Compact Modeling of Multiple-Gate MOSFETs

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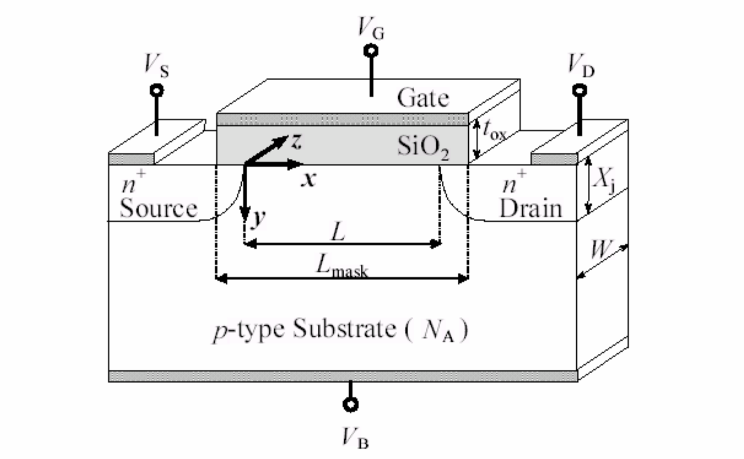
Outline

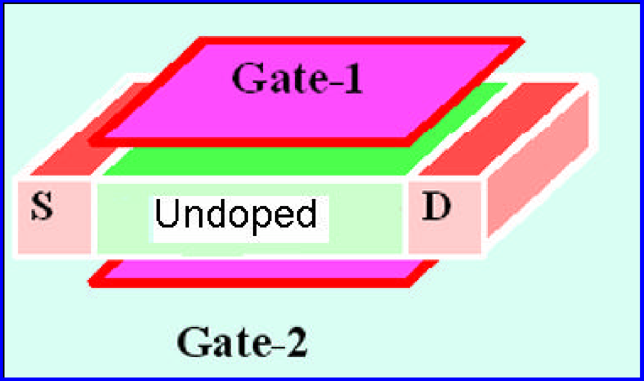
- INTRODUCTION
- SYMMETRIC MG MOSFETS: PLANAR AND CYLINDRICAL
- A COMPLETE DOUBLE-GATE COMPACT MODEL WITH HARDWARE CALIBRATION
- GENERALIZATION OF CORE MODEL TO MULTIPLE-GATE MOSFETS
- OTHER APPROACHES
- CONCLUSION

Impending Limit of Bulk MOSFETs

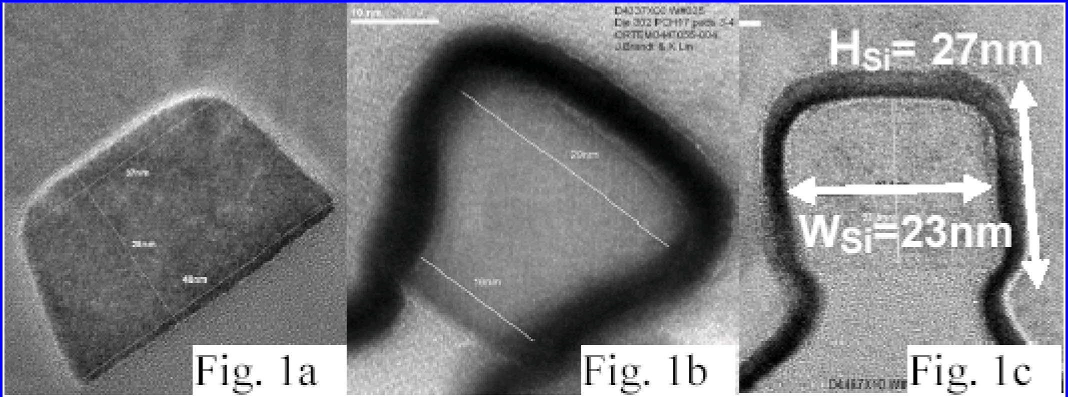
- Impending limit of bulk MOSFETs
  - Gate oxide tunneling
  - Threshold voltage limit
  - Band-to-band tunneling
  - Dopant fluctuation



- Potential promise of double-gate MOSFETs
  - Better control of SCEs
  - Ideal subthreshold slope
  - Better mobility without doping
  - No dopant fluctuation

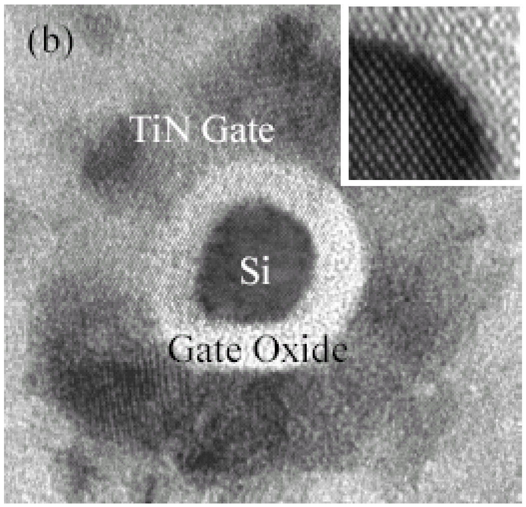


Fabrication Difficulty and Structural Variety



Tri-gate

J. Kavalieros et al, *VLSI Tech. Symp.,* 2006)

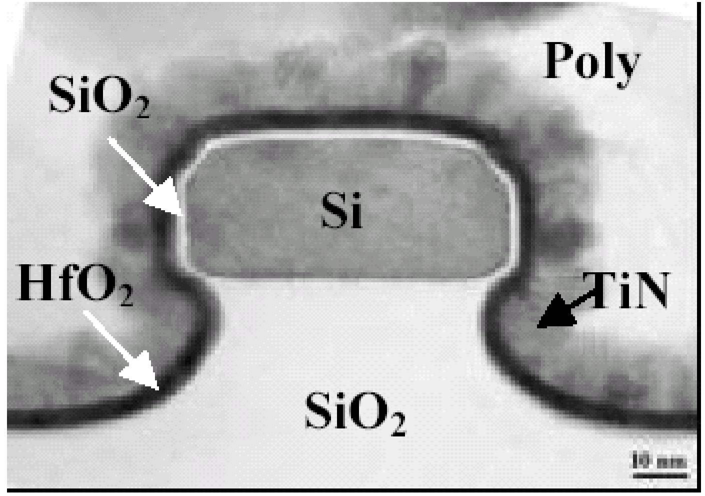


Nanowire

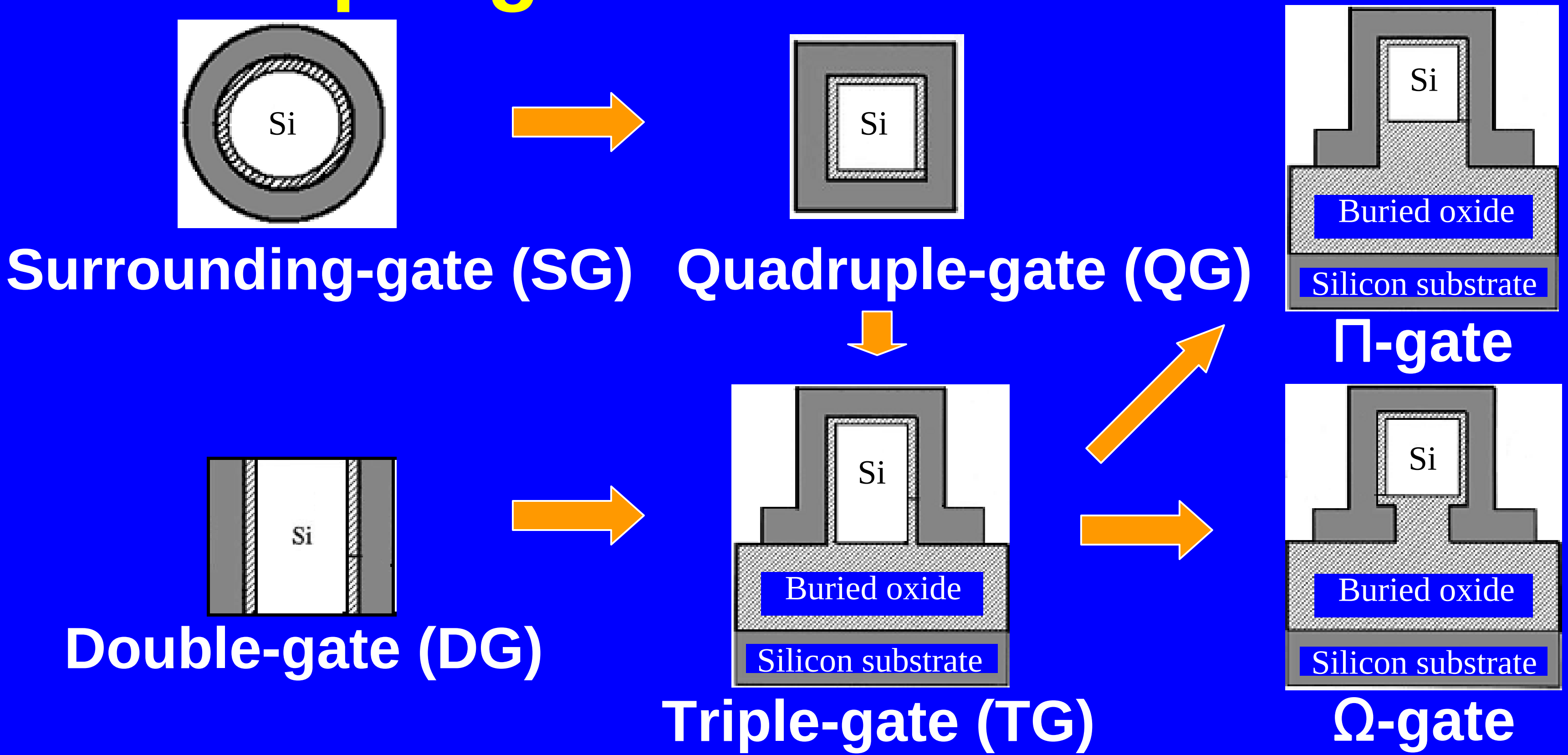
K. Cho et al, *IEDM,* 2006

Ω-gate

H. Lee et al, *VLSI Tech. Symp.,* 2006



Unified Approach to Compact Modeling of Multiple-gate MOSFETs



Surrounding-gate (SG)

Quadruple-gate (QG)

Π-gate

Double-gate (DG)

Triple-gate (TG)

Ω-gate

- DG and SG MOSFETs can be exactly solved
- QG model based on SG model
- TG approximated by the average of QG and DG
- Π-gate and Ω-gate models from TG model

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Compact Modeling

- **Bulk MOSFETs --- All based on the charge sheet model.**
  - **Poisson's equation cannot be solved analytically due to presence of depletion and mobile charge.**
  - **Charge sheet approximation is necessary in all bulk compact models.**

- **DG/SG MOSFETs --- Analytic potential function without charge-sheet approximation.**
  - **Exact solution to Poisson's equation is possible due to the absence of depletion charge.**
  - **Volume inversion in subthreshold region requires a non-charge-sheet based model.**

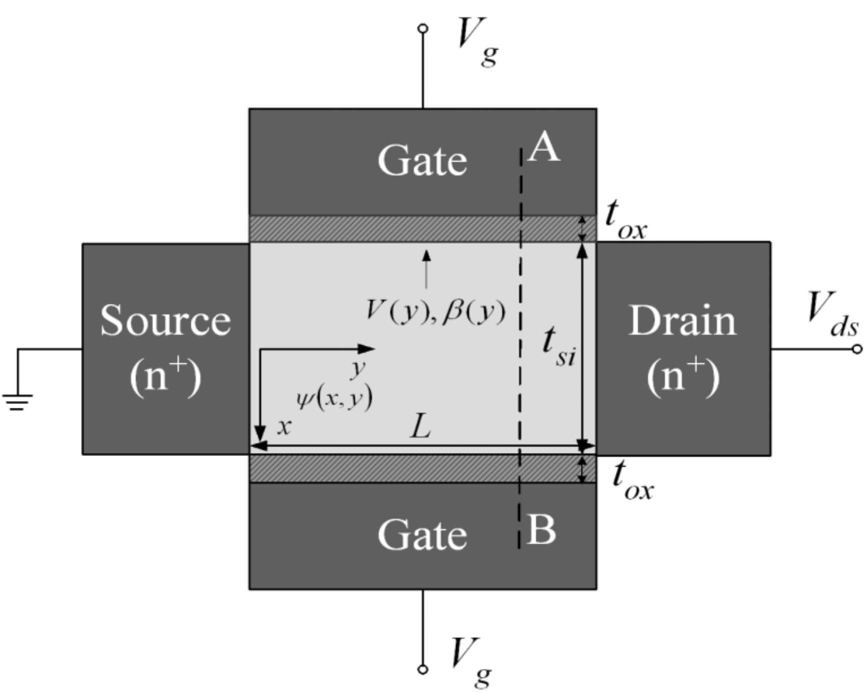
Analytic Solution for DG MOSFETs

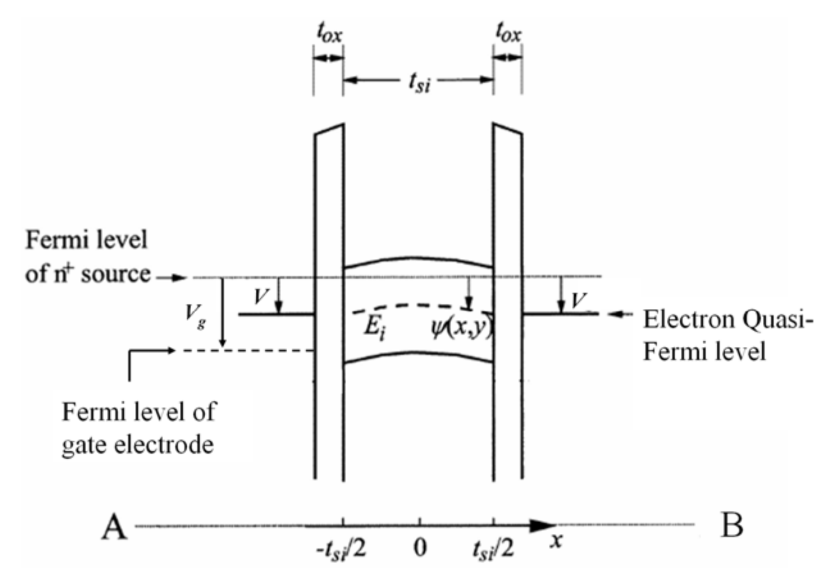
- **Poisson's equation under gradual channel approximation (GCA) with only mobile charge**

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_{si}} n_i e^{q(\psi - V)/kT}$$

***V*: quasi-Fermi potential**

- **Current continuity equation:** $I_{ds} = \mu W Q_i dV / dy = const.$





Drain Current for Planar DG MOSFETs

- **Drain current model**

$$I_{ds} = \mu \frac{W}{L} \frac{8\varepsilon_{si}}{t_{si}} \left(\frac{kT}{q} \right)^2 \left[-2\beta \tan\beta + \beta^2 - 2r\beta^2 \tan^2\beta \right] \Big|_{\beta_s}^{\beta_d}$$

$$\boxed{r = \varepsilon_{si} t_{ox} / \varepsilon_{ox} t_{si}}$$

β_s and β_d are given by the boundary condition

$$\frac{q(V_g - \Delta\varphi - V)}{2kT} - \ln\left[\frac{2L_{Di}}{t_{si}}\right] = \ln\beta - \ln[\cos\beta] + 2r\beta\tan\beta$$

for V=V$_s$, V$_d$

$$\boxed{L_{Di} = \sqrt{2\varepsilon_{si} kT / q^2 n_i}}$$

- **Continuous for all regions of MOSFET operation**
- **Implicit equations for β_s and β_d**

Taur et al., EDL 2004

Continuous Model

- **All regions of MOSFET operation are covered under one continuous function**

☐ Linear region: β_s, $\beta_d \sim \pi/2$

$$I_{ds} = 2\mu C_{ox}\frac{W}{L}(V_g - V_t - V_{ds}/2)V_{ds}$$

☐ Saturation: $\beta_s \sim \pi/2$, $\beta_d \ll 1$

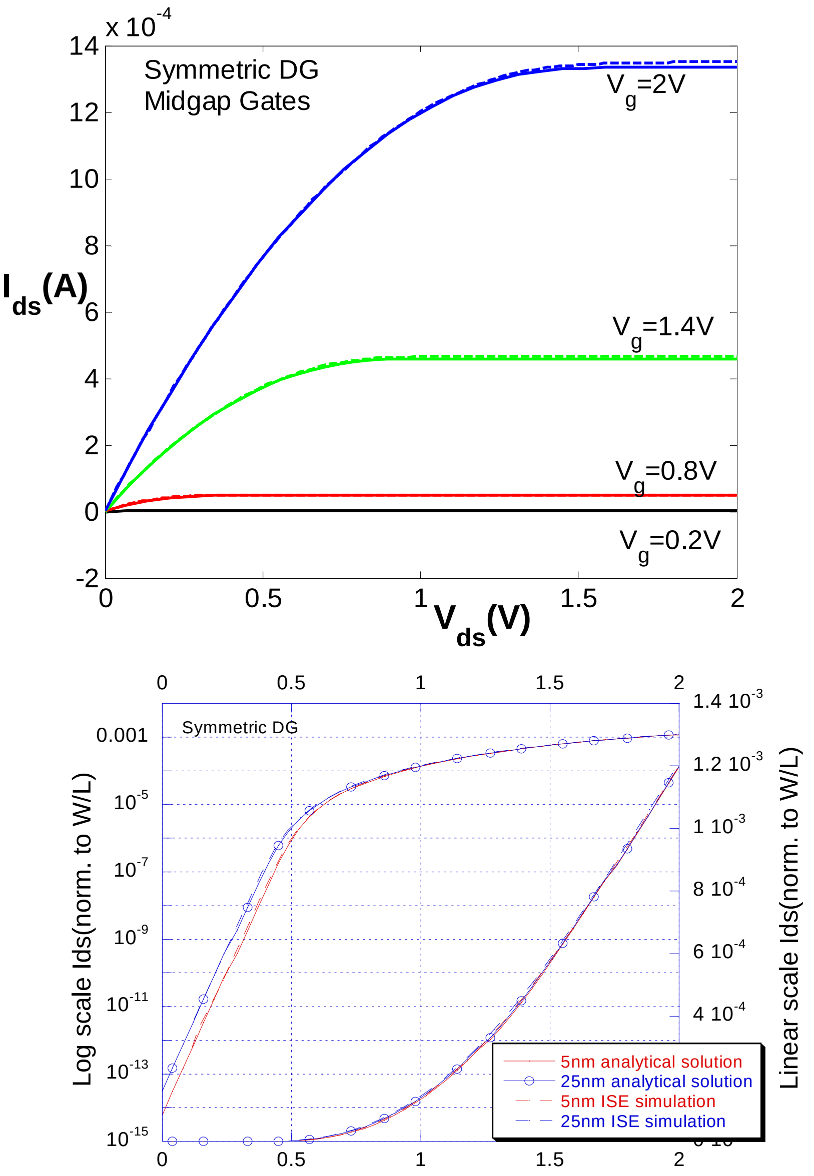
$$I_{ds} = \mu C_{ox}\frac{W}{L}\left[(V_g - V_t)^2 - \frac{8rk^2T^2}{q^2}e^{q(V_g - V_t - V_{ds})/kT}\right]$$

☐ Subthreshold: β_s, $\beta_d \ll 1$

$$I_{ds} = \mu\frac{W}{L}kTn_i t_{si}e^{q(V_g - \Delta\varphi)/kT}\left(1 - e^{-qV_{ds}/kT}\right)$$

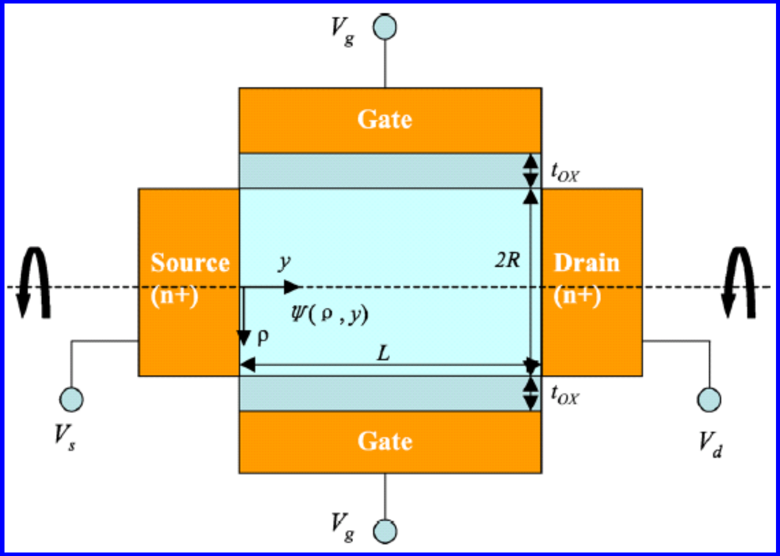
Saturation and "volume inversion" in subth. come naturally from the analytic, non-charge-sheet model.

Comparison with 2-D Numerical simulation



Analytic Solution for SG MOSFETs

- **Poisson’s equation under GCA**



$$\frac{d^2\psi}{d\rho^2}+\frac{1}{\rho}\frac{d\psi}{d\rho}=\frac{q}{\varepsilon_{si}}n_i e^{q(\psi-V)/kT}$$

- **Drain current model**

$$\boxed{s=2\varepsilon_{si}\ln\left(1+t_{ox}/R\right)/\varepsilon_{ox}}$$

$$I_{ds}=\mu\frac{8\pi\varepsilon_{si}}{L}\left(\frac{2kT}{q}\right)^2\left[\left(-2/\alpha-\ln\alpha\right)+s\left(-1/\alpha^2+2/\alpha\right)\right]\Big|_{\alpha_s}^{\alpha_d}$$

α_s and α_d are given by the boundary condition

$$\frac{q(V_g-\Delta\varphi-V)}{2kT}-\ln\left(\frac{2L_{Di}}{R}\right)=\ln\sqrt{1-\alpha}-\ln\alpha+s\frac{1-\alpha}{\alpha}$$

for V=V_s, V_d

Jimenez et al., EDL 2004

Explicit Solutions for DG and SG MOSFETs --- General Method

Solve $f(x;a,b,c)=0$; *x* is an implicit function of *a, b, c*; employ a general method similar to PSP's explicit solution of surface potential in the charge-sheet model.

1. Guess a continuous starting function $x_1 = g(a,b,c)$

- **Has the appropriate asymptotic behaviors**

2. Modify the starting function with correction term

$$x_2 = g(a,b,c) + h(a,b,c)$$

$$h = -\frac{f_{g0}}{f_{g1}}\left(1+\frac{f_{g0}\,f_{g2}}{2\,f_{g1}{}^{2}}+\frac{f_{g0}{}^{2}(3\,f_{g2}{}^{2}-f_{g1}\,f_{g3})}{6\,f_{g1}{}^{4}}\right)$$

$$f_{gn} = \left.\frac{\partial^n f(x;a,b,c)}{\partial x^n}\right|_{x=g(a,b,c)}$$

3. Another correction may be needed to further improve accuracy

$$x_3 = g(a,b,c) + h(a,b,c) + w(a,b,c)$$

Explicit Solutions for DG and SG MOSFETs --- Accuracy

- **Simplified equation for SG**

$$g(z;s,G)=\ln(\sqrt{z+z^2})+sz-G=0$$

1. $$z_1=\sqrt{\left(\frac{1}{2s^2}\right)^2+\left(\frac{1}{s}\right)^2\ln^2\left(1+e^G\right)}-\frac{1}{2s^2}$$

2.

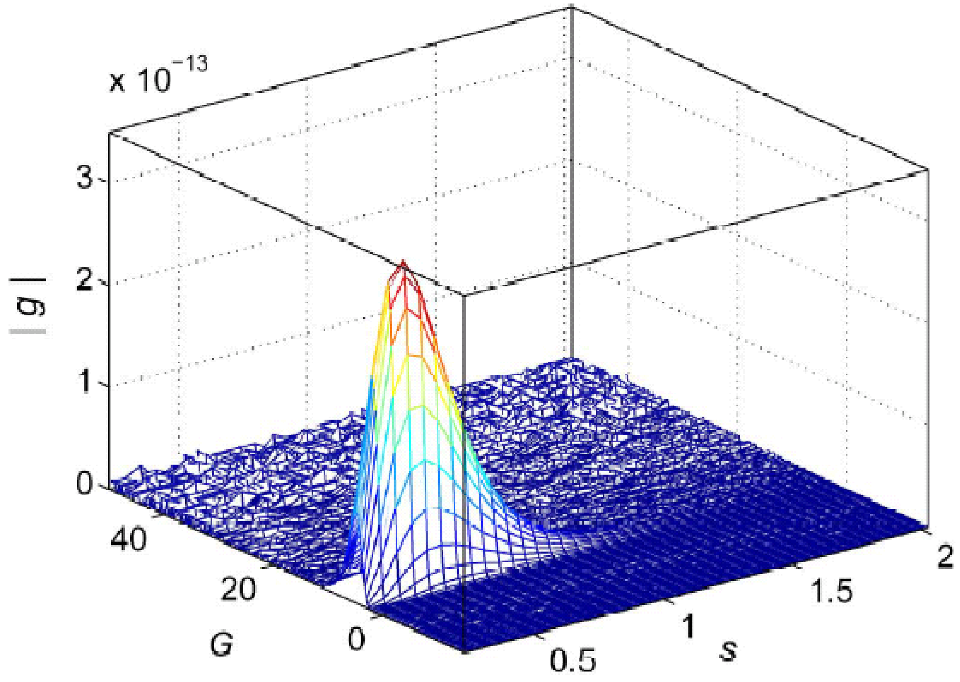
$$\eta_0=\frac{1}{2}\ln\left(z_1+z_1^2\right)+sz_1-G$$

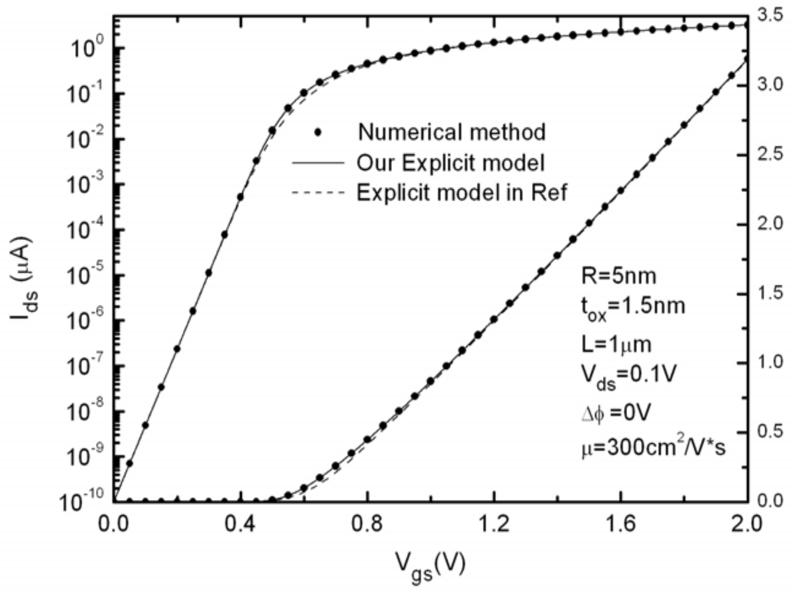
$$\eta_1=\frac{1}{2z_1}+\frac{1}{2\left(1+z_1\right)}+s$$

$$\eta_2=-\frac{1}{2z_1^2}-\frac{1}{2\left(1+z_1\right)^2}$$

$$\eta_3=\frac{1}{z_1^3}+\frac{1}{\left(1+z_1\right)^3}$$

$$z_2=z_1-\frac{\eta_0}{\eta_1}\left(1+\frac{\eta_0\eta_2}{2\eta_1^2}+\frac{\eta_0^2\left(3\eta_2^2-\eta_1\eta_3\right)}{6\eta_1^4}\right)$$





Charge and Capacitance Model

- **Ward-Dutton partition method**

$$Q_g = -W\int_0^L Q_i(y)\,dy \qquad Q_d = W\int_0^L \frac{y}{L} Q_i(y)\,dy \qquad Q_s = W\int_0^L \left(1-\frac{y}{L}\right) Q_i(y)\,dy$$

- **Definitions of transcapacitances**

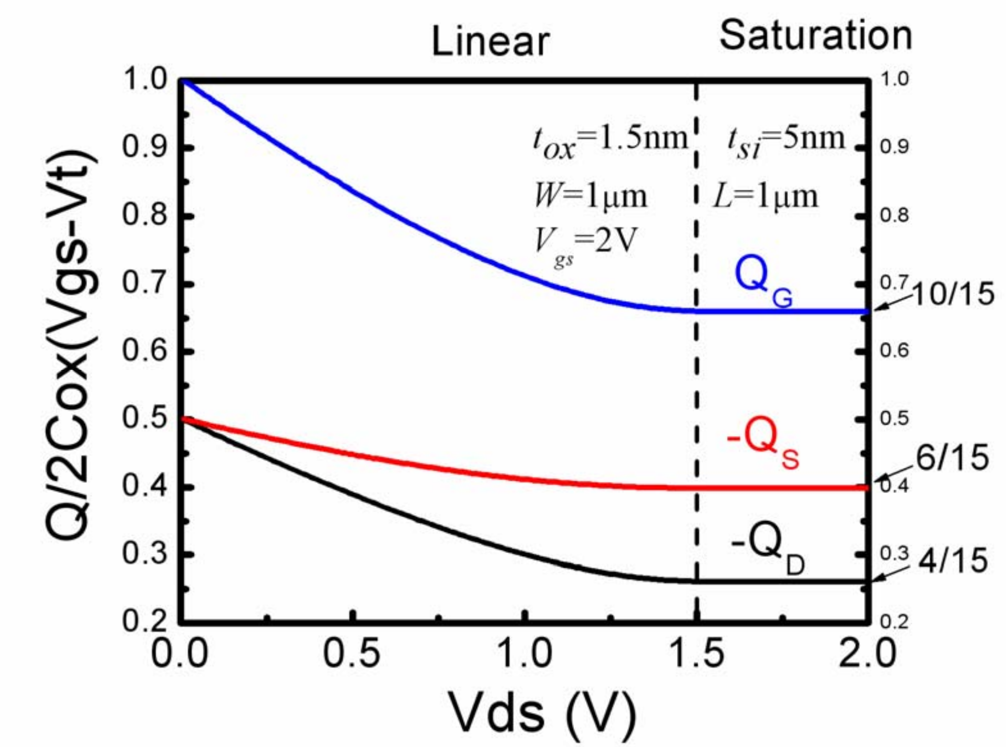
$$C_{kk} = \frac{\partial Q_k}{\partial V_k} \qquad C_{kl} = -\frac{\partial Q_k}{\partial V_l} \qquad l \neq k$$

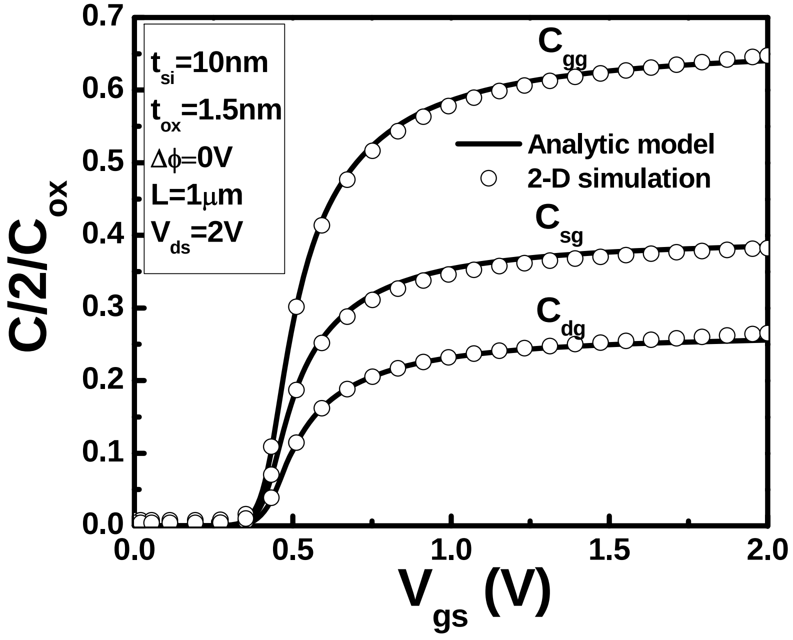
- **Capacitances can be derived as a function of Q, g and I**

$$C_{dd} = -\frac{L^2 g_{ds}^2}{\mu I_{ds}} - \frac{2Q_D}{I_{ds}} g_{ds} \qquad C_{dg} = -\frac{L^2 g_{ds}^2}{\mu I_{ds}} + \frac{Q_G}{I_{ds}}(g_m + g_{ds}) + \frac{2Q_D}{I_{ds}} g_m$$

$$C_{gd} = -\frac{L^2 g_{ds}^2}{\mu I_{ds}} + \frac{Q_G}{I_{ds}} g_{ds} \qquad C_{gg} = -\frac{L^2\left[\left(g_m + g_{ds}\right)^2 - g_{ds}^2\right]}{\mu I_{ds}} + \frac{Q_G}{I_{ds}} g_m$$

Charge/Capacitance Model Validation





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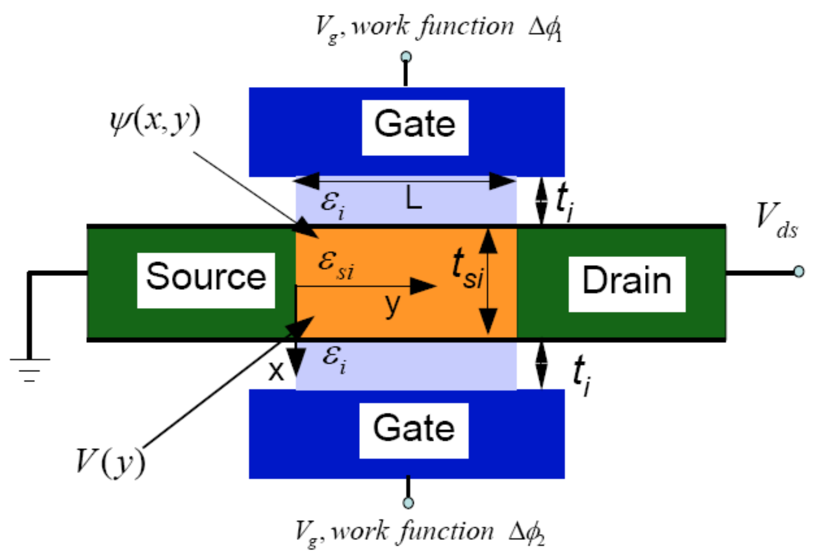
Short Channel Effect

- **2D Poisson's Eq. in subthreshold:**

$$\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\psi}{\partial y}\right)=0$$

- **Solve 2-D boundary value problem analytically:**



Top gate :	$\psi(-t_{si}/2-t_i,y)=V_g-\Delta\varphi_1$	$0<y<L$
Bottom gate:	$\psi(t_{si}/2+t_i,y)=V_g-\Delta\varphi_2$	$0<y<L$
Source :	$\psi(x,0)=E_g/2q$	$-t_{si}/2<x<t_{si}/2$
Drain :	$\psi(x,L)=V_{ds}+E_g/2q$	$-t_{si}/2<x<t_{si}/2$

Short Channel Effect

- **2-D potential function in the subthreshold region**

$$\psi(x,y)=\frac{\Delta\varphi_1-\Delta\varphi_2}{t_{si}+2t_i\varepsilon_{si}/\varepsilon_{ox}}x+V_g-\frac{\Delta\varphi_1+\Delta\varphi_2}{2}+$$

$$\frac{b_1\sinh\left[\pi\left(L-y\right)/\lambda_1\right]+c_1\sinh\left(\pi y/\lambda_1\right)}{\sinh\left(\pi y/\lambda_1\right)}\cos\left(\pi x/\lambda_1\right)$$

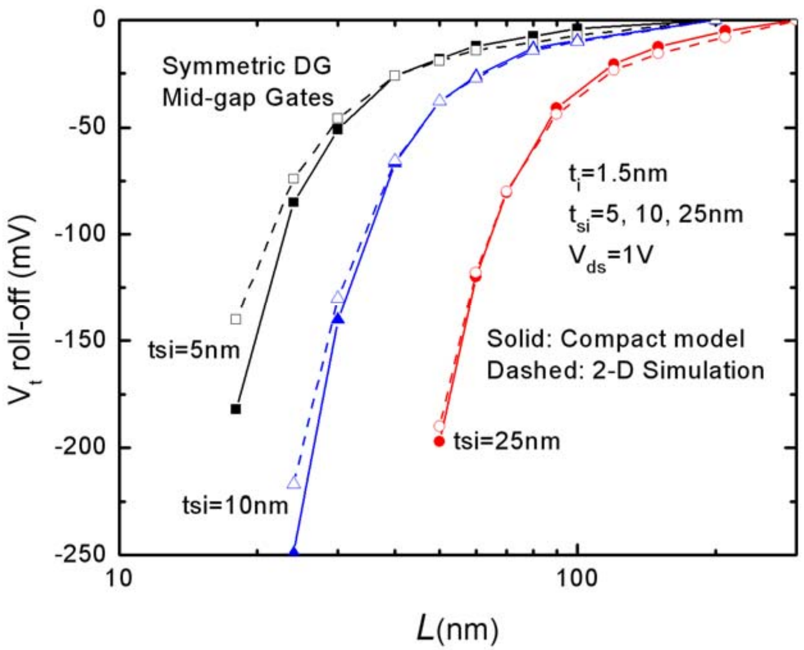
$$\boxed{b_1,c_1=B\,(E_g/2q+V_{s,d}+\frac{\Delta\phi_1+\Delta\phi_2}{2}-V_{g0})}$$

- **Scale length**

$$\tan(\frac{\pi t_{ox}}{\lambda_1})\tan(\frac{\pi t_{si}}{2\lambda_1})=\frac{\varepsilon_{ox}}{\varepsilon_{si}}$$

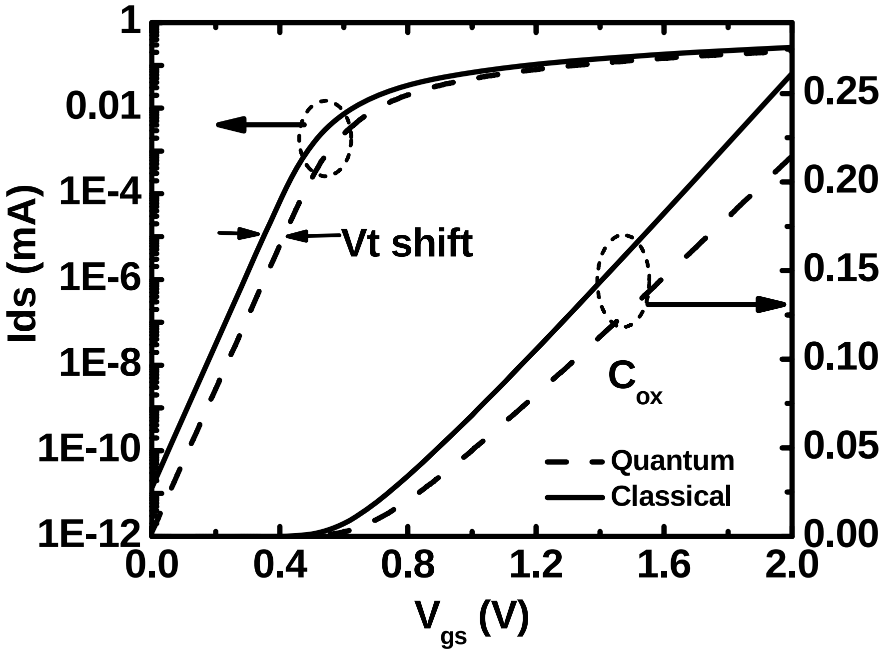
- **Subthreshold current**

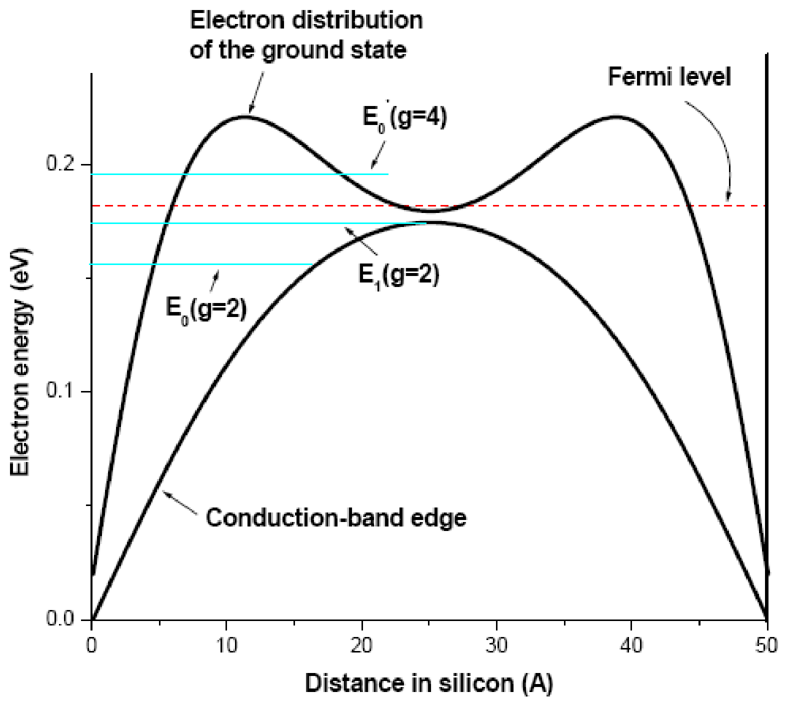
$$I_{ds}=\frac{\mu W(kT/q)[1-\exp(-qV_{ds}/kT]}{\int_0^L\frac{dy}{\int_{-t_{si}/2}^{t_{si}/2}n_i e^{q\psi(x,y)/kT}dx}}$$



Quantum Effect: Poisson + Schrodinger eqs.

- Slope: QM effect degrades the inversion capacitance because of deeper charge centroid.





- Vt shift: QM effect causes higher Vt due to higher ground state energy.

Quantum Effect

- **Compact model for Vt shift**

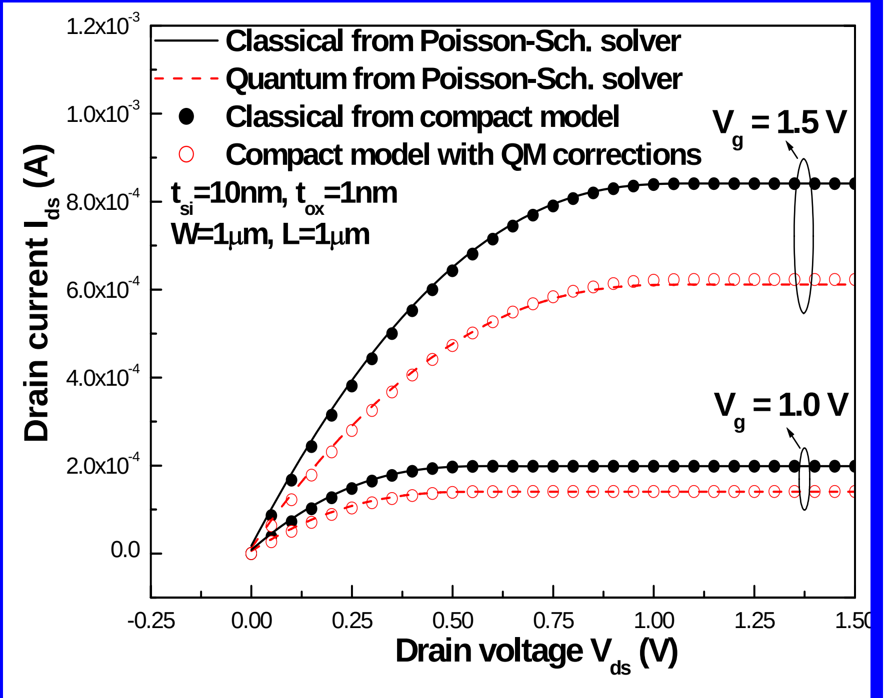
$$\Delta V_t = \frac{E_1}{q} = \frac{\hbar^2 \pi^2}{2qm^* t_{si}{}^2}$$

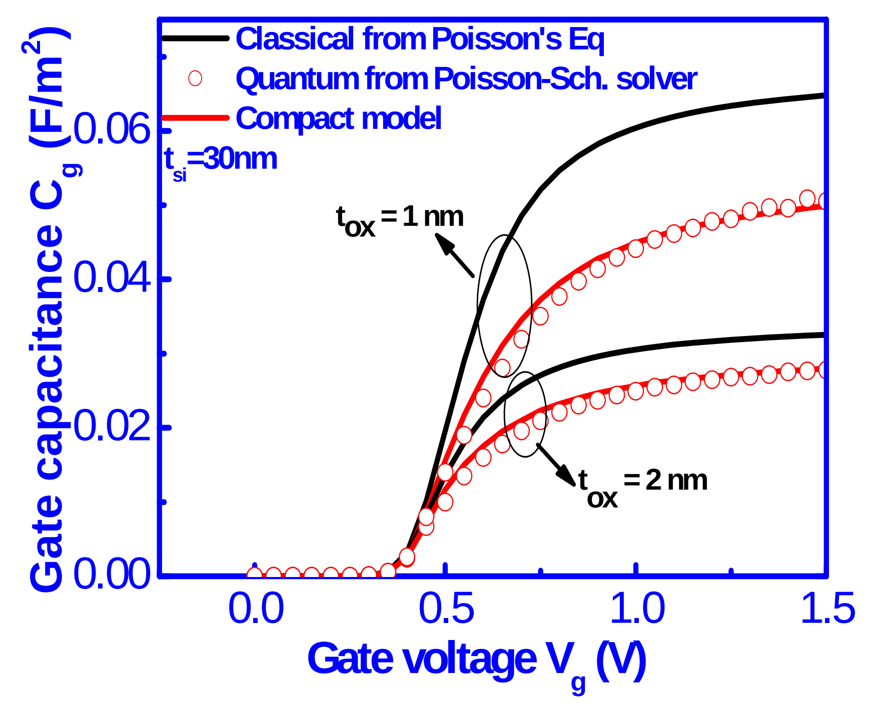
- **Compact model for gate capacitance degradation**

Effective oxide thickness

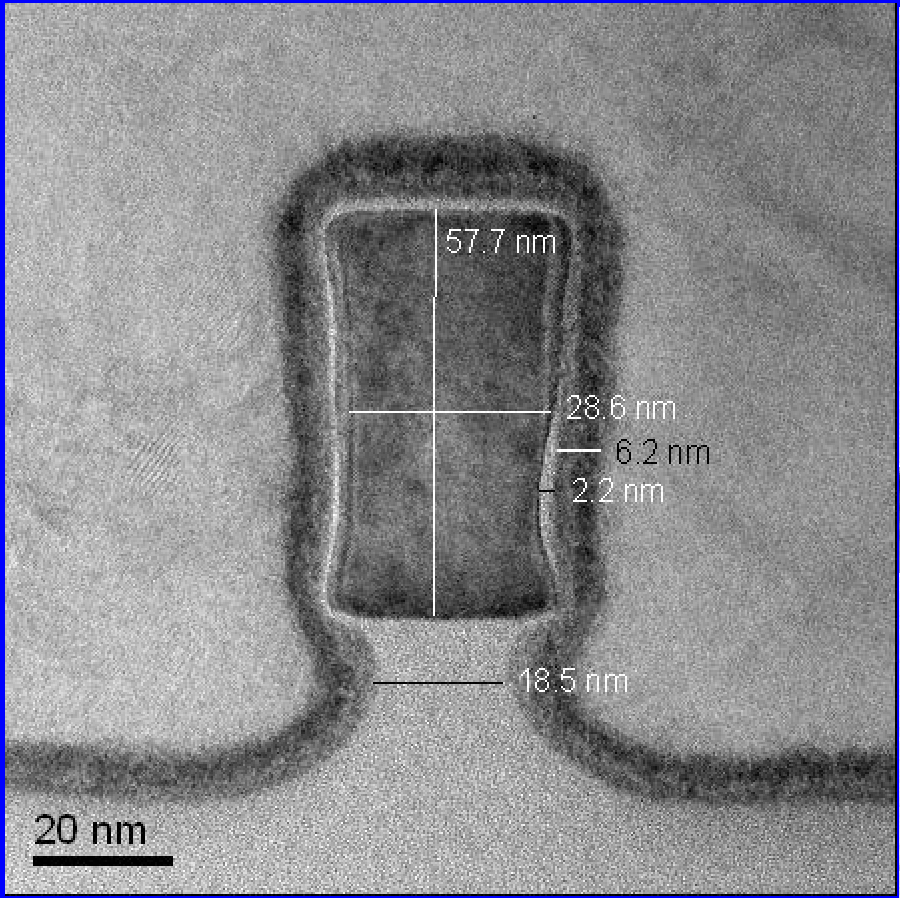
$$t_{oxeff} = t_{ox} + \frac{\varepsilon_{si}}{\varepsilon_{ox}} \delta t_{inv}$$

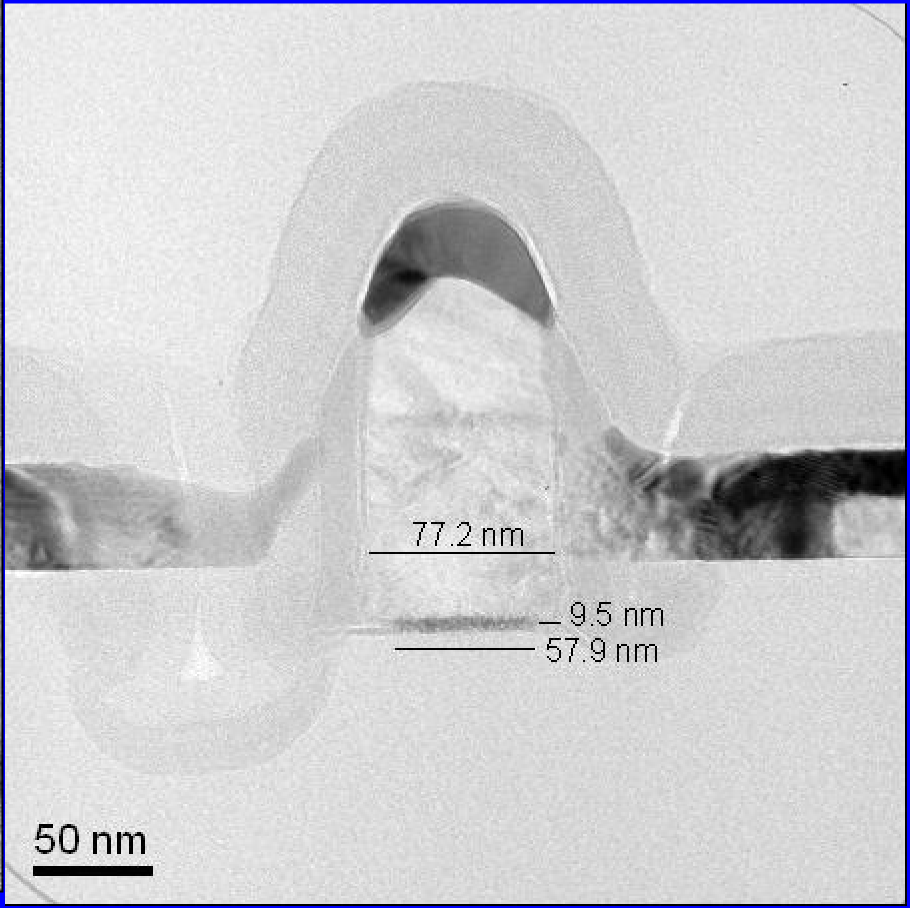
$$\boxed{\delta t_{inv} = \left(\frac{21 \varepsilon_{si} \hbar^2}{2m^* q Q_i} \right)^{1/3}}$$





FinFET Hardware Calibration





Acknowledgement: Wade Xiong, Texas Instruments.

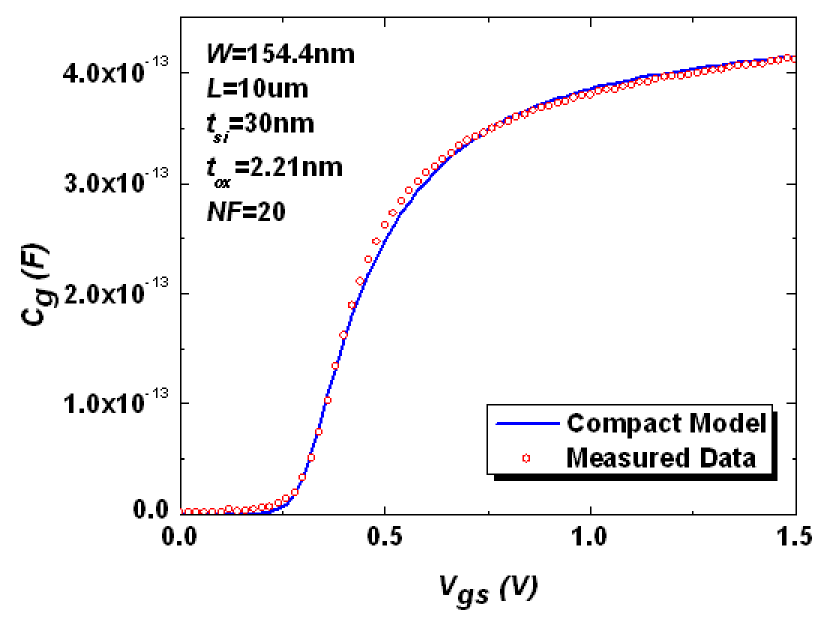
Hardware Calibration Procedure

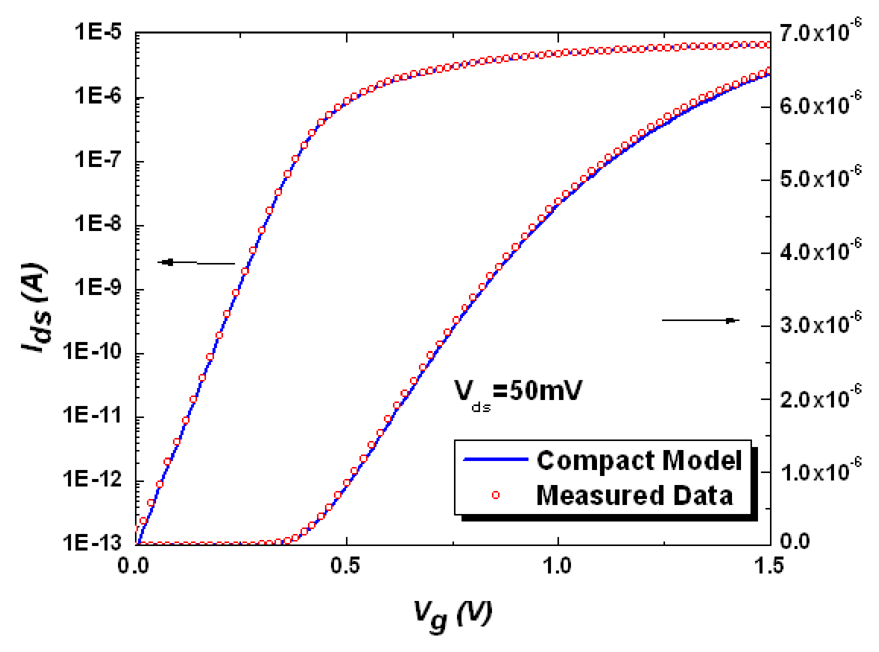
- **Start with C-V Data:**
- ➢ **Calibrate Oxide Thickness, Effective Device Width, with Quantum Mechanical Effects.**

- **Long Channel Devices:**
- ➢ **Calibrate Mobility from low drain-bias I_{ds}-V_g data.**

- **Short Channel Devices:**
- ➢ **Low Drain I_{ds}-V_g --- Calibrate Source-Drain series resistance.**
- ➢ **SCE: V_t rolloff, DIBL, subthreshold slope --- Determine effective channel length consistent with data.**
- ➢ **High Drain I_{ds}-V_g and I_{ds}-V_d --- Calibrate saturation velocity, channel length modulation.**

FinFET hardware calibration

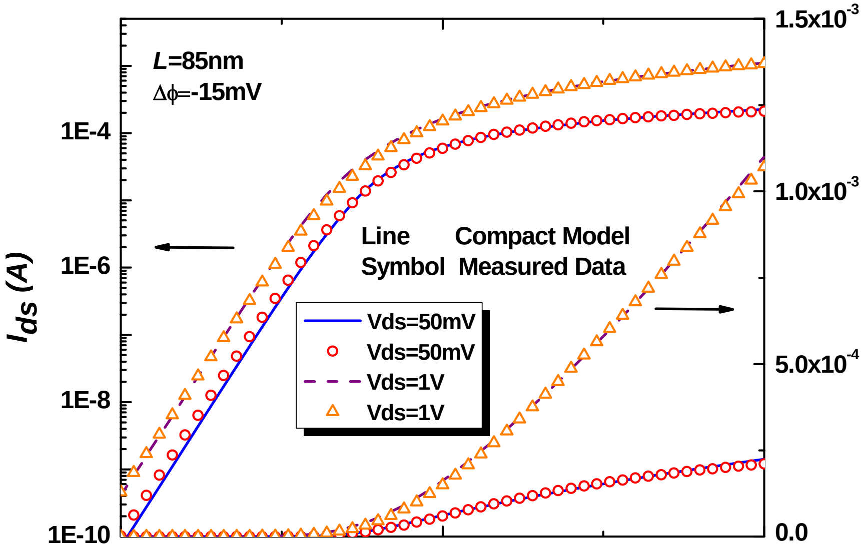


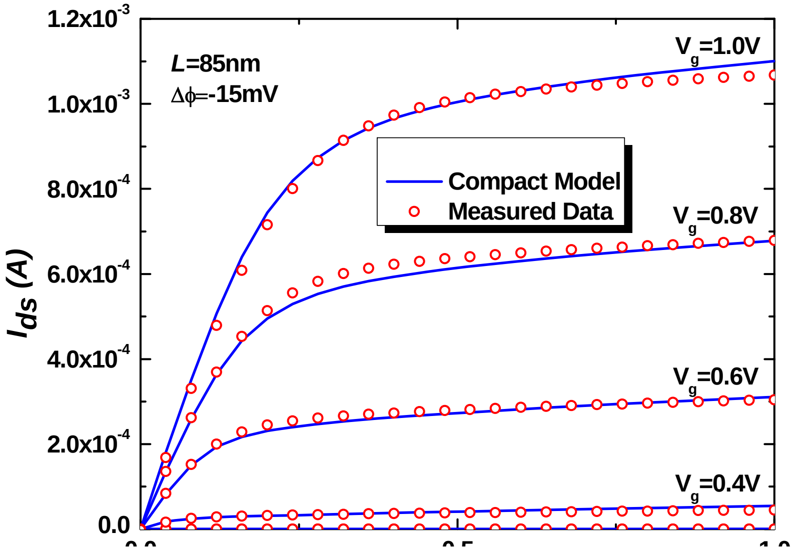


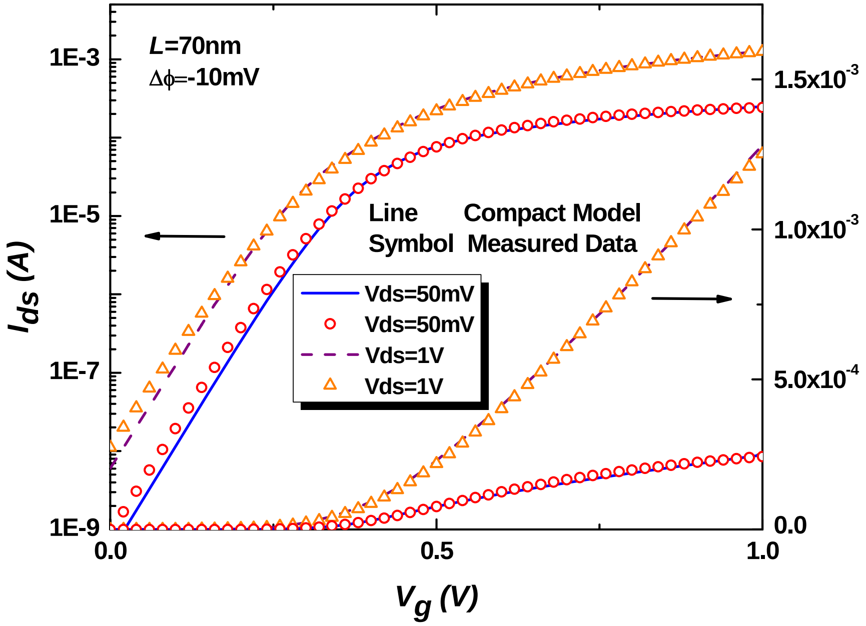
- Quantum-mechanical models are necessary to obtain agreement with experimental C-V data.
- Mobility model consists of impurity scattering and phonon scattering terms.

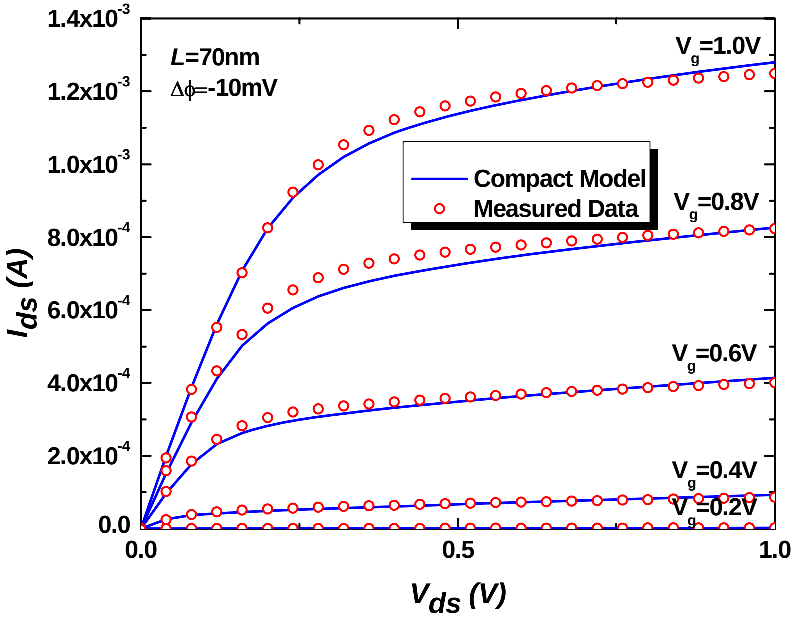
$$\mu = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2} \quad \textbf{where} \quad \mu_1 = \frac{740}{1 + \left(\frac{E_{eff}}{0.52e6}\right)^{1.25}} \quad \mu_2 = 540\left(1 + \left(\frac{E_{eff}}{0.075e6}\right)^{1.55}\right)$$

FinFET hardware calibration









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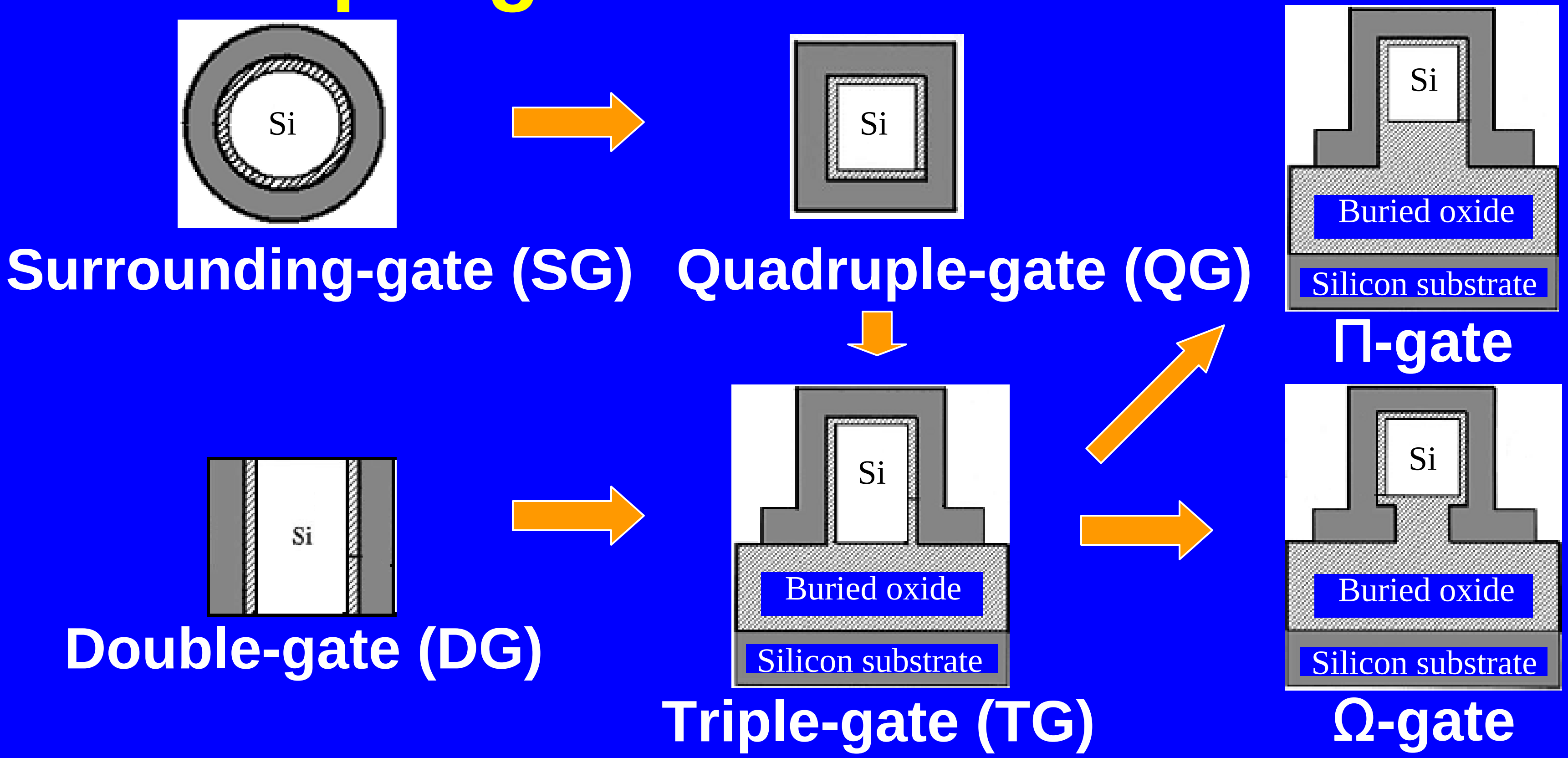


Surrounding-gate (SG)

Quadruple-gate (QG)

Π-gate

Double-gate (DG)

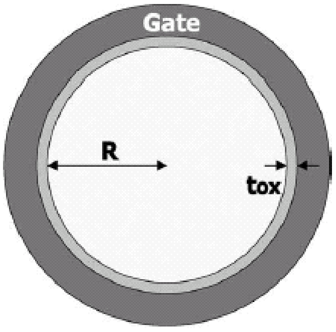
Triple-gate (TG)

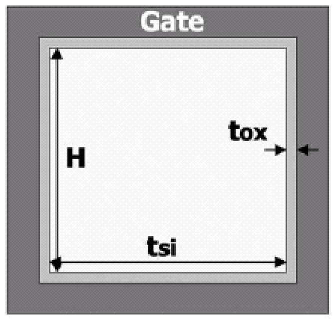
Ω-gate

- **DG and SG MOSFETs can be exactly solved.**
- **QG model based on modified SG model.**
- **TG represented by the average of QG and DG.**
- **Π-gate and Ω-gate models extended from TG model.**

From SG to QG

- **Asymptotic behaviors:**
  - **Volume inversion: In subthreshold, Q_m is proportional to the cross section area of silicon body.**
  - **Surface inversion layer: Far above threshold, Q_m is proportional to the perimeter of silicon body. (no visible corner effect)**

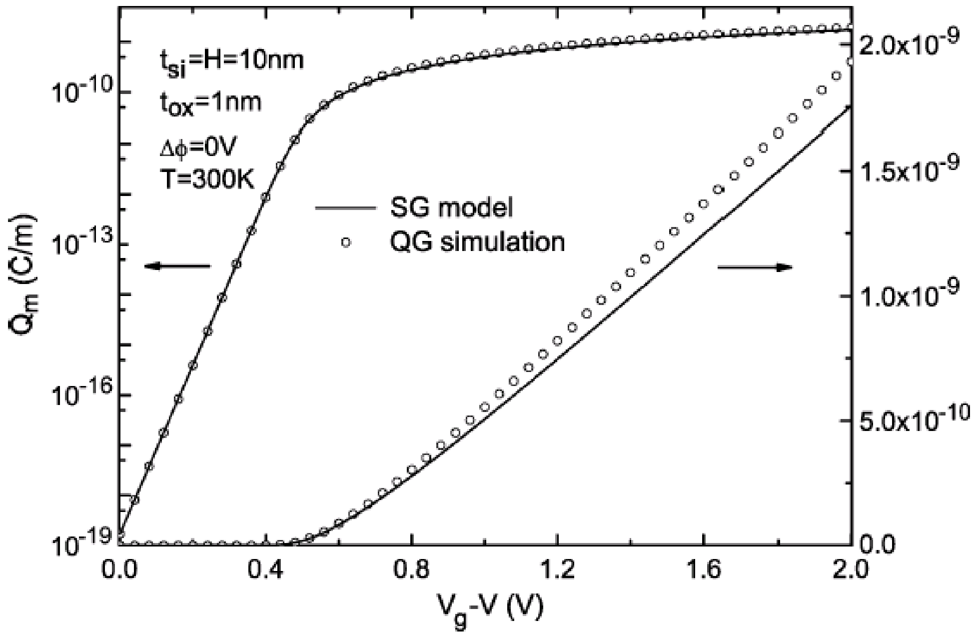




- **Compare Qm-Vg Curves of QG and SG:**
  - **QG parameters: t_{si}, H, and t_{ox}.**
  - **SG parameters:**

$R = \sqrt{t_{si} H / \pi}$ **and t_{ox}.**

Same cross section area as QG



From SG to QG

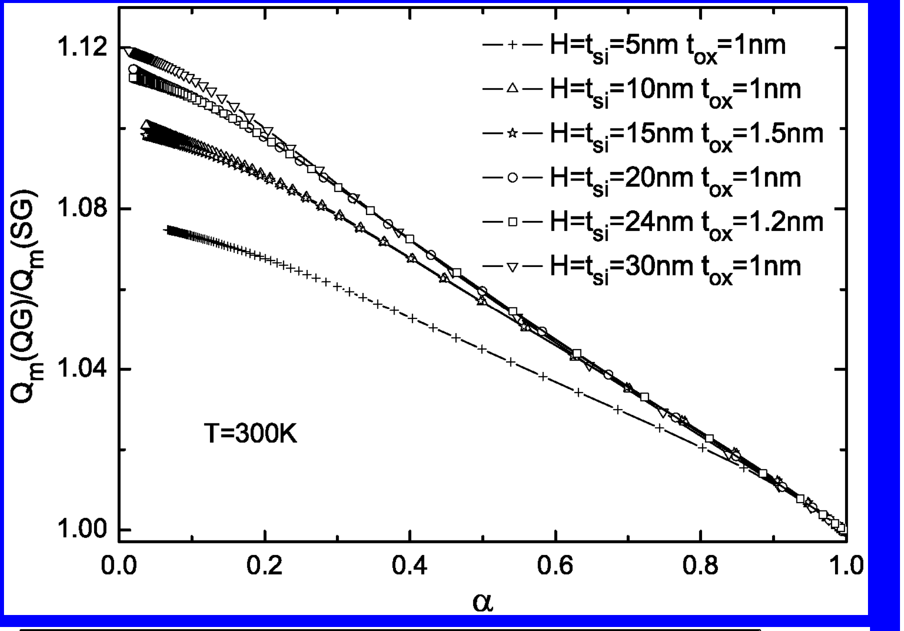
- $Q_m(QG)/Q_m(SG)$ vs α:
  - ➢ Almost linear behavior

$$Q_m(QG)/Q_m(SG) \cong 1 + C_1(1-\alpha)$$

$\alpha \approx 1$ → **subthreshold**

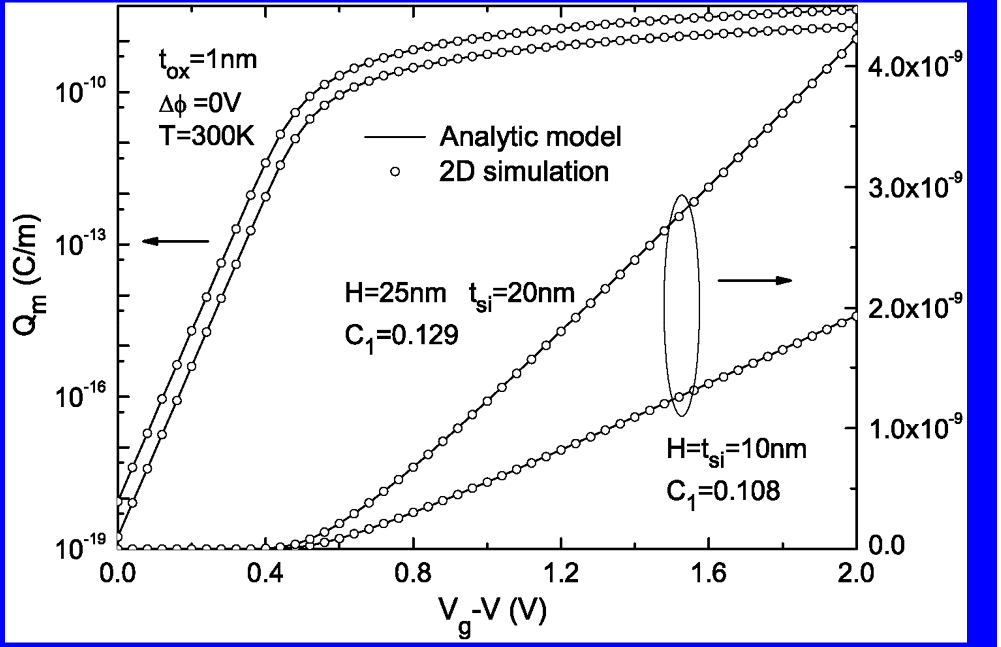
$\alpha << 1$ → **above threshold**

  - ➢ C_1 is a function of t_{si}/t_{ox} and H/t_{ox}, determined by 2D simulation.



- Analytic model for QG MOS:

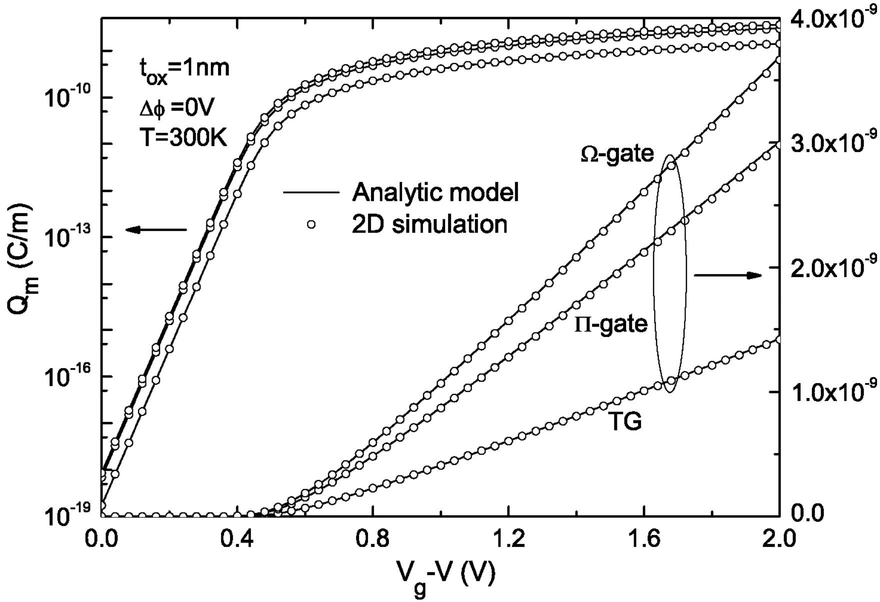
$$Q_m(QG) = 8\pi\varepsilon_{si}\frac{kT}{q}\frac{1-\alpha}{\alpha}\times\left[1+C_1(1-\alpha)\right]$$



Analytic Model for TG and TPG MOS Capacitor

- Approximated by [(1+C_2)*QG+(1-C_2)*DG]/2; C_2 determined by simulation

$$Q_m = (1+C_2)4\pi\varepsilon_{si}\frac{kT}{q}\frac{1-\alpha}{\alpha}\left[1+C_1(1-\alpha)\right]+(1-C_2)4\varepsilon_{si}\frac{H}{t_{si}}\frac{kT}{q}\beta\tan\beta$$



Device	H	t_{si}	t_{ox}	C_1	C_2
DG	W	t_{si}	t_{ox}	0	-1
SG	$\sqrt{\pi}R$	$\sqrt{\pi}R$	t_{ox}	0	1
QG	H	t_{si}	t_{ox}	C_1	1
TG	H	t_{si}	t_{ox}	C_1	0
TPG	H	t_{si}	t_{ox}	C_1	C_2

π-gate: C_2=0.218

Ω-gate: C_2 =0.440

Unified Analytic Drain Current Model

- **Based on MOS capacitor behaviors, one can carry out Pao-Sah integral to obtain current**

$$I_{ds} = \frac{\mu}{L}\int_{V_s}^{V_d} Q_m(V_g, V)\,dV$$

$$I_{ds} = \mu\frac{4\pi\varepsilon_{si}}{L}\left(\frac{kT}{q}\right)^2 (1+C_2)\left[f(\alpha)+C_1 d(\alpha)\right]\Big|_{\alpha_s}^{\alpha_d}$$

$$+\mu\frac{4\varepsilon_{si}}{L}\frac{H}{t_{si}}\left(\frac{kT}{q}\right)^2 (1-C_2)\left[p(\beta)\right]\Big|_{\beta_s}^{\beta_d}$$

$$\boxed{f(\alpha) = \left(-2/\alpha - \ln\alpha\right) + s\left(-1/\alpha^2 + 2/\alpha\right)}$$

$$\boxed{p(\beta) = -2\beta\tan\beta + \beta^2 - 2r\beta^2\tan^2\beta}$$

$$\boxed{\begin{aligned} d(\alpha) = &\left(-\frac{2}{\alpha} - 3\ln\alpha + \alpha\right) \\ &+ s\left(-\frac{1}{\alpha^2} + \frac{4}{\alpha} + 2\ln\alpha\right)\end{aligned}}$$

α and β at source/drain are calculated from the boundary conditions

A core long-channel compact model for DG, SG, QG, TG, and TPG MOSFETs

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- INTRODUCTION
- SYMMETRIC MG MOSFETS: PLANAR AND CYLINDRICAL
- A COMPLETE DOUBLE-GATE COMPACT MODEL WITH HARDWARE CALIBRATION
- GENERALIZATION OF CORE MODEL TO MULTIPLE-GATE MOSFETS
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BSIM-CMG

- **BSIM-CMG includes doping effect with a perturbation term to the undoped Poisson’s Eq. :**

$$\frac{d^2\psi}{dx^2}=\frac{qn_i}{\varepsilon_{si}}\left(e^{\frac{q\psi}{kT}}e^{\frac{-q\phi_B}{kT}}e^{\frac{-qV_{ch}}{kT}}+e^{\frac{q\phi_B}{kT}}\right)$$

- **Three coupled implicit equations need to be solved**

$$\psi_{inv}=\psi_1\big|_{x=-\frac{T_{si}}{2}}=\psi_0-\frac{2kT}{q}\ln\left(\cos\left(\sqrt{\sqrt{\frac{q^2}{2\varepsilon_{Si}kT}\frac{n_i^2}{N_A}e^{\frac{-qV_{ch}}{kT}}e^{\frac{qV_0}{2kT}}\frac{T_{Si}}{2}}}\right)\right)$$

$$\psi_C=\frac{2qn_i}{\varepsilon_{Si}}\frac{e^{\frac{q\phi_B}{kT}}}{a}\left(\frac{e^{\frac{T_{Si}}{2}\frac{\sqrt{a}}{2}}-1}{2e^{\frac{T_{Si}}{2}\frac{\sqrt{a}}{2}}}\right)^2 \quad \textbf{where} \quad a=\frac{q^2n_i}{\varepsilon_{Si}kT}e^{\frac{q(\psi_{inv}-V_{ch}-\phi_B)}{kT}}$$

$$V_g=V_{FB}+\psi_s+\frac{\varepsilon_{Si}}{C_{ox}}\sqrt{\frac{2qn_i}{\varepsilon_{Si}}\left(\frac{e^{\frac{q\psi_s}{kT}}-e^{\frac{q\psi_0}{kT}}}{q/kT}e^{\frac{-q\phi_B}{kT}}e^{\frac{-qV_{ch}}{kT}}+e^{\frac{q\psi_B}{kT}}(\psi_s-\psi_0)\right)}$$

Charge-Sheet Current Expression

- **Charge Sheet Current used in BSIM-CMG and others**

$$I_{ds,CSA} = -\mu W\left(Q\frac{d\psi_s}{dy} - \frac{kT}{q}\frac{dQ}{dy}\right)$$

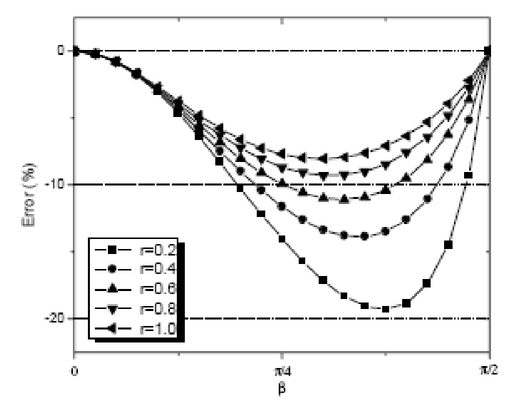
  - **Not generally consistent with the rigorous integration of drift-diffusion current density because *dψ/dy* is a function of *x*.**

$$I_{ds} = -\mu W\int_{t_{Si}/2}^{-t_{Si}/2} qn\left(\frac{d\psi}{dy} - \frac{kT}{qn}\frac{dn}{dy}\right)dx = -\mu W\left(\int_{t_{Si}/2}^{-t_{Si}/2} qn\frac{d\psi}{dy}dx - \frac{kT}{q}\frac{dQ}{dy}\right)$$

- **From the Analytic Potential Model**
  - **Current Expressions in terms of *β***

$$I_{ds,CSA} = \mu\frac{W}{L}\frac{8\varepsilon_{si}}{t_{si}}\left(\frac{kT}{q}\right)^2\left|-\beta\tan\beta - 2r\beta^2\tan^2\beta\right|_{\beta_s}^{\beta_d}$$

$$I_{ds} = \mu\frac{W}{L}\frac{8\varepsilon_{si}}{t_{si}}\left(\frac{kT}{q}\right)^2\left|-2\beta\tan\beta + \beta^2 - 2r\beta^2\tan^2\beta\right|_{\beta_s}^{\beta_d}$$



  - **As much as ~20% error in the transition region.**

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Compact Modeling of Multiple-Gate MOSFETs

- For highly symmetric planar DG and cylindrical SG MOSFETs, Poisson's and current continuity eqs. have been solved analytically to derive continuous I–V models for all regions without the charge-sheet approximation. Accurate explicit solutions have been developed for both DG and SG MOSFETs.

- The DG core model has been expanded into a complete compact model with short-channel and quantum effects, charge and capacitance models. The full DG model has been calibrated with FinFET hardware data.

- Based on the asymptotic behavior of inversion charge density and with combinations of the core DG and SG models, a unified approach has been presented to develop core models for the less symmetric quadruple-gate, triple-gate, Π-gate, and Ω-gate MOSFETs.