IEEE

Santa Clara Valley -Reliability

Are You Analyzing Reliability Data Correctly?

Repairable Vs. Non-Repairable Systems: There Is a Difference

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Topics

Case Study Analysis Example
Properties of Repairable Systems
Renewal Processes
Multiple Systems Considerations
Graphical Analysis and MCF
HPP
MTBF Issues
Non-Renewal Processes
Time Dependent Reliability (TDR)
Detecting Trend with RAT
Redo Analysis of Case Study Example
Summary

Objectives

To highlight some major distinctions characterizing reliability data from **repairable** and **non-repairable systems**

To show simple **graphical** and **analytical techniques** for the analysis and modeling of repairable system data

Case Study Example

Repairable system:

Equipment used in a manufacturing process. System had a single, replaceable board. Repairs made by replacing failed board with new board from the same population in stockpile.

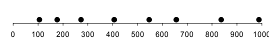
Analysis:

Engineers wanted to model the reliability of the system based on failure data obtained during the first 1000 hours of operation.

Case Study: System Repair History

Repairs (board replacements) were done at system ages (in hours) 108, 178, 273, 408, 548, 658, 838, and 988.

A time plot of repairs is shown below.



Case Study Analysis

The engineers performed the reliability analysis by treating the **times between repairs** (that is, individual time to failure for each replaced board) as a group of *independent and identically distributed (iid) observations* arising from a ***single*** *population* of failure times.

The **order** in which these times occurred was ignored.

Analysis methods used were:

- Weibull probability plotting of data
- Parameter estimation
- Model fitting

Weibull Analysis of Data

The times ***between*** repairs, called the **interarrival times,** are calculated below:

Repair Time (System Age)	Time Between Repairs (Interarrival Times)
108	108
178	178-108 = 70
273	273-178 = 95
408	408-273 = 135
548	548-408 = 140
658	658-548 = 110
838	838-658 = 180
988	988-838 = 150

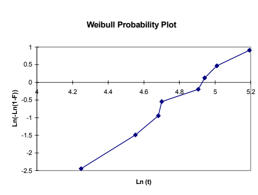
For plotting, the ***sorted*** interarrival times are: 70, 95, 108, 110, 135, 140, 150, 180.

Median ranks are used for the plotting positions in the Weibull plot.

Weibull Probability Plot of Repair Times

The Weibull probability plot showed a reasonable fit to a straight line.

The ***parameters*** of the Weibull distribution were estimated graphically.



Weibull Parameter Estimation

The **shape parameter**, ***m***, estimated by the slope of the straight line, is.

$$\hat{m} = 3.65$$

The **characteristic life**, ***c***, is estimated from the graphical intercept, ***I***, using the equation:

$$\hat{c} = e^{-I/m}$$

Here, the ***c*** estimate in hours is

$$\hat{c} = 137$$

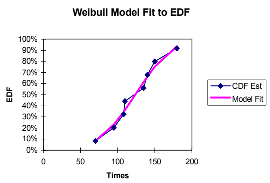
Thus, the Weibull CDF model is:

$$F(t) = 1 - e^{-(t/c)^m}$$

$$F(t) = 1 - e^{-(t/137)^{3.65}}$$

Weibull Model Fit to Data

The graph below shows the Weibull model fit to the empirical distribution function, EDF, of the actual data viewed as a single population of failure times.



Engineering Interpretation of Analysis

Engineers concluded times between repairs follow a Weibull distribution.

Of concern to the engineers was the fact that the estimated shape parameter, ***m***, was greater than 1, indicating an ***increasing*** "failure rate."

The equipment engineers thus felt the machine needed to be brought down for additional repair and maintenance.

In fact, the engineers analyzed the boards as ***non-repairable*** components. Were these conclusions justified or misleading?

We shall return to this example later.

Definition of a Repairable System

A system is ***repairable*** if, following a failure at some time ***t***, it can be **restored to satisfactory operation** by any action.

We must be careful to specify exactly what we mean by the term "***failure***." Some failures may involve no downtime or decrease in operational capability. For example, if the radio in your car fails, your car can still operate. Is that a system (car) failure?

What constitutes "satisfactory operation"?

It is very important to distinguish different types of failures from ***outages*** when analyzing reliability data.

Systems and Repair Actions

Repairable Systems include:

- Network Servers
- Personal Computers
- TV's
- Automobiles
- Production equipment
- Software programs

Examples of restoring actions:

- Replacing a circuit board
- Rebooting a computer
- Changing adjustable settings
- Swapping of parts
- Automatic switchover to a redundant component
- Resuming electrical power
- Sharp blow with a hammer
- Fixing a software bug (maybe/maybe not)

Why Analyze Repairable Systems?

Most products are repairable and involve issues such as:

- Detect trends
- Estimate and improve reliability and availability
- Specify burn-in requirements
- Provide for servicing and spare parts
- Forecast repair and warranty costs
- Satisfy regulatory requirements
- Size safety and security concerns
- Upgrade existing systems
- Design better future systems

Reliability Measures for Repairable Systems

Key Considerations:

1. **Times Between Repairs**
2. **Number of Repairs Over Time**

Function of many factors

- Basic system design
- Operating conditions
- Environment
- Applications
- Load
- Software robustness
- Type of repairs
- Quality of repairs
- Materials used
- Suppliers
- Human behavior

System Age

System age is the total running hours, that is, the elapsed time, on a system. Also called power-on hours (**POH**) or operating hours.

Often called the **uptime**

Carefully distinguish age from times **between** failures and device-hours or unit-hours.

We'll assume repair times are negligible, that is, availability issues are not the concern here.

Time Dependent Reliability

Key Property of Repairable Systems:

<u>Failures occur **sequentially** in time.</u>

Critical Question:

Are the times between failures **independent and identically distributed (i.i.d.)** observations from a single population? If so, we call this a ***renewal*** process.

In a renewal process, there is ***no trend***, since we're sampling from a single distribution with a ***fixed mean***.

Sequence of Failure Times Key Points!

If the times between successive failures are getting **longer**, then the system reliability is **improving**.

Conversely, if the times between failures are becoming **shorter**, the reliability of the system is **degrading**.

Thus, the ***sequence*** of system failure times can be very important.

If the times show no trend (relatively **stable)**, the system is neither improving or degrading. In this case, the occurrence ***order*** of the times is not important.

Renewal Process for a System

For a system, restoration to "**like new**," such as replacement of a failed component with one from same population, ***implies*** a **renewal process** (i.i.d.). However, even replacement with an identical component is no guarantee of a renewal process!

The assumption of a renewal process must be checked for validity.

If the times between repairs are not i.i.d., a renewal model is not valid and special techniques for analysis are required.

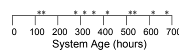
Analysis of Renewal Process

Consider a **single** system for which the **times to make repairs are ignored.**

Ten failures are reported at the system ages (in hours):

106, 132, 289, 309, 352, 407, 523, 544, 611, 660.

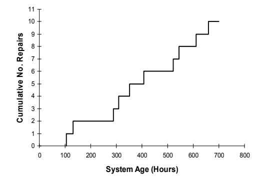
The pattern of repairs is



Cumulative Plot

A very revealing and useful data graph is called the **cumulative plot**: the **cumulative number** of repairs, ***N(t)***, is plotted against the system **age**, ***t***, at repair.

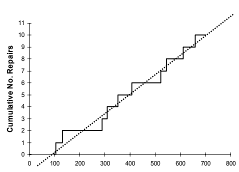
For the renewal data, the cumulative plot is:



Analysis of a Renewal Process

Under a renewal process, the times between failures are i.i.d., that is, from a single population having a ***constant*** mean time between repairs (average or **MTBF**).

Consequently, the cumulative plot should appear to follow a **straight line**.

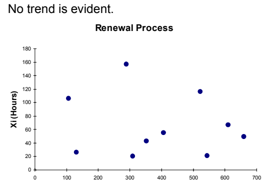


Interarrival Times Versus System Age

A plot of the times **between** sequential failures (that is, the interarrival times) versus the system age is a very useful chart for revealing trends. Large values are desired.

For the renewal data, the interarrival times are 106, 26, 157, 20, 43, 55, 116, 21, 67, 49.

No trend is evident.



Number of Repairs in Time is a Counting Process

We'd like to **estimate *M(t)***, the **mean number of repairs per system for the population**, by system age ***t.***

M(t) is also called the ***mean cumulative function*** or ***MCF***.

For a single system, the actual **number of repairs *N(t)*** by time *t* is an unbiased estimate of *M*(*t*).

The **repair rate** or **recurrence rate, *m(t)***, is the derivative of ***M***(***t***).

Analysis of Many Repairable Systems

The data may consist of **many identical or similar systems** possibly subjected to multi-censoring.

Example: servers installed in the field at different dates throughout the year.

Within a system, there may be a collection of **identical** and **independent subsystems.** The system behavior is the combined repair activity for all subsystems.

Example: a server with many identical boards.

Reliability Issues for Multiple Systems

*What's the **mean number** of repairs, MCF, by time *t*?

*What's the **mean repair rate** for all systems?

What's the expected **variation** in the mean number of repairs at a given time?

*What's the **distribution** of failures across the identical systems.

*What's the expected **time to first repair**? *To ***k*th** repair?

What is the mean repair **cost**?

Are costs of repairs **increasing** or **decreasing**?

Are **spare parts** adequate?

Is run-in necessary? How long? How costly? Is it cost effective?

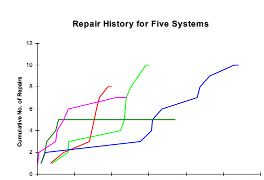
How serviceable is the system for repairs?

How is qualification of new systems done?

* Topic covered.

Graphical Approach to Multi-System Analysis: MCF

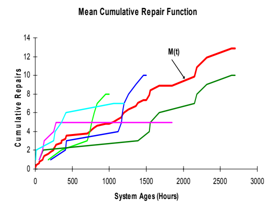
Consider a sample of systems subject to repair actions. Represent **individual** repair **histories *N*(*t*)** using connecting lines between repairs, referencing all starting times back to zero. Example plot for **five** systems is shown below.



Mean Cumulative Repair Function: MCF

- We can envision a **single** MCF curve denoted by ***M*(*t*)** that gives the **average or mean number of repairs per system** at time ***t***.
- For **uncensored** data, the MCF *M*(*t*) is estimated by the average cumulative number of repairs among all systems by time ***t***, that is, a **vertical slice** of the combined cumulative plot.

MCF for Five Systems



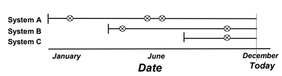
Nelson (1994) has also developed confidence intervals for this approach.

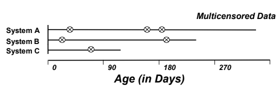
How do we estimate *M*(*t*) when **censoring** occurs, for example, after about 1000 hours?

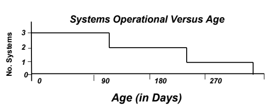
Multicensoring Issues

- Because systems are installed at different dates throughout the year, system ages will differ, resulting in ***multicensored data.***
- ***Right censored*** data has no information beyond a specific system age.
- ***Left censored*** (truncated) data has no information before a specific date.
- Analysis must account for the number of systems operational (at risk) at any system age.

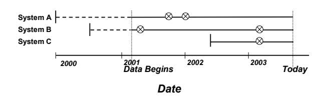
Right Censored Data Example

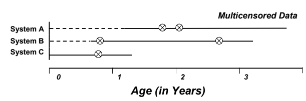


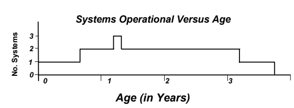




Left Censored Data Example

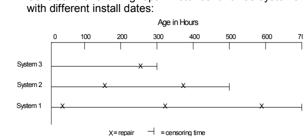






MCF Estimation Under Right Multicensoring
(Pooled Summation)

Consider the following repair histories for three systems with different install dates:



The pooled summation method calculations are

Repair and Censoring Times (Hrs)	33	135	247	300|	318	368	500|	582	700|
Number of Systems Operational	3	3	3	3	2	2	2	1	1
Reciprocal for Pooled Method	1/3	1/3	1/3		1/2	1/2		1/1	
Pooled Summation Estimate	1/3	2/3	3/3	3/3	3/3+1/2	3/3+2/2	3/3+2/2	3/3+2/2 +1/1	3/3+2/2 +1/1

Limitations of Graphical Analysis

- The ***M*(*t*)** graph alone does not tell us about the **distribution** of the **number of repairs** at that time.
- How **accurate** is the estimate of ***M*(*t*)**? Confidence intervals are needed.
- Can we identify anomalous systems?
- What is the **distribution** of the ***time*** to a specific number of repairs? (horizontal slice)
- What is the **distribution** of the ***number*** of repairs across the systems at time ***t***? (vertical slice)
- Graphical analysis is important, but we need additional analytical and modeling tools.

Importance of a Model

- Helps us to understand current results.
- Allows for prediction of future behavior.
- May prevent reaction to noise.
- Helps identify potential failure mechanisms or anomalous behavior.

George Box:
"All models are wrong. Some are useful."

What models are useful for repairable system?

Model of a Renewal Process Replacement Example

- Single component system: light bulb.
- Light bulb is replaced upon failure with a light bulb from the same population as the one replaced.
- Stock of spare parts all basically identical
- Single distribution of failure times
  - Independent
  - Identically distributed

Renewal Process Variables

Two variables are of key interest:

M*(*t*)** the **mean number of repairs** by time *t*, that is, the ***MCF

***T*(*k*)** the **time to reach the *k*th failure**

For a renewal process, ***M*(*t*)**, the MCF, is also called the **renewal function**, which is the **expected (or average) value** of ***N*(*t*)**, the number of repairs by time ***t*** for a single system.

Renewal Process Single System

For a renewal process, the **single distribution** of **failure times between repairs** defines the expected pattern of repairs.

Let X_i denote the **interarrival time** between the *i*th and the (*i*-1) repair.

The **time to the *k*th repair** can be written as the sum of *k* interarrival times

$$T(k)=\sum_{i=1}^{k} X_i$$

For example, if the first three interarrival times are 100, 150, and 75 hours, then the time to the third repair is 100+150+75 = 325 hours.

Knowing the **probability distribution** (**pdf**) of X_i, we can theoretically find distributions for ***N*(*t*)** and ***T*(*k*)** along with ***M*(*t*)** and the renewal rate ***m*(*t*) =** *dM*(*t*)/*dt*

Poisson Model for N(t) Renewal Process

Suppose the interarrival times X_i are i.i.d. with **exponential** pdf having constant failure rate **λ**, that is,

$$f(x)=\lambda e^{-\lambda x}$$

Then, we can show that ***N*(*t*)** has a **Poisson distribution** with **constant rate λ** (intensity). The **expected number** of repairs in time ***t*** is ***λt*.**

Note that ***λ*** is a **rate** (i.e., repairs/time) that is multiplied by time ***t*** to give the **number** of repairs by time ***t***.

Homogeneous Poisson Process Model

Consequently, the probability of observing **exactly *N*(*t*) = *k*** failures in the **interval** (0,***t***) is

$$P[N(t)=k]=\frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

We call this renewal process a ***homogeneous Poisson process*** (**HPP**).

MTBF for HPP

For a HPP, the **mean time between failures** (MTBF) is constant and

$$MTBF=\theta=1/\lambda$$

The **expected number** of repairs in time ***t*** is

***M*(*t*) = *λt* = *t*/*θ*.**

The **mean time to the *k*th** repair is

***k*/*λ* = *kθ*.**

HPP in Terms of MTBF

We can rewrite the Poisson distribution for the HPP in terms of the MTBF, *θ*:

$$P[N(t)=k]=\frac{(t/\theta)^k e^{-t/\theta}}{k!}$$

Example: The MTBF is 10,000 hours. What's the probability of one failure in 3 months?

The expected number ***λt*** is

t/*θ* = (91days x24hrs/day)/10,000 hrs = 0.218

The probability of exactly one failure is

$$P[N(t)=1]=\frac{(0.218)e^{-0.218}}{1!}=0.0878$$

HPP for Multiple Systems

By multiplying the calculated HPP Poisson distribution probabilities by the number of systems, we can estimate the expected distribution of failures across many systems.

We'll illustrate this concept later.

Summary: Homogeneous Poisson Process

A **renewal** process in which the **interarrival distribution** is **exponential** is called a **homogeneous Poisson process** (**HPP**).

The **expected** number of repairs, *M*(*t*), in time *t* is **λ*t* = *t*/*θ*.**

The mean time between repairs (**MTBF**) *θ* = 1/ **λ.**

The mean time to the *kth* repair is ***k* /λ = *kθ*.**

MTBF Estimate for HPP

An estimate of *θ* for a **single** HPP repairable system is the total running hours (age) of a system divided by the number of failures.

For example, a system has run 5100 hours with failures at system ages 150, 1800, and 3790 hours. The MTBF estimate is

5100/3 = 1700 hr.

Note that for a single repairable system, the MTBF estimate above is the **same as the average of the interarrival times** *including the last censoring time* (hence the term MTBF), assuming repair times are negligible.

Qualification Issue: How Long to Run a Test for MTBF for HPP

***Applied Reliability**, 2nd ed., by Tobias and Trindade,* provides Table 10.5, which is a guide for determining test length.

Example:

To demonstrate an MTBF of at least 168 hours at 90% confidence, how many hours should we run the test if we allow up to three failures?

From Table 10.5, we the factor 6.68. Hence,

6.68x168 = 1,122 hours

are needed for a time censored test on a single system.

Misguided Qualification Testing

Warning!

It is potentially **dangerous to assume** that accumulated hours on multiple systems can replace actual running hours on a single system.

For example, running three systems for 1122/3 = 374 hours provides equivalent system hours **ONLY** if the identical HPP holds for all three systems.

A common HPP model often does not hold. For example, in the presence of early life failures, such an assumption is invalid.

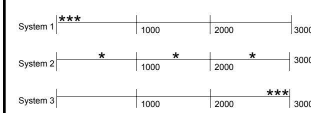
Caution: MTBF is a Summary Statistic – Hides Information

Consider three systems operating for 3000 hours, each with MTBF of 1000 hours

System 1 had three failures at 30, 70, 120 hours and no further failures

System 2 had three failures at 720, 1580, and 2550 hours

System 3 had three failures at 2780, 2850, and 2920 hours



Are these systems behaving the same?

MTBF Misinterpretation
Not an Entitlement

"Expected or Typical Lifetime of a System"

During the years 1996-1998, the average annual death rate in the US for children ages 5-14 was 20.8 per 100,000 resident population.

The average failure rate is thus 0.02%/yr

The MTBF is 4,800 years!

Assumptions for HPP One More Time!

Renewal Process ("Good as New")

-Times ***between*** failures are independent and identically distributed

-***Single*** distribution of times between failures

Exponential distribution for interarrival times

-***Constant*** failure rate (time independent: aged systems fail at the same rate as new systems)

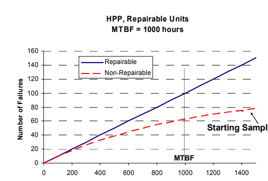
-***No trend***

-MTBF has meaning

MTBF
(HPP Repairable Systems)

Assume a random sample of 100 units from a exponentially distributed population having a MTTF of 1,000 hours is tested for 1,500 hours. The units are **repaired or replaced** upon failure in a renewal process. We have a HPP with MTBF = MTTF.

The expected number of failures versus time is:



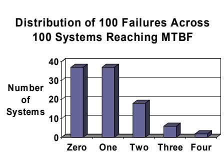
Note at the MTBF we still expect 63.2% failures or 36.8% survivors among the **starting** sample.

MTBF Implications

For a 100 repairable systems, ***by the time they all reach the MTBF***, on the average there will be 100 failures, not however, one per system.

37 systems will have no failures

63 systems will have at least one failure: 37(1), 18(2), 6(3), 2(4).



Only those 37 systems with one failure will match the expected MTBF. Customers with multiple failures will see low MTBFs.

MTBF can be an Inadequate Measure of System Reliability

Valid only for a renewal process.

Treating all system hours as equivalent and all failures as equal ignores potential and likely real age effects.

Users of MTBF calculations seldom checked for validity of HPP.

A better and less assuming approach to measure reliability is to analyze the data versus system age, that is, apply time dependent reliability TDR analysis. We'll discuss TDR later.

Example of a Non-Renewal Process

Consider a light bulb which is replaced upon failure but now the cooling fan inside the equipment is degrading, causing a gradually rising temperature.

Times of failures will get shorter.

There is not a single distribution of independent failure times (no constant MTBF).

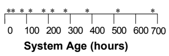
Now important to determine the occurrence order of failures to analyze system behavior properly.

Graphical Analysis of Non-Renewal Processes

Suppose the observed consecutive repairs times were

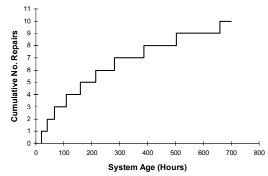
20, 41, 67, 110, 159, 214, 281, 397, 503, 660.

A line sketch of the pattern of repairs shows:



Cumulative Plot

The cumulative plot for this set of data is shown below.



The curvature suggests a decreasing frequency of repairs, that is, an **improving** failure rate.

Interarrival Times

For this set of data, the interarrival times are 20, 21, 26, 43, 49, 55, 67, 106, 116, 157.

For the renewal data, the interarrival times are 106, 26, 157, 20, 43, 55, 116, 21, 67, 49.

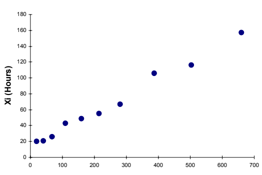
Comparing the two sets of data, we see the **interarrival times** are exactly the **same**, but in a **different order**!

Now the **order** in which the interarrival times appear is **important**.

Cannot use standard non-repairable methods, such as probability plotting, to analyze this data. We'd get same results!

Look at a plot of interarrival times versus system age.

Interarrival Times Versus System Age, Improving



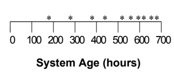
Larger is better.

Another Repairable System History

Suppose the repairs occurred at the following times

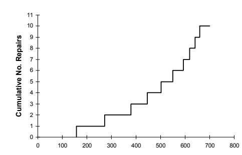
157, 273, 379, 446, 501, 550, 593, 619, 640, 660.

The line sketch is



Cumulative Plot

The cumulative plot is shown below.



The curvature shows the frequency of repairs increasing in time, indicating system **degradation**.

Interarrival Times

The interarrival times are

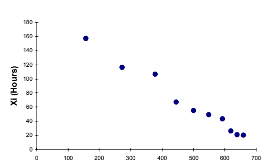
157, 116, 106, 67, 55, 49, 43, 26, 21, 20.

Note these are exactly the **same** times as the last two sets of data! Only the order is different.

Again, it is not correct to analyze the time between repairs as if they were independent observations from a single population.

Let us view the plot of interarrival times versus system age.

Interarrival Times Versus System Age, Degrading



Larger is better.

MTBF Comparisons

Stable Process

106, 26, 157, 20, 43, 55, 116, 21, 67, 49.

MTBF = 66

Improving Process

20, 21, 26, 43, 49, 55, 67, 106, 116, 157

MTBF = 66

Degrading Process

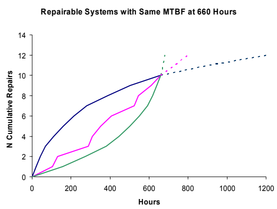
157, 116, 106, 67, 55, 49, 43, 26, 21, 20

MTBF = 66

The data are the same! Only the order has changed. MTBFs are identical!

Yet, the behavior is vastly different!

MTBF Comparisons



MTBF may tell us on the average where we are at some time, but MTBF may not reveal how we got there or where we're headed.

What's the Point?

Using a summary statistic like the MTBF is ***dangerous*** if we do not distinguish between stable or trending processes.

Study the **ordered** times between failures of systems versus the system age to determine if our assumptions are correct.

Average Repair Rates

Since an MTBF may be a meaningless number (e.g., the MTBF holds only for a specific time interval and is considerably larger than that interval), we recommend using more realistic measures such as average failure, outage, or repair **rates** referenced to a **relevant time period**.

Example:

We can define an average annual repair (AARR) or outage rate (AAOR) as the average number of repairs per year specified for any year: first, second, etc. A plot of the AAOR as a function of system age may be very useful to make comparisons.

TDR Analysis: Customers Want More than an MTBF Number for Reliability

What is the reliability of servers? What should it be?

What are the causes of downtime?

What can we expect going forward?

The answers can be provided by time dependent reliability (TDR) analysis.

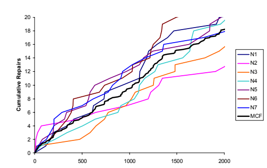
TDR Overview Mean Cumulative Function

Recall previous definitions:

Cumulative Plot: A plot of the ***cumulative*** number of recurrences (repairs) for a ***single*** system versus the system ***age***

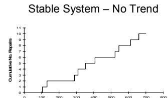
Mean Cumulative Function (MCF): A vertical slice at a given system age that is the ***average number of repairs across all systems.***

Cumulative Plots & MCF Example

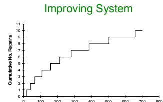


MCF Vs. Age Reveals Any Trends in Reliability

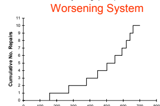
Stable System – No Trend



Improving System



Worsening System



Estimating Recurrence Rates

(Repair Rates or Failure Rates)

By numerical differentiation of the MCF, it is possible to estimate repair rates

The degree of smoothing is controlled by the number of data points captured in each tangent to the MCF (ruler method)

Slope is plotted at midpoint age of group of data points

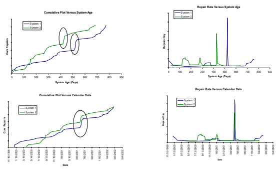
MCF: System Age Versus Calendar Date

MCF versus ***system age*** is expected to reveal cumulative repairs per system that **depend on the system operating hours**

MCF versus ***calendar date*** may reveal repairs common across systems and **associated with specific time periods** (day, month, etc.)

Plotting MCF versus age may highlight effects not as easily seen by plotting MCF versus calendar date and vice versa.

Example of Date Effect for Two Systems



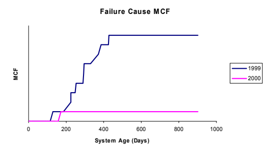
Simulated data: Two systems. One installed 1/1/2001and second 4/1/2001. Software upgrades on both systems on 6/1/2001.

TDR Analysis by Failure Cause Code

Individual MCF plots can be constructed for individual cause codes to reveal failure trends.

TDR analysis of cause codes provides time related information not available through static Pareto analysis.

Plotting system MCF with cause codes shows the time effects of different failure types on reliability.



MCF Comparisons

- By platform
- By customer
- By vintage
- By age (left and right censoring)
- By calendar date
- By location
- By failure cause
- By supplier
- By technology
- By payload or applications

Data Needs

For ***each and every*** system type (by serial number) at a specific customer site:

Configuration information

Date installed

Date data capture began

Date of each failure, ***if any***

Failure cause for each failure

Description of repairs

Current or removal dates

Special comments (applications, load, etc.)

TDR Analysis

MCF and recurrence rate plots coupled with time dependent plots of failure causes are powerful tools for understanding system reliability over time. Anomalous systems can be identified and failure clustering revealed.

These techniques provide us with the ability to compare systems across platforms, customers, and vintages, as well as showing us what are the reliability, associated causes, and early trends.

Testing for Trends and Randomness

To develop appropriate models, **assumptions** should be **verified**.

There are possible steps to follow in coming up with a suitable model :

Plot the data several ways.

Check for trend. (See RAT.)

If there is trend, employ non HPP or other **nonstationary** models

If no trend and i.i.d. (renewal process), check if exponential distribution holds for interarrival times

If yes, assume HPP

If not, try other models or analyze using distribution free methods

A Simple Test to Check for Trend

Reverse Arrangement Test (RAT)

Nonparametric Test

Consider set of **interarrival times** occurring in the sequence

$$X_1, X_2, \ldots, X_n$$

Test is based on whether X_i are stable, growing, or getting smaller.

A ***reversal*** is defined as each instance of an earlier interarrival time being smaller than a later interarrival time, that is, we count each occurrence where $X_i < X_j$, for every $i < j$.

RAT Example
Four Repairs

Assume the times between repairs are 15, 20, 5, 10, **in some order**. There are 4! = 24 ways of arranging these four times.

If the order is 5, 10, 15, 20, the times are **trending longer**, indicating improvement. The probability of this sequence by chance alone is 1/24 or 4%, which is considered statistically significant at 95% confidence.

We count 6 reversals.

Similarly, if the order is 20, 15, 10, 5, the times are **trending shorter**, indicating degradation. The probability of this sequence by chance alone is 1/24 or about 4%, again statistically significant at 95% confidence.

There are zero reversals.

Table* of Critical Values for RAT

Table 10.6 Critical Values $R_{n,\%}$ of the Number of Reversals for the Reverse Arrangement Test

Sample Size	*Single-Sided Lower Significance Level*			*Single-Sided Upper Significance Level*		
n	**1%**	**5%**	**10%**	**10%**	**5%**	**1%**
4		0	0	6	6	
5	0	1	1	9	9	10
6	1	2	3	12	13	14
7	2	4	5	16	17	19
8	4	6	8	20	22	24
9	6	9	11	25	27	30
10	9	12	14	31	33	36
11	12	16	18	37	39	43
12	16	20	23	43	46	50
	Too few reversals. (degradation)			Too many reversals. (improvement)		

From *Applied Reliability*, 2nd ed., by Tobias and Trindade

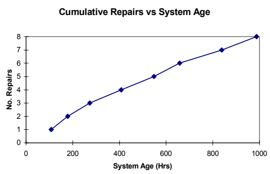
Return to Case Study Example

Recall the system experienced repairs at the following ages

108, 178, 273, 408, 548, 658, 838, 988

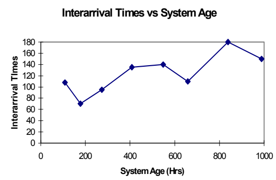
Is there any evidence of a trend?

We show below the important graph to reveal repairable system behavior: the cumulative repair plot, ***N(t)*** vs. ***t***.



Interarrival Times Versus System Age Plot

We plot the consecutive times between repairs versus the system age at repair.



There appears to be a trend visible in either plot. How significant is it?

What is the probability of the **observed sequence of repair times** occurring **by chance alone** under a renewal process?

RATS on Case Study Example

Solution:

The interarrival times are 108, 70, 95, 135, 140,110, 180, 150.

There are 5+6+5+3+2+2+0=23 reversals

Comparison to critical table shows 22 reversals in 8 items is significant at the 5% level. Hence, we reject the renewal process, and in particular, the HPP, as a suitable model.

With at least 95% confidence, we state that the system is **improving** in time.

Results/Implementation

The **correct analysis** showed an **improving** trend in the repairable system history.

Incorrect, non-repairable board analysis lead to the belief that maintenance was necessary to restore reliability. **Unnecessary repairs** might have made the reliability **worse.**

By not performing unnecessary maintenance considerable **savings** in money and cycle time was possible.

Discovering the source of the improvement lead to adoption of new techniques for repair.

The result was improved reliability and cost savings for the existing system and future systems.

Another Example of How Wrong Analysis Can Mislead

Suppose a manufacturer of computers or servers has introduced a new product in January. The reliability is stable for the first six months.

In July, process improvements and efficiencies result in dramatically improved MTBFs for the new product.

Suppose field repairs are now done with the new, improved product.

Repairable system analysis using the MCF **will show** the improvement with system ages at the end of the MCF curves.

Since the new product beginning at time zero will have fewer hours than existing product with poorer MTBF, non-repairable component analysis can result in the product appearing worse with time, the complete opposite of what is occurring!

Brief Recap for Analysis of Repairable Systems

Study the times between failures of systems for trend to determine if HPP assumptions are correct.

Use graphical and statistical tools for analysis.

Understand the difference between the analysis of repairable systems and non-repairable systems.

Remember…

Reliability is **age specific. TDR** methods reveal trends.

Use **TDR** measures for reliability metrics.

Think **distributions**, **mean cumulative plots**, and **recurrence rates**.

Display results **graphically**.

Track failures and downtime **by system** versus system **age**

Use the **analysis** to drive appropriate preventive and corrective action