Post-FFT Carrier Frequency Offset Compensation in OFDM

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Organization

- OFDM versus single-carrier
- Effects of Carrier Frequency Offset (CFO)
- Post-FFT CFO Compensation
- High Data Rate Extension via Advanced Coding and Diversity
• Advantages
  – Multi-path tolerance
  – Tolerance to narrowband interference
  – Channel capacity via adaptive bit loading

• Disadvantages
  – Sensitive to carrier frequency offset (CFO)
  – Sensitive to phase noise
  – High peak to average ratio \(\rightarrow\) PA nonlinearity
OFDM Applications

- High Speed Wireless LAN (IEEE802.11a)
- Digital Audio/Video Broadcasting (DAB/DVB)
- ADSL
- Fixed Broadband Wireless Access
OFDM Basics

**single-carrier**
Time-domain symbol interval is small
– delay spread is significant portion of $T_1$

**OFDM**
Time-domain symbol interval is large for individual subcarrier – delay spread is small portion of $T$. 

$\frac{1}{T_1} = \frac{4}{T}$

$4 \times \frac{1}{T} = \frac{4}{T}$
Time Gap and Cyclic Prefix in OFDM

- Time gap prevents ISI in delay spread.
- Cyclic prefix maintains orthogonality in delay spread.
Measured TX Constellation

- 64 QAM
- 54 Mbps
- Agilent E 8408A Vector Signal Analyzer
- EVM < -26dB
- 802.11a EVM spec = -25dB
Measured RX Constellation

- Data after equalization/phase compensation

64 QAM constellation for 54 Mbps data rate.
Rx sensitivity test via wired connection to a waveform generator.
Multi-path Capabilities of OFDM:
150 ns rms delay spread (Regular Mode)

Per, 1500 Byte Packets

Pr [dBm], Trms=150ns, 6 Hz Doppler, NF=8dB
Low Power Mode, -90dBc/Hz@100kHz
SFO/CFO=40ppm, I/Q Match=1dB, 2deg.
Multi-path Capabilities of OFDM:
150 ns rms delay spread (Performance Mode)

Pr [dBm], Trms=150ns, 6 Hz Doppler, NF=8dB
High Performance Mode, -90dBc/Hz@100kHz
SFO/CFO=40ppm, I/Q Match=1dB,2deg.
Trms = 250ns Multipath Channel

**Power Profile and Single Channel Taps**

- Time (ns)
- Magnitude Response of 4 Channels (dB)
Multi-path Capabilities of OFDM: 250 ns rms delay spread (Regular Mode)

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Multi-Path Capability of 11Mbps CCK Mode

- rms delay spread=0nsec
- rms delay spread=50nsec
- rms delay spread=150nsec
- rms delay spread=250nsec

Eb/No (dB) vs. PER (log10) for Rake and Rake+ISI/ICI cancel.
Block Diagram for 802.11a OFDM PHY
Effect of CFO

- Ideal sampling positions - no ICI
- CFO-induced position shift - ICI
Effect of CFO on signal constellation

CFO = 1ppm

CFO = 2ppm

CFO = 3ppm
Performance sensitivity to CFO

![Graph showing SNR (dB) vs. Carrier frequency offset (ppm) with curves for 1024 QAM, 256 QAM, and 64 QAM. Symbol error rate (log10) is also shown.]
Conventional CFO Estimation Using Preamble

Supposedly \( x(t_0 + P) = x(t_0) = A \)

With CFO,

\[
x(t_0) = A e^{j2\pi f_e t_0}
\]

\[
x(t_0 + P) = A e^{j2\pi f_e (t_0 + P)}
\]

\[
\frac{x(t_0 + P)}{x(t_0)} = e^{j2\pi f_e P} \rightarrow \text{extract } \hat{f}_e
\]
Modeling of CFO Effect

Tx signal: \[ s(t) = \sum_{k=0}^{K-1} a_k g(t) e^{j2\pi ft_k} e^{j2\pi f_e t}, \]

FFT output at Rx: \[ r_k = \frac{1}{T} \int_0^T s(t) \gamma^*(t) e^{-j2\pi ft_k} dt + w_k, \]

\[ r_k = \sum_{m=0}^{K-1} a_m \left\{ \frac{1}{T} \int_0^T g(t) \gamma^*(t) e^{-j2\pi (k-m) ft} e^{j2\pi f_e t} dt \right\} + w_k \]

\[ = \sum_{m=0}^{K-1} a_m I_{k-m}^{f_e} + w_k \]

\[ = y_k(f_e, \mathbf{a}) + w_k. \]
Post-FFT CFO Estimation/Compensation

\[ \hat{a} \]

Known \( f_e \)

\[ r \rightarrow \text{Viterbi detector tuned to } y \rightarrow \hat{a} \]

\[ \hat{r} \rightarrow \text{Viterbi detector tuned to } y \rightarrow \hat{a} \]

\[ \hat{f}_e \rightarrow \text{D} \rightarrow \text{CFO estimator} \rightarrow \hat{f}_e \]
E-M Algorithm Perspective

**Estimation Step:** Estimate $\mathbf{a}$ based on $\mathbf{r}$ and $f_e^i$.

**Maximization Step:** Maximize $-\sum_k |r_k - y_k(f_e, \hat{a})|^2$ with respect to $f_e$. The resulting estimate on $f_e$ is labeled as $f_e^{i+1}$.
Root-raised-cosine pulse used for symbol shaping
Frequency domain pulses for different CFO values
Variance and Convergence Range of the CFO estimate

![Graph showing variance of CFO estimate versus Es/N0 (dB) and f_0/F for different values of f_0/F.](Image)

- MCRB
- f_0/F = 0.1
- f_0/F = 0.2
- f_0/F = 0.4
BER performance for the joint CFO-data estimator

BER

\( E_b/N_0 \) (dB)

\( f_c/F = 0.1 \)

\( f_c/F = 0.4 \)

synchronous
### Extended Data Rates and Spectral Efficiencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Data bit rate (Mbit/s)</th>
<th>Spectral efficiency (bit/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK 3/4</td>
<td>18</td>
<td>1.5x.6=.9</td>
</tr>
<tr>
<td>16-QAM 1/2</td>
<td>24</td>
<td>2x.6=1.2</td>
</tr>
<tr>
<td>*64-QAM 3/4</td>
<td>54</td>
<td>4.5x.6=2.7</td>
</tr>
<tr>
<td>256-QAM 3/4</td>
<td>72</td>
<td>6x.6=3.6</td>
</tr>
<tr>
<td>1024-QAM 1/2</td>
<td>60</td>
<td>5x.6=3</td>
</tr>
<tr>
<td>1024-QAM 3/4</td>
<td>90</td>
<td>7.5x.6=4.5</td>
</tr>
</tbody>
</table>
OFDM with LDPC Coding and Antenna Diversity

Diagram:
- LDPC Encoder
- Conv. Encoder
- Interleaver
- Mapping
- IFFT
- Delay
- FFT
- FFT
- Demapping (Soft)
- Deinterleaver
- Viterbi Decoder
- MP Decoder
- Info. bit
- Decoded bit
HYPERLAN/2 Channel B – NLOS, 100 ns rms delay spread.
Channel independent from one OFDM symbol to next.
HYPERLAN/2 Channel B – NLOS, 100 ns rms delay spread. Channel independent from one OFDM symbol to next.
OFDM Performance with Coding and Diversity

PER of rate-3/4 coded-OFDM over 802.11a type quasi-static fading channel, with antenna diversity, packet size=512 bytes

Eb/No (dB) vs. PER

- 1x1 64QAM CC
- 1x2 256QAM CC
- 2x2 256QAM CC
- 2x2 256QAM PCPC

50 ns rms delay spread

9/28/02
Yong Li
Summary

- 5GHz OFDM PHY superior to 2.4 GHz SS/CCK PHY in indoor multi-path environments
- Post-FFT compensation feasible up to 50% CFO
- Antenna diversity and iterative coding promising for high data rate extension

- Contributions from H. Jin and Y. Li of ECE, U of MN and entire PHY and System Integration Groups at Bermai acknowledged