
Seminar

Modeling and Simulation of Dynamical Systems

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Numerical Physical Simulation

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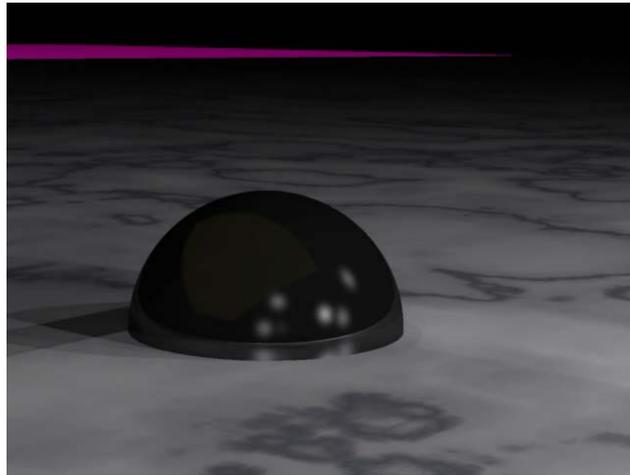
Numerical Physical Simulation

- Initial Value Problem:
 - Given some initial state $q(t)$
 - Want the state at some point in the future $q(t+dt)$
 - The change in state should convergently follow the desired set of physical laws.
 - PDEs, $F=ma$, Collisions, Constraints, etc...

Simulation Materials

- Continuum Mechanics
 - Deal with volumes rather than infinitesimal particles
- Range of different materials
 - Rigid \leftrightarrow Elastic \leftrightarrow Plastic \leftrightarrow Fluid
- Different set of challenges with each material

Cloth Simulation



Steps to Simulation

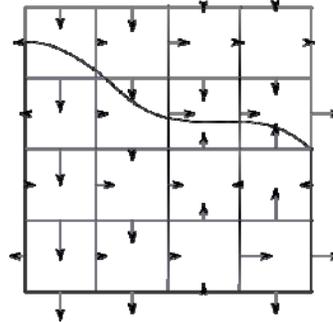
- Spatial Discretization
- Temporal Discretization
- Material Specific Issues
- Solvers

Spatial Discretization

- How to represent state (positions, velocities, mass) throughout space
- Eulerian
 - Store a grid fixed in space and change values
 - Mass moves between degrees of freedom
- Lagrangian
 - Discretization moves with material
 - Mass is fixed to degrees of freedom
- Arbitrary Lagrangian Eulerian (ALE)

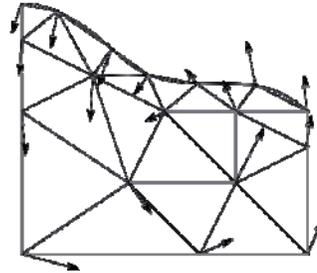
Eulerian Simulation

- Pros:
 - Accurate numerical derivatives
 - Lower memory usage with better cache coherency
- Cons:
 - Adaptivity is difficult
 - Lose memory and accuracy benefits
 - T-junctions when adjacent cells are misaligned
 - Can't accurately represent boundaries unaligned with grid
 - Represent as implicit surfaces



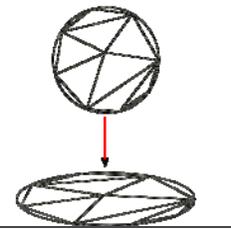
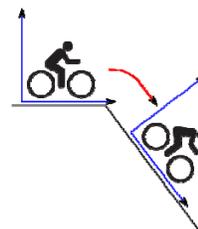
Lagrangian Simulations

- Pros:
 - Adaptivity is easy
 - Boundaries are easy to resolve
- Cons:
 - Higher memory requirements
 - Remeshing is necessary to guarantee accuracy of derivatives
 - Collisions
 - Topology changes are nontrivial



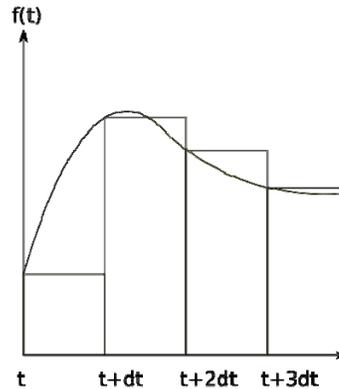
Common Spatial Discretizations

- Rigid Bodies (lagrangian)
 - 3D positions and orientation (6 DOF space)
 - Geometry stored in local frame
- Elastic Bodies/Plastic Materials (lagrangian)
 - Plastic bodies need to be remeshed when rest state is too deformed
- Fluids (lagrangian/eulerian)
 - Particles in a lagrangian scheme
 - Velocities stored on grid faces in eulerian scheme



Temporal Discretization

- Use a numerical integration scheme to compute values at new time
- Many schemes
 - Taylor expansions (forward/backward euler, bdf, etc...)
 - Runge Kutta,...
 - Quadrature
- Explicit/Implicit
- Stability/Accuracy

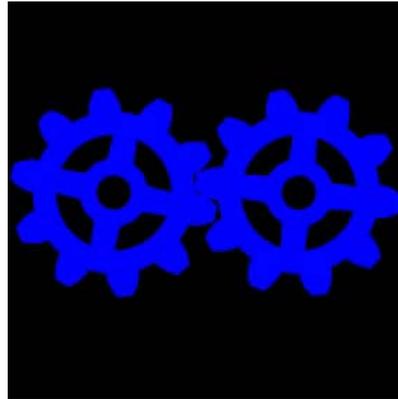


Rigid and Deformable Body Integration

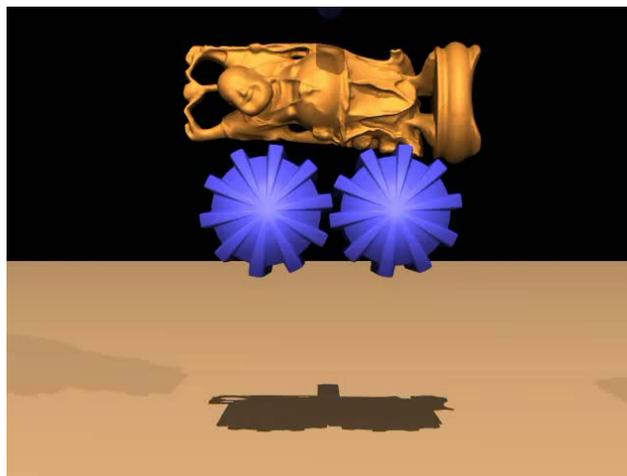
- Rigid body motion is hard to solve analytically
 - Approximate by assuming rotation is constant over timestep
- Deformable motion needs to be solve implicitly to be stable
 - New positions are used when computing forces
 - Backward euler, trapezoidal rule...
- Elastic and Inelastic collisions are handled as a post process

Finite Element Method

- The finite element method approximates solutions to PDEs on discretization
- Forces arising due to the solution of these PDEs are then incorporated into the numerical integration scheme
- Heat, Wave Equations, Elasticity etc...



Robust Finite Element Simulation

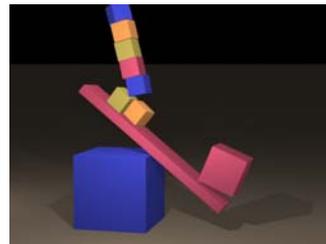
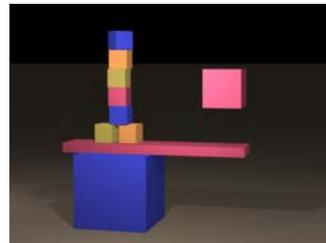


Elastic Collisions

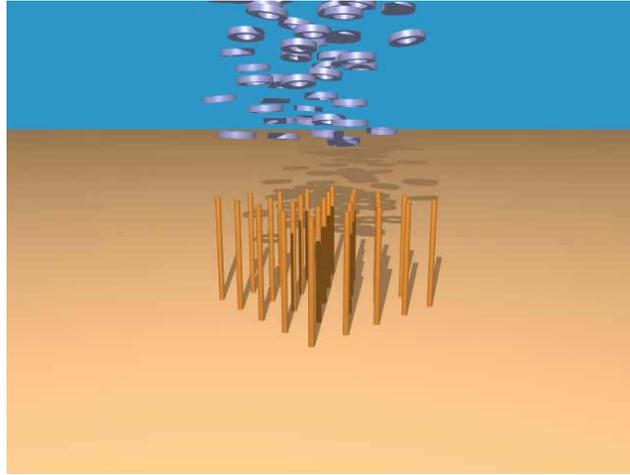
- Iteratively process each collision one at a time
 - New collisions may occur and need to be processed
- Thin shell objects need to be collision free
 - Any objects still colliding after a certain number of iterations are combined and evolved rigidly for the current time step
- For speed allow volumetric objects to penetrate slightly

Contact (Inelastic Collisions)

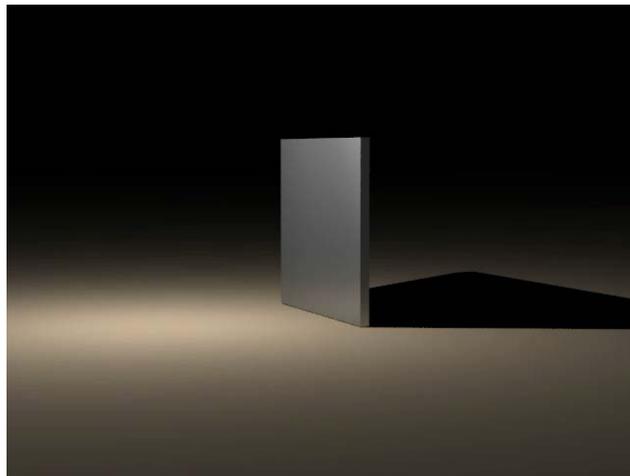
- When many objects are stacked need to solve contacts accurately to prevent penetration and without rigidifying
- Impose inequality constraints in monolithic system
- Generally requires the solution of a linear complementarity problem



Large Scale Stacking



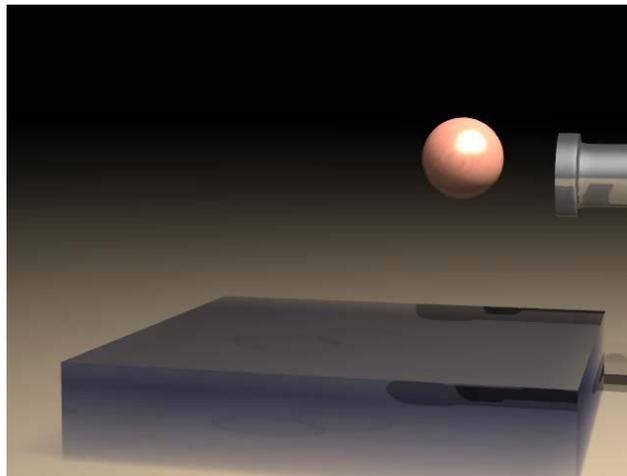
Plasticity and Fracture



Fluid Integration

- Typically split integration into two steps
 - Advection – moves velocities and mass around (computed explicitly)
 - Semi-lagrangian advection traces backwards from velocity samples back and interpolates new velocities from locations
 - Pressure and Viscosity Solve (computed implicitly)
 - Incompressible flows project the advected velocity field to be divergence free
- Other field values are also advected
 - Levelset implicit surface, density, temperature, etc...
- Solid fluid coupling
 - Straightforward when purely lagrangian
 - Eulerian frameworks need to handle nonconforming boundaries
 - Usually handled in implicit solve

Multiphase Reacting Flow



Solvers

- Large sparse linear systems
 - Direct
 - Large sparse matrix factorizations (Cholesky, LU)
 - Iterative
 - Jacobi, Gauss Seidel, Conjugate gradient
- Collisions (Nonlinear nonsmooth systems)
 - Direct
 - Pivoting methods: Lemke, Dantzig (simplex method)
 - Iterative
 - Projected Gauss Seidel

Controller Driven Systems

- Simulations can have external forces due to controller output
 - Articulated body joints
 - Point attached muscles
- Both mechanical and biomechanical systems
- Numerical optimization of parametric designs
 - Aircraft, automobiles,...



Ongoing Research

- Energy conservation
 - Not necessarily more accurate but stable (large dt)
 - More meaningful results when not converged
- Resolve all collisions - interpenetration free simulation (continuous collision detection)
- Monolithic integration schemes
 - Solve for both collision and constitutive model forces in the same system
- Scalability – solvers which scale better than $O(n^2)$
 - Optimal solvers - $O(n)$ or $O(n \log(n))$ time (Multigrid)

Annotated bibliography

<http://physbam.stanford.edu> Dr. Ron Fedkiw group website. Contact information, papers, videos, images.

Q & A

