Design and Analysis of Accelerated Reliability Tests with Piecewise Linear Failure Rate Functions

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Problem Solving Tools

DART Abstract

- Part 1 proposes piecewise linear failure rate (PLFR) functions, for modeling simplicity and resemblance to the left-hand end of the bathtub curve. It shows how to estimate the PLFR, reliability, infant mortality, and MTBF. It proposes acceleration alternatives, including one that accelerates testing greatly.
- Part 2 describes how to design and analyze accelerated reliability tests, assuming a piecewise linear failure rate function and power law acceleration. It shows how to obtain credible results, with limited sample size and test time, *at one accelerated stress level*. It provides estimators for parameters, reliability, MTBF, confidence intervals, and it shows how to test model assumptions and verify MTBF. It is inspired by the observation that failure rates are not constant, often because of infant mortality.

Part 1 Contents

- Motivation for PLFR
- MTBF and reliability for PLFR
- Acceleration of PLFR and RAF

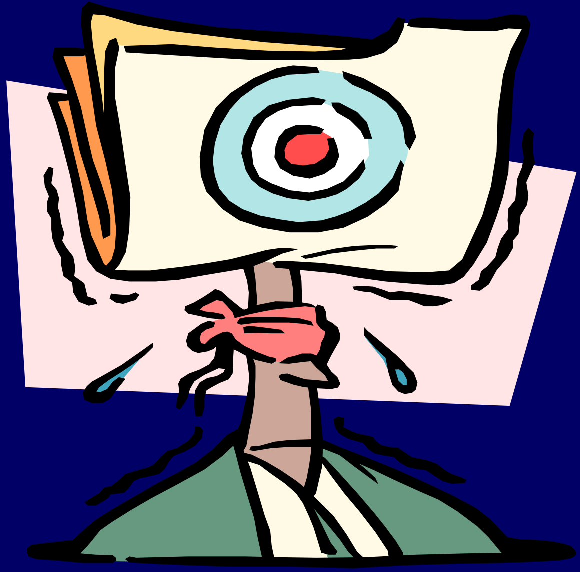
Vision Statement

- Know and use reliability, "the probability of successful function [according to customers] to specified ages [DoA, warranty, PM, useful life] under specified conditions [field conditions]."
- Management wants reliability before field data are available
  - So predict and test
- ***No fair to say you don't have ages at failures! [http://www.fieldreliability.com]***

DART Objectives

- Make *credible* MTBF, reliability, and failure rate function estimates
  - (*Credible Reliability Prediction,* http://www.asq-rd.org/publications.htm and http://www.fieldreliability.com/Preface.htm)
  - Quantify infant mortality
  - Verify MTBF
- Use accelerated tests with only one, high stress level
- Use available information early in life cycle

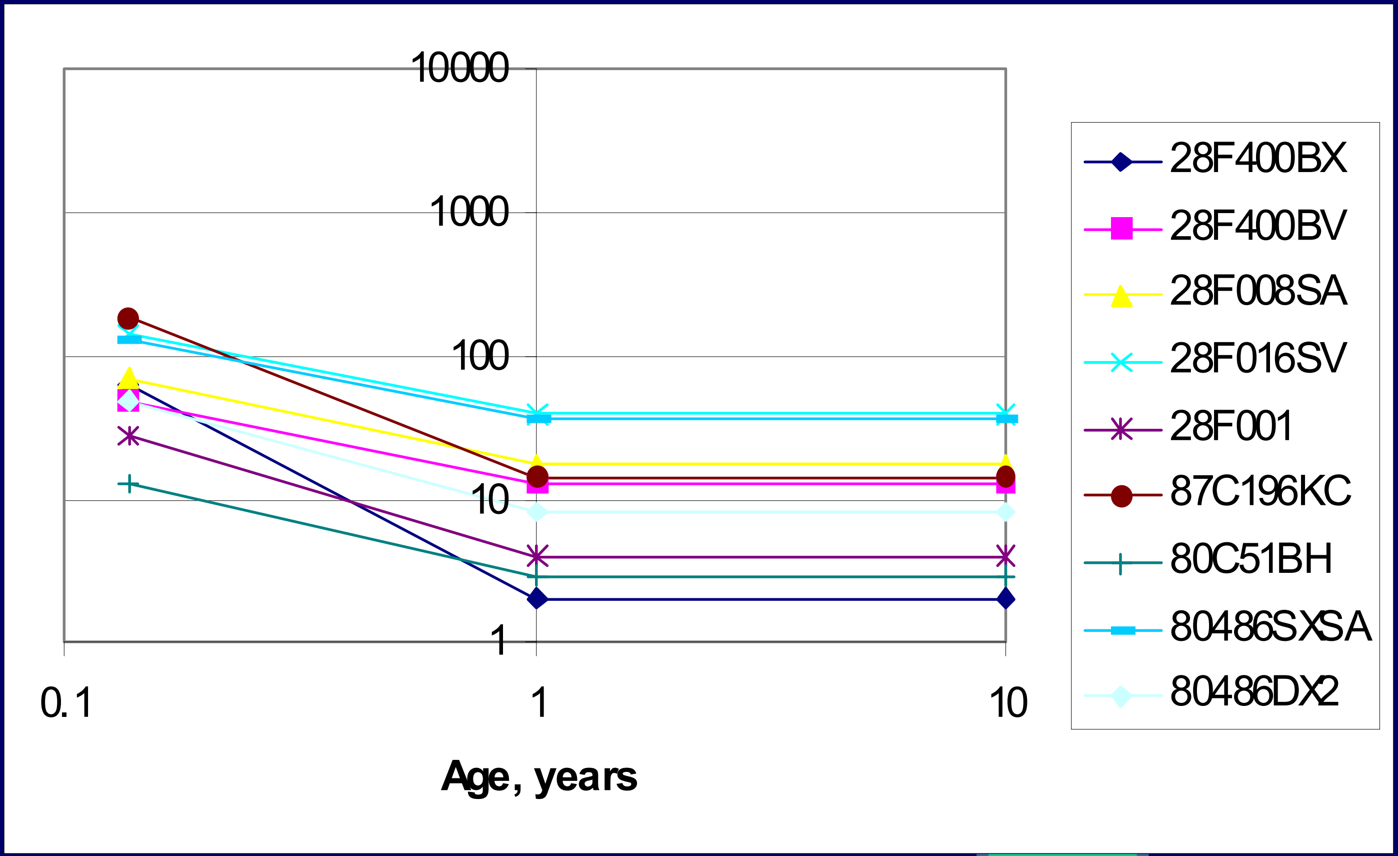
Today's Situation?



- Verify MTBF with tests that end << MTBF, even accelerated, without failures?
- Verify P[Life > useful life] > 0.9 with high confidence with small samples and short tests?
  - Does management ever agrees to statistical sample size and test time?
- Extrapolate accelerated tests, at high stress, to working stress, with few failures << MTBF?
  - NIST [http://www.itl.nist.gov/div898/handbook/index.htm] ASQ [Meeker and Hahn], and others [Nelson, Bagdonavicius et al, Viertl] recommend at least two accelerated stress levels

Intel FITS have IM

Data used to be at http://www.intel.com/support

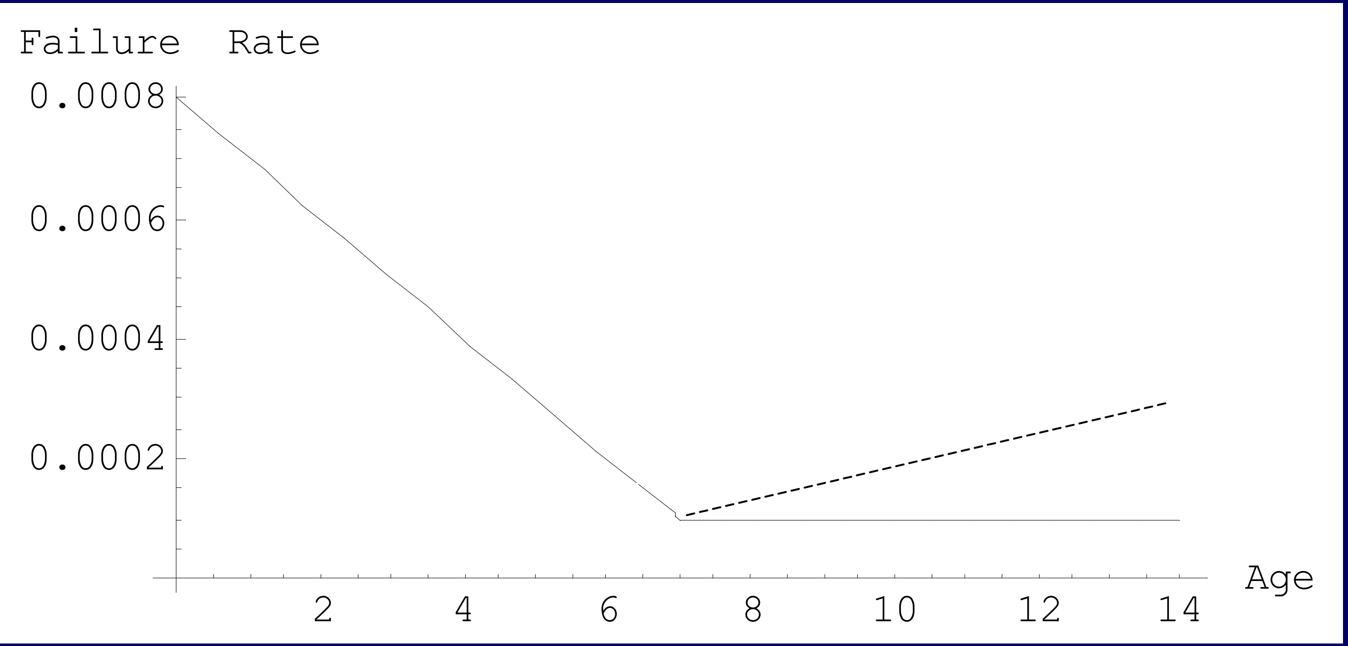


Invalid Assumptions

- Constant failure rate
  - Products have infant mortality, initially ↓ failure rate.
- Monotonic ↑ or ↓ failure rate
  - Products often have both (rules out Weibull) [George 1995]
- Acceleration doesn't affect Weibull shape parameter
  - It does, usually [http://www.esc.auckland.ac.nz/Organisations/ORSNZ/Newsletters/dec99.pdf]
- Can't extrapolate to normal stress with only one accelerated stress level (one hand clapping)
  - We will.

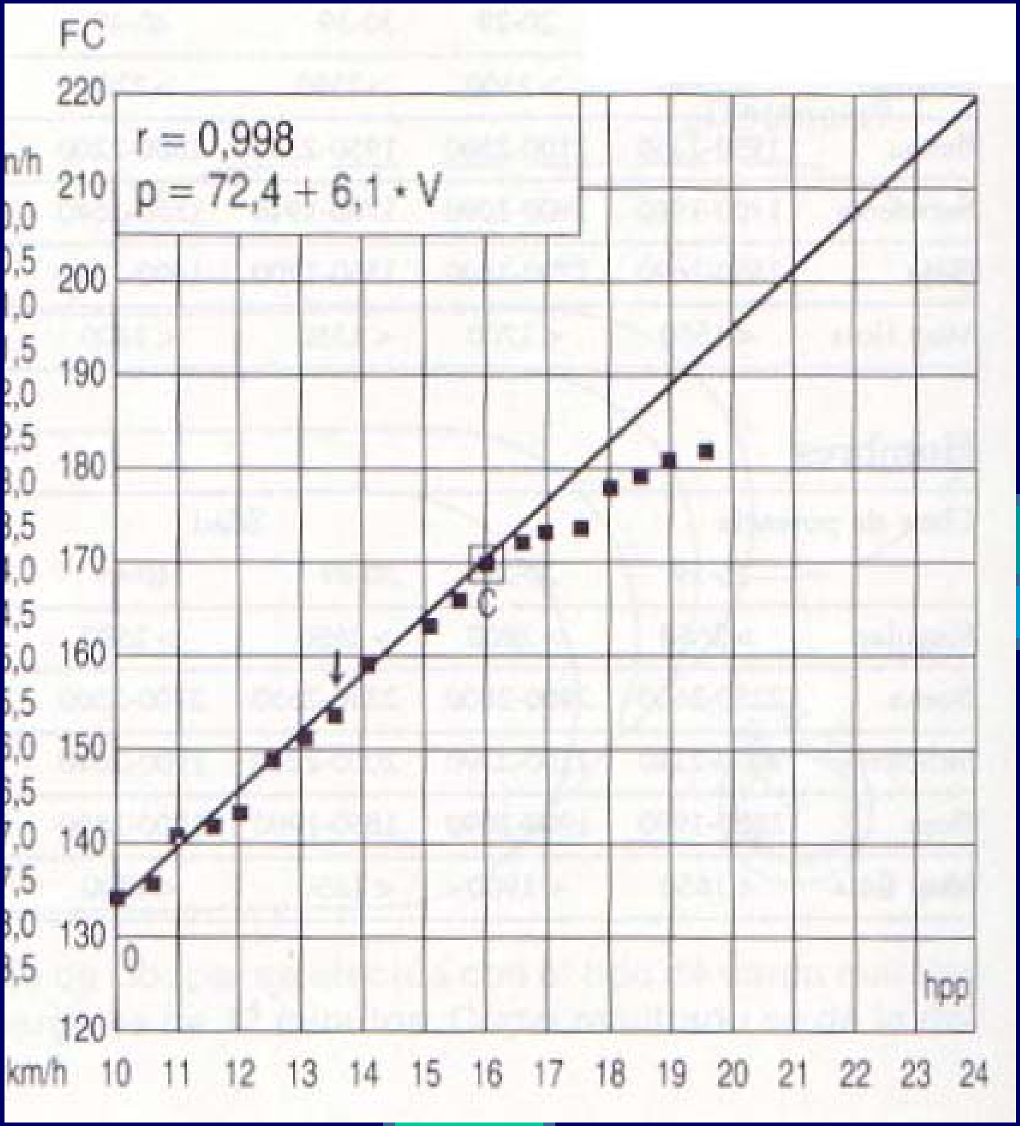
Piecewise Linear Failure Rate

- $a(t) = 0.0001+0.0001(7-t)^{+}$
- Dotted line is a possibly ↑ failure rate



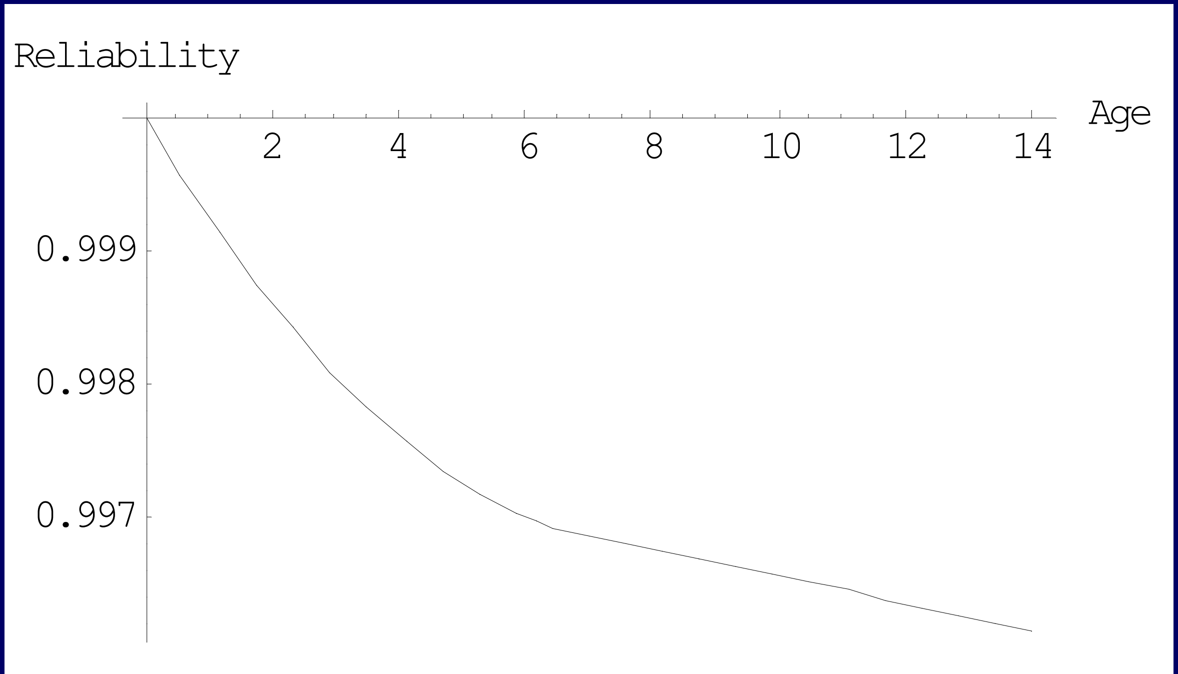
Test Conconi

- Aerobic threshold is the heart rate at which the slope of work rate vs. heart rate decreases



Reliability with PLFR

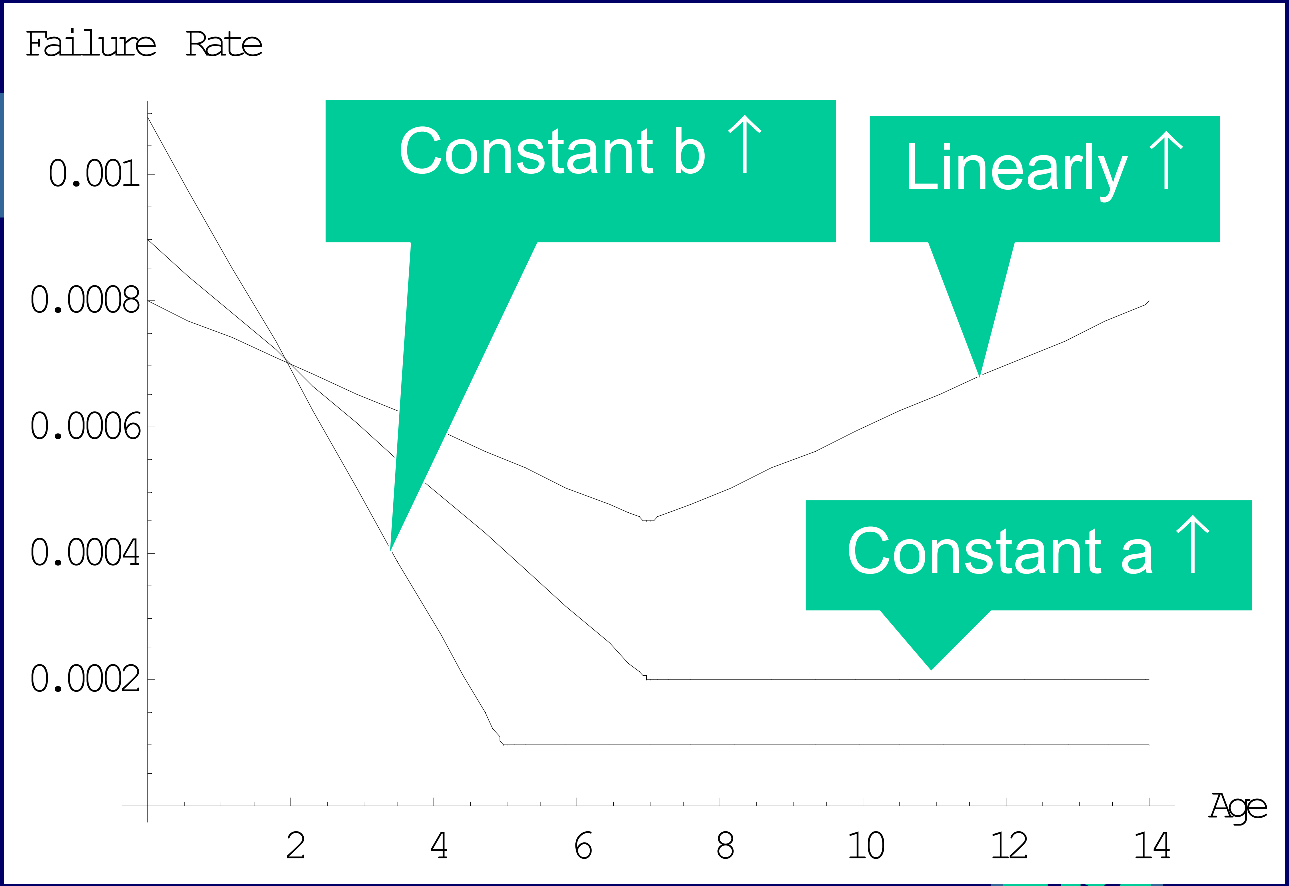
- Reliability function has two parts, IM and after:
  - $\text{Exp}[(0.0001t^2)/2 - t(0.0001 + 0.0001t_o)]$ for $t < t_o$
    $\text{Exp}[-0.0001t - (0.0001t_o^{\,2})/2]$ for $t \geq t_o$
  - P[Fail in IM] $\sim bt_o^{\,2}/2$
- $\text{MTBF} \sim (1 - t_o^{\,2}b)/2 + t_o^{\,2}b/6 - at_o^{\,4}b/24 = 9975.5$



Acceleration alternatives

- Constant segment increases to greater constant
- Constant segment becomes linearly increasing; acc. induces premature wearout, (limit of step stress)
- IM Slope increases and perhaps t_o, the age at the end of IM, decreases as acc.exacerbates process defects
- ***System acceleration is not same as part accelerations! (unless parts are iid in series)***

Acceleration alternatives



Reliability Acceleration Factor

- $RAF(t) = (1-R_{Unacc}(t)/(1-R_{acc}(t))$
  - RAF(60) = 1.705 for double constant failure rate 2a
  - RAF(60) = 1.288 for double infant mortality, b, increases from 0.0001 to 0.0002
  - RAF(60) = *11.350* for changing from constant, a, to linearly increasing failure rate, a+0.0005t!

Fairly General Acceleration Model

- $a_{Acc}(t) = a_{UnAcc}[t/\theta(x)]/\theta(x)$ [Xiong and Ji]
  - $\ln\theta(x) = \alpha + \beta x$
  - x is stress factor, (stress-normal)/(max stress-normal)
  - Continuous version of step stress
- Multiplies failure rate by a factor and rescales age t
- Includes Arrhenius and Eyring models, [Shaked], motivated by Miner's rule
- Apply it to constant, IM slope, or entire piecewise linear failure rate function

Part 2

- Designs and examples
  - |D|-optimal and other statistical designs
  - Moderately credible design
- Need only one acceleration level
- Examples: estimate parameters, LR test of MTBF
  - Unacc.
  - Acc.
- Freebies

Alternative Designs

- |D|-optimal is versatile, but recommends tests at 0, t_o, and anywhere thereafter
  - DoE expects every design point to yield age at failure. Reliability tests often don't.
- Neyman design for multiple strata [Neyman, George 2002]
- Min variance design, must specify how much? [Nelson, Meeker and Hahn]
- Moderately credible design gives 50% probability of at least one failure in infant mortality and one thereafter, sufficient to estimate piecewise linear parameters

Moderately Credible Design

- 50% probability of ≥ 1 failure in IM and ≥ 1 after IM before end of test, t

Parameters	Case 1	Case 2	Case 3
a (guess)	0.01	0.01	0.01
b (guess)	0.01	0.01	0.01
t_o (guess)	2	2	2
n (choose)	29	34	31
t (choose)	7	5.6	6.4
P[failure < to]	0.039	0.039	0.039
P[failure in [to, t)]	0.047	0.034	0.041
P[failure < to|n]	0.371	0.356	0.366
P[≥ 1 failure in [to, t)|n-1]	0.739	0.680	0.718
P[Both, all]	0.501	0.499	0.504

Example Data

Sample	Age at failure	Survivors' ages
1	1	
2	2	
3	15	
4	30	
5	45	
6		45
19		45
20		45

Example Result

Parameter/Model	a	a+b(t–to)	ct	b(t–to)+ct	a+ct	a+b(t–to)+ct
a	0.007	0.004			0.008	0.000
b		0.016		0.018		0.018
c			0.000	0.000	0.000	0.000
to		3.319		3.346		3.346
MTBF	154	215	73	83	125	84
ln likelihood	-30.17	-28.21	-34.97	-27.78	-30.27	-27.78
LR statistic		3.919		14.389		4.989
Sig level		10%		10%		10%
χ^2		6.251		6.251		7.779

Best model

Put all your eggs in one basket for acceleration

- $a(t) = x^p(a+b(t_o-t)^+ + ct)$
- Test at highest reasonable stress
- Predict MTBF or use specification
- Find mle of parameters *constrained* to specified MTBF at working stress, x=1
- Use LR to test MTBF
  - $-2\ln[L(MTBF)/L(unconstrained)] \sim \chi^2$

Example Data

Sample	Age at failure	Survivors’ age
1	1	
2	1	
3	2	
4	2	
5	10	
6	15	
7	20	
8	25	
9	30	
10	35	
11	40	
20		45

Example Result, x = 1.5

Parameter	$x^p(a+ct)$	$x^p(a+b(t-to)+ct)$
a	0.001452	0
b		0.018298
c	7.79E-05	0.000180
to		3.345768
p	5.149690	5
MTBF	125	125
Log likelihood	-53.84	-56.17
LR test statistic		-4.65
Sig level		10%
Chi-square		9.23634

Better model

Switch Example

- Demonstrate MTBF > 39,500 hours with 75% confidence
- Test 7 switches for 6 weeks (1008 hours) at 60° C with MTBF AF = 14.6 (Arrhenius) to give χ^2 LCL of ~39,000 hours
- Xcvrs failed at 486 and 660 hours (16 xcvrs per unit), after IM

Real Example Data

Parameter	Value
c	3.56E-8 per hour per hour
Stdev c	$c/\sqrt{(2n)}$ = 2.38E-9 per hr^2
MTBF	$\sqrt{(\pi/2c)}$ = 6645 hours
25^{th} %ile of MTBF	6584 hours
MTBF of 16 xcvrs acc.	$\sqrt{(\pi/32c)}$ = 1661 hours
25^{th} %ile of 16-xcvr MTBF	~1000 hours
25^{th} %ile of 16-xcvr MTBF, unacc.	1000*35 = 35,000 hours

Recommendations

- For simplicity, use the PLFR to approximate left-hand end of bathtub curve…
- Approximate acceleration with power law, rescale age if necessary and if Miner's rule fits
- Use one, high level of acc. and MTBF
- Send data to me for free samples

References

- Bagdonavicius, Vilijandas and Mikhail Nikulin, *Accelerated Life Models, Modeling and Statistical Analysis,* Chapman and Hall, New York, 2002
- George, L. L., "Design of Ongoing Reliability Tests (DORT)," ASQ *Reliability Review,* Vol. 22, No. 4, pp 5-13, 28, Dec. 2002
- Kalbfleisch, John D. and Ross L. Prentice, *The Statistical Analysis of Failure Time Data,* Second Edition, Wiley, New York, 2002
- Meeker, William Q. and Gerald J. Hahn, *How to Plan an Accelerated Life, Test: Some Practical Guidelines,* Vol. 10, ASQ, 1985
- Nelson, Wayne, *Accelerated Testing,* Wiley, New York, 1990
- NIST, *Engineering Statistics Handbook,* Ch. 8.3.1.4, "Accelerated Life Tests," http://www.itl.nist.gov/div898/handbook/apr/section3/apr314.htm
- Shaked, Moshe, "Accelerated life testing for a class of linear hazard rate type distributions," *Technometrics,* Vol. 20, No. 4, pp 457-466, November 1978
- Viertl, Reinhard, *Statistical Methods in Accelerated Life Testing,* Vandenhoeck & Ruprecht, Göttingen, 1988
- George, L. L., "What MTBF Do You Want?" *ASQ Reliability Review,* Vol. 15, No. 3, pp 23-25, Sept. 1995
- Neyman, J., "On the Two Different Aspects of the Representative Method: The Method of Stratified Sampling and the Method of Purposive Selection," *J. of the Roy. Statist. Soc.,* Vol. 97, pp 558-606, 1934
- Xiong, Chengjie, and Ming Ji, "Analysis of Grouped and Censored Data from Step-Stress Life Test," *IEEE Trans. on Rel.,* Vol. 53, No. 1, pp. 22-28, March 2004