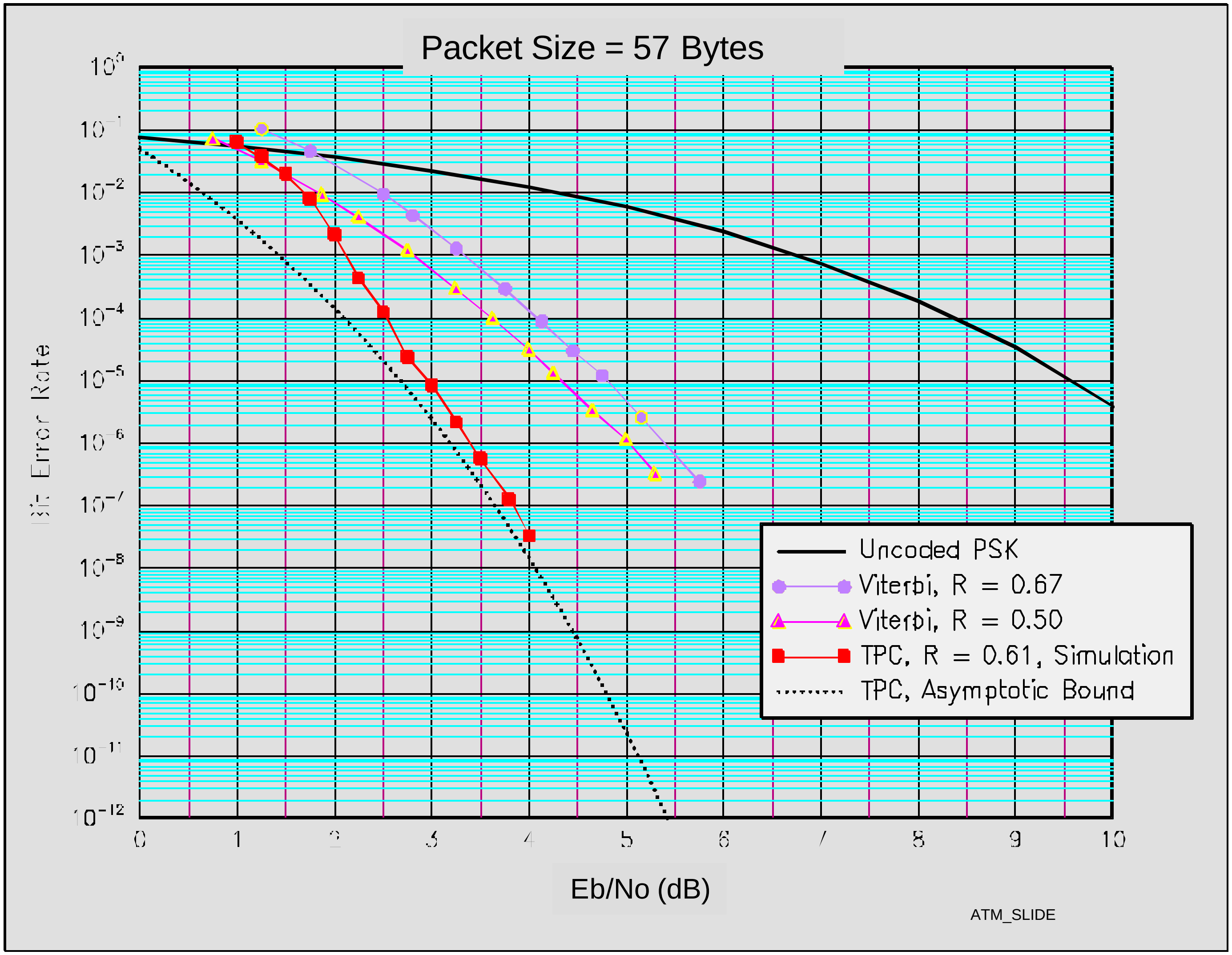
ATM Performance



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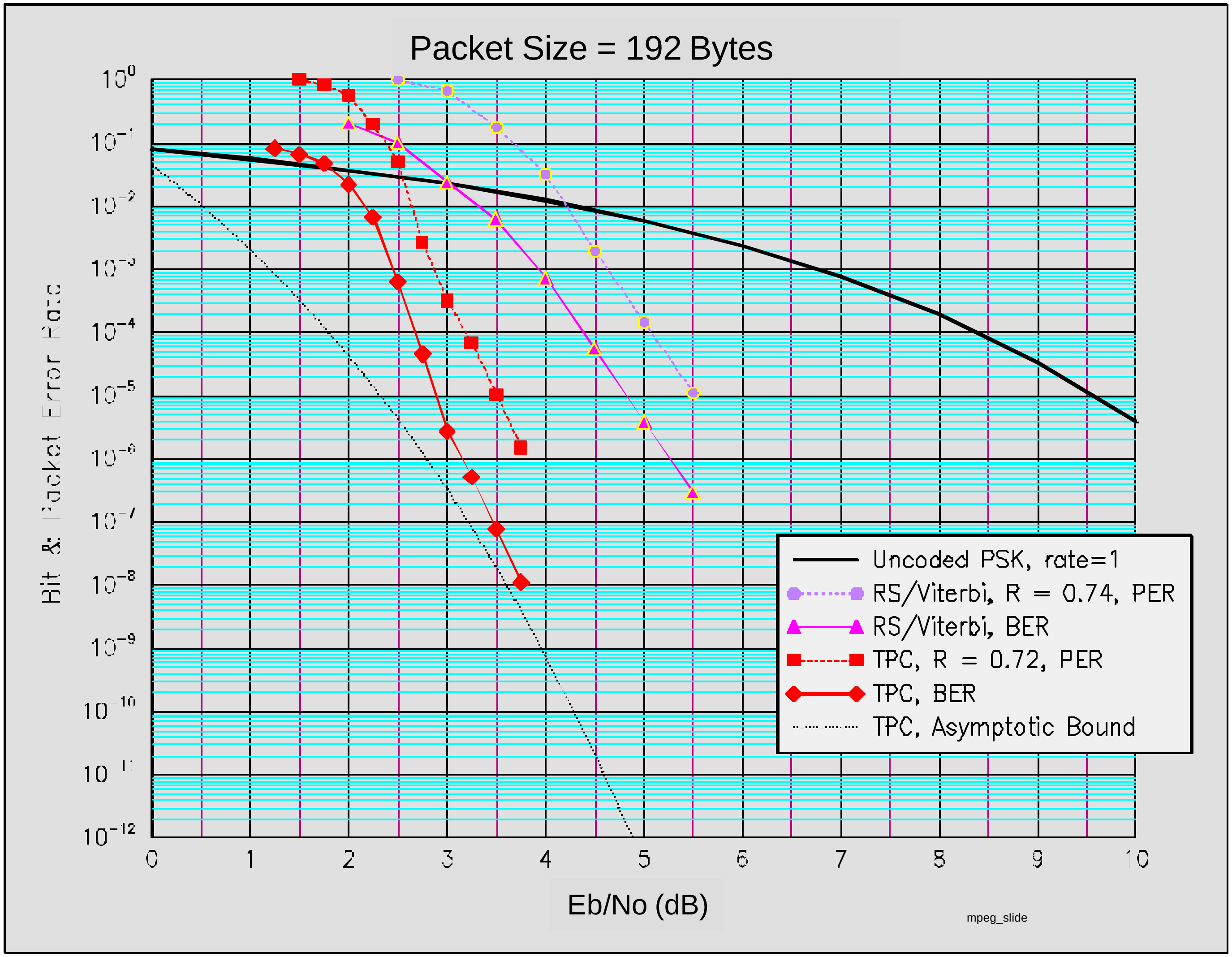
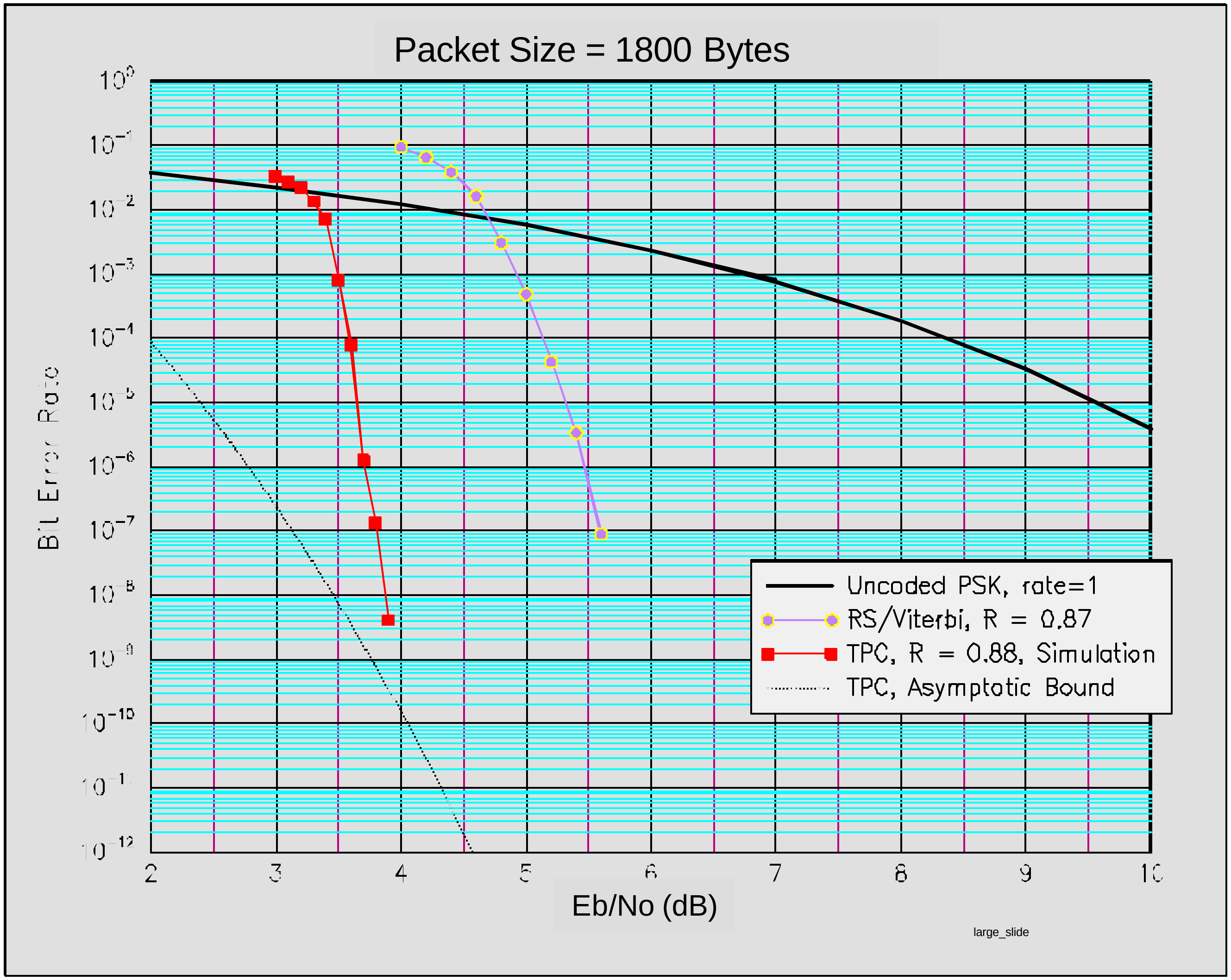
MPEG Performance



Large Packet Size Plot



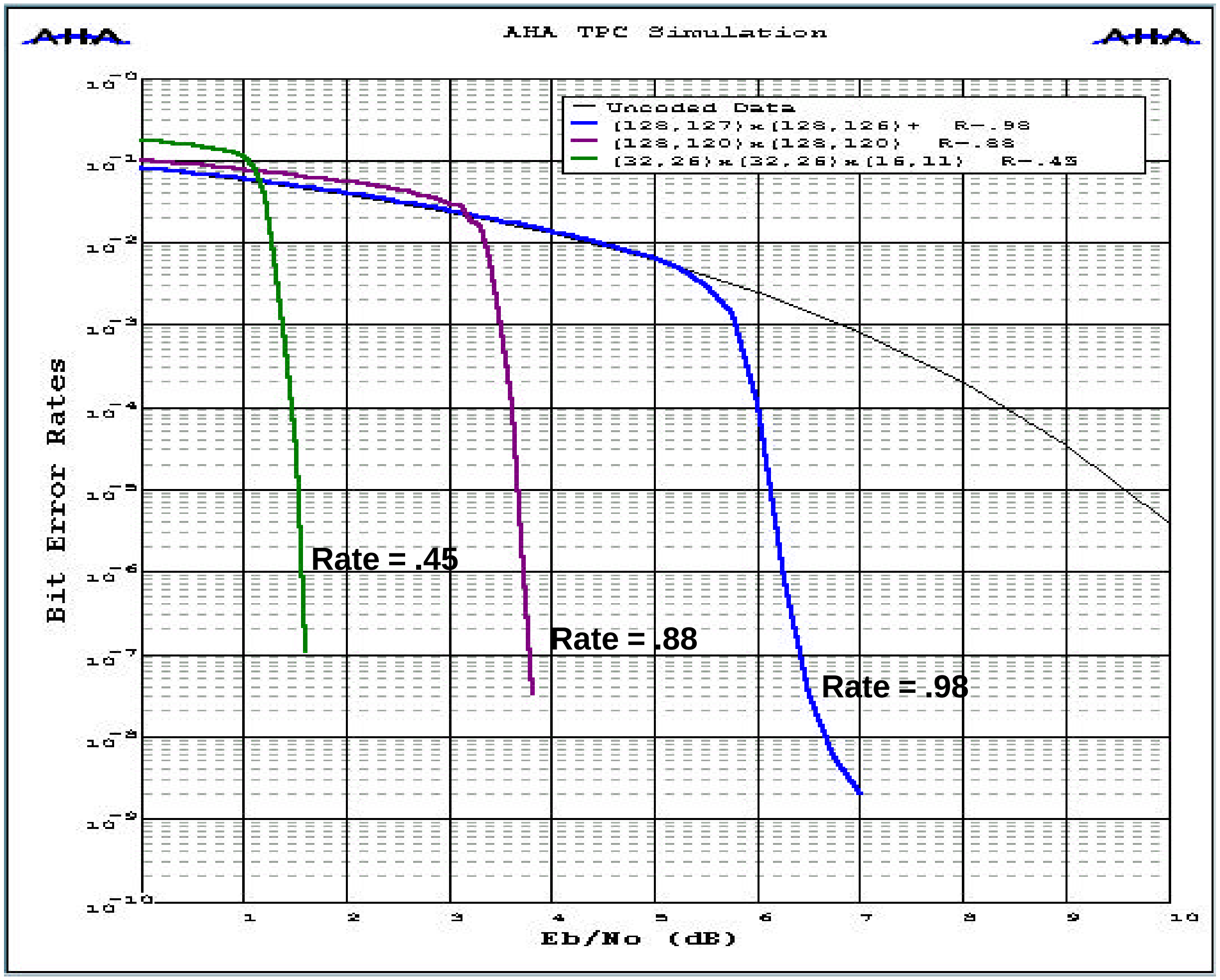
TPC Applications

- Broadband wireless access
- VSAT modems
- Wireless internet access
- Wireless LANs
- Free Air Optical

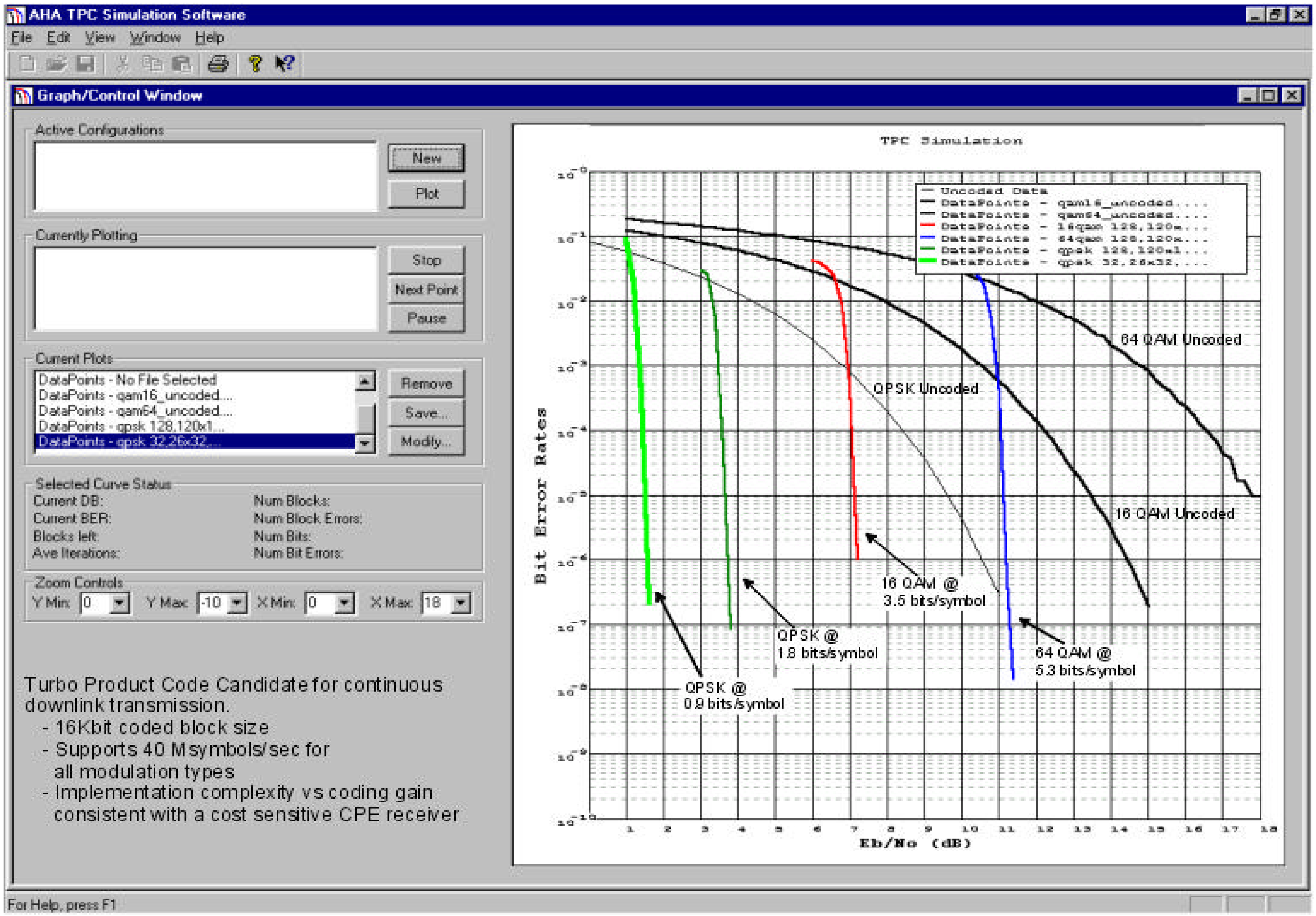
TPC Products

- AHA4501
  - 36 Mbit/sec channel rates
  - 4K blocks
  - Introduced November, 1998
- AHA4540
  - Supports OC-3 data rates (155 Mbit/sec)
  - Block size up to 16K bits
  - Protos: November, 2000

AHA4540 - Example Codes



TPC+Modulation – BPSK, QPSK, QAM16, QAM64



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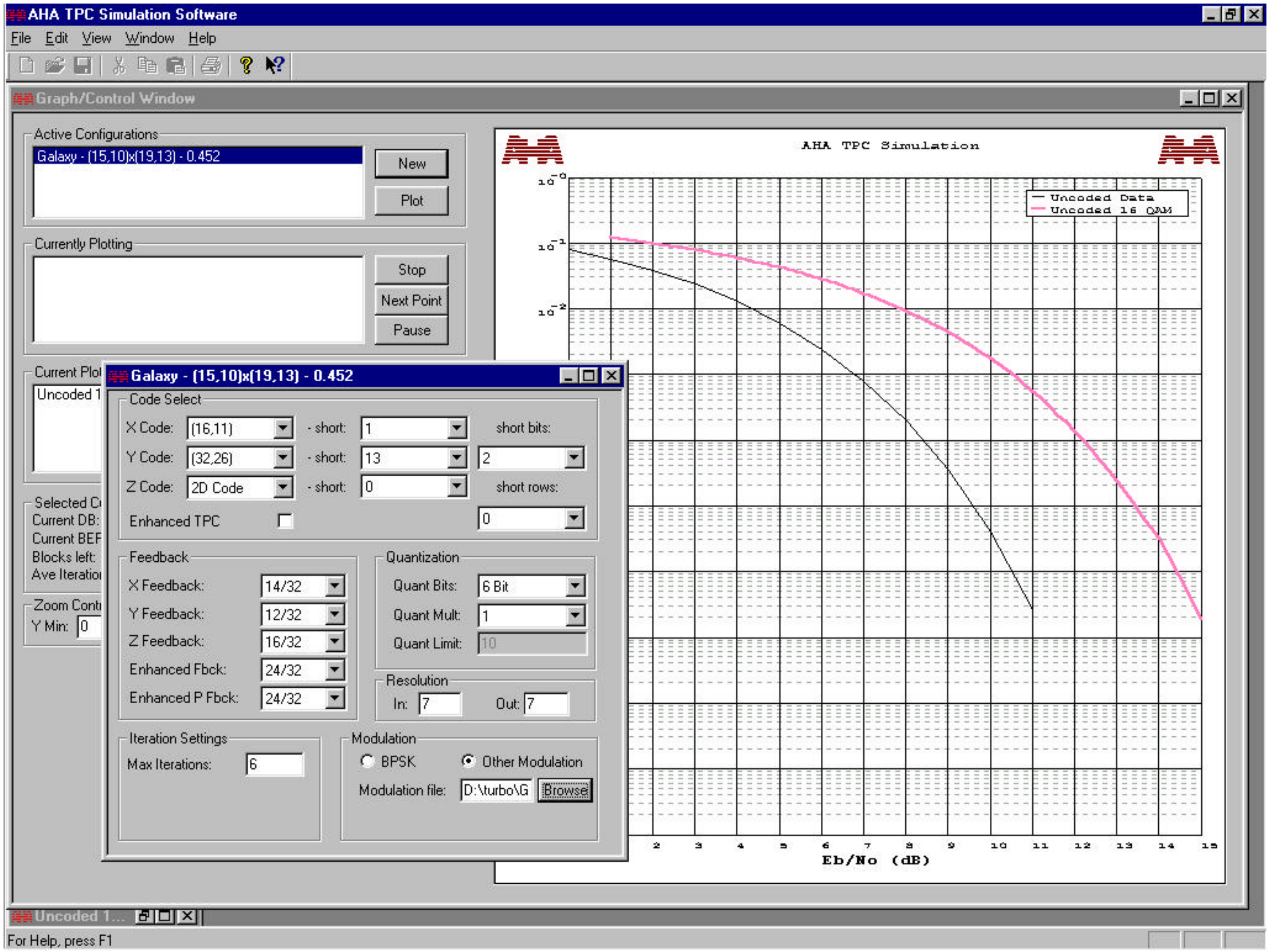
TPC Evaluation Software

- AHA4501 Evaluation software (v2.73)
  - download from: http://www.aha.com
- AHA Galaxy Simulation Toolkit
  - Windows GUI
  - Matlab API
  - C/C++ API
  - contact: sales@aha.com
- ECC, Inc. TPC Simulation software
  - info at: http://www.eccincorp.com

Galaxy Simulation Software



Conclusions

- TPCs provide a superior FEC solution with performance approaching theoretical limits
- TPCs operate over a wide range of data rates, block sizes, code rates for a wide range of applications
- TPCs are available now (ICs, cores, eval software & eval boards)

TPC Hardware Demonstration

- AHA4501 TPC encoder/decoder board
- Video Data - 8 bit grayscale
- TPC encode
- AWGN channel
- TPC decode

Contact Information

- Website: www.aha.com
- Phone: 509-334-1000
- email: tpc@aha.com

- for copies of this presentation contact:
  - bthomson@aha.com