Sensorless PM Brushless Drives

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Outline

• Review of sensorless techniques
• Zero-crossing detection of back-emf waveform
• 3rd back-emf detection
• Flux observer
• Rotor saliency
• Extended Kalman filter
• Design of high-speed (>100krpm) BLDC motor for sensorless operation
• Vector control
• Flux weakening control
• Direct torque control
Sensorless Techniques

Why sensorless?

- Reduced component count
- Improved reliability
- Eliminates mechanical/hysteresis problems of discrete sensors

Key consideration:

- Simple algorithm
- Accurate rotor position estimates to dynamic load disturbances
- Robust to parameter variations
Sensorless Techniques

• **Brushless DC:**
  - Back-emf zero crossing detection
  - Third harmonic voltage detection
  - Freewheel diode approach
  - ...

• **Brushless AC**
  - Flux/position observer
  - Inductance variation
  - Kalman filter
  - …
Sensorless Techniques

Existing problems

- Sensitive to parameter variation
- Poor performance at low speed
- Initial position not identifiable
- May not work at zero speed

Rotor saliency based approaches

- Operational at zero & low speed
- Rotor saliency required

Key issues: - two zeros

- Zero crossing of back-emf waveform for BLDC
- Zero speed for both BLDC & BLAC
(1) Detection of Zero-Crossing of Back-EMF Waveform

- Most common technique for sensorless operation of brushless DC motors
- Appropriate switching devices commutated 30°elec. after detection of zero-crossing of back-emf waveform when phase is unexcited
- Conduction angle of free-wheeling diodes must <30°elec.
  - may be problem at high speed or high load condition
  - not suitable for flux-weakening operation
- Starting and low speed operation problems (due to absence of emf)
Detection of zero-crossing of back-emf waveform

Example:

Mode of operation:
- Initial alignment
- Synchronous open-loop run-up
- Sensorless close-loop

Various commercial ICs, e.g.
- Micro-linear 4425/4426/4428 Sensorless PWM motor controller
Design of 120krpm high-speed motor for sensorless operation

Optimal design: within same space envelope, maximum efficiency, diode conducting duration significantly <30°elec.

Motor A
- Longer stator core
- Fewer turns/coil
- Shorter end winding
- More iron, less copper
- Relatively high unbalanced magnetic pull.

Motor B
- Shorter stator core
- More turns/coil
- Longer end winding
- Less iron, more copper
- Lower unbalanced magnetic pull
Design of 120krpm high-speed motor for sensorless operation

Motor A
- Sinusoidal back-emf waveform

Motor B
- High conduction angle, almost continuous current waveform
- Low diode conduction angle
Design of 120krpm high-speed motor for sensorless operation

Motor A
- Suitable for sensorless control

Motor B
- Unsuitable for sensorless control

Phase terminal voltage

Line terminal voltage
Sensorless high-speed PM brushless motors

Current waveforms on no-load, 125,000rpm
Pros & cons of back-emf zero-crossing detection

Simple, fast, commercial IC chips available

Cases in which zero-crossing of back emf is not detectable:

• BLDC - High speed operation (high reactance)
• BLDC - High load
• BLDC - Flux-weakening operation
• BLAC - Brushless ac operation

➢ Current is continuous or almost continuous
➢ Alternative sensorless technique is required
(2) 3rd Harmonic Back-EMF Detection

3 ways of detecting $e_3$ in literature:

$u_{sn}, u_{hn}, u_{hs}$

$$u_{sn} \approx e_3 = -E_{m3} \sin 3\theta_r$$
Features of 3rd harmonic back-EMF detection

• 3rd harmonic back-EMF can be extracted from voltage
• Only voltage $u_{sn}$ is suitable for extracting 3rd harmonic back EMF in both BLDC and BLAC drives
• Independent of motor operation mode
• Applicable to both BLDC and BLAC operations
• Open-loop starting & close-loop operation as conventional back-emf detection
• Most suitable for high-speed application
• Example: 18 slots, 6 poles, surface-mounted magnet rotor, overlapping winding, 1 slot pitch skew
Detection of 3\textsuperscript{rd} harmonic back-EMF - Voltage $u_{sn}$
BLDC operation with/without commutation advance

Low speed 320rpm, 4.62Nm;
without advanced commutation, $\theta_{ad} = 0^\circ$

High speed 1950rpm, 0.25Nm;
with advanced commutation, $\theta_{ad} = 45^\circ$
BLDC operation with/without commutation advance
BLAC operation with/without flux-weakening control

Low speed 320rpm, 4.63Nm; without flux-weakening control

High speed 2010rpm, 0.27Nm; with flux-weakening control
BLAC operation with/without flux-weakening control
Restriction of 3rd harmonics back-emf detection

\[ E_{m3} \propto \omega \cdot B_3 \cdot k_{w3} = \omega \cdot B_3 \cdot (k_{p3} \cdot k_{d3} \cdot k_{sk3}) \]

Absented \( E_{m3} \)

\( B_3 = 0 \) - Sinusoidal shaped magnet, 120°elec. pole arc magnet, Halbach magnetised motor

\( k_{p3} = 0 \) - Conventional 3 slot / 2 pole BLDC (120°elec. coil pitch)

Reduced \( E_{m3} \)

\( k_{d3} \rightarrow 0 \) - Distributed winding

\( k_{sk3} \rightarrow 0 \) - Skewed winding/magnet

Low speed, as conventional back-EMF based technique
(3) Based on rotor saliency

- Applicable to PM motors with rotor saliency (interior and inset magnet rotors)
- Winding inductance is rotor position dependent

**Method 1:**
- Inject high frequency signal into motor terminals
- AC current component resulting from injected signal $i_i \sim \sin(2\Delta\theta)\sin(\omega_it) \approx 2\Delta\theta\sin(\omega_it)$
- $\Delta\theta =$ instantaneous difference between estimated rotor position and actual rotor position
- $\Delta\theta$ fed into observer that updates velocity and position to force error to zero

**Method 2:**
- Current variation from hysteresis current PWM controller
- Inductance $\sim 1/(\Delta I/\Delta t)$
- $\Delta I =$ current variation over $\Delta t$
- $\Delta t =$ current rise or decay time
- Rotor position obtained from variation of winding inductance
(4) Flux observer and rotor position estimation

- Suitable for brushless ac machines
- Based on machine model
- Influenced by parameter variations due to temperature & saturation
- Speed obtained from differentiation of estimated rotor position
- Filtering necessary

1. Voltage and current vectors - measured: $\dot{U}, \dot{I}$
2. Stator flux-linkage vector - observed: $\dot{\Psi}_s = \int (\dot{U} - R \cdot \dot{I}) dt + \Psi_s(0)$
3. Excitation flux-linkage vector - observed: $\dot{\Psi}_f = \dot{\Psi}_s - L_s \dot{I}$
4. Rotor position - calculated: $\theta_{r\_est.} = \arctan \frac{\Psi_{f\beta}}{\Psi_{f\alpha}}$
High pass filter to eliminate influence of DC offset on flux observer

Stator flux-linkage vector

\[ \dot{\Psi}_s = \int_0^t (\dot{U} - R \cdot \dot{I}) dt + \Psi_s(0) \]

Flux locus without high pass filter

Flux locus with high pass filter
Low pass filter on observed excitation flux-linkage vector

Rotor position

- calculated from observed excitation flux-linkage vector

\[ \theta_{r\_est.} = \arctan \frac{\psi f_\beta}{\psi f_\alpha} \]

With low pass filter:

✔ Smooth locus of flux-linkage vector.
✔ Reduced ripple in estimated position.
✘ Time delay in position estimation.
✘ High frequency position error still exists, causes ripple in estimated speed.
Low pass filter on observed excitation flux-linkage vector

Without low pass flux-filter

- High frequency ripple exists in flux locus
- Large position error exists

With low pass flux-filter

- Smooth flux locus
- Significant phase shift exists
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Speed estimation 1 - Differential of estimated rotor position

\[ \omega_p = \frac{d\theta_r\_est.}{dt} \approx \frac{\Delta\theta_r\_est.}{\Delta t} = \frac{\Delta\theta_r\_act. - \Delta\theta_r\_err.}{\Delta t} \]

† _est.: estimated value; _act.: actual value; _err.: error.

\( \Delta t \) small, \( \Delta\theta_{r\_act.} \) small \( \Rightarrow \) comparable to error \( \Delta\theta_{r\_err.} \).

Comparison of Estimated and Actual Speeds

Sensorless operation

\( \times \) Error in estimated position causes ripple in estimated speed.

\( \times \) System maybe unstable if estimated speed used as feed-back.
Speed estimation 2 - Average speed estimation

\[ \omega_d = LPF(\omega_p) = LPF\left(\frac{\Delta \theta_r}{\Delta t}\right) \]

† LPF: low pass filter.

Comparison of Estimated and Actual Speeds

![Graph showing comparison of estimated and actual speeds](image)

**Sensored operation**
- Accurate estimation during steady-state operation.
- Time delay in estimated speed during transient operation.
- System may be unstable if estimated speed used as feedback.

**Sensorless operation**
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Speed estimation 3 - from induced EMF & excitation flux-linkage

\[
\begin{align*}
E_m &= \omega \Psi_f \\
u_q &= R_i + (L_s \cdot p_i + \omega L_s i_d) + E_m
\end{align*}
\]

\[
\omega_e = \frac{u_q - R_i \cdot p_i}{\Psi_f + L_s i_d} \approx \frac{u_q - R_i}{\Psi_f}
\]

**Comparison of Estimated and Actual Speeds**

- **Sensored operation**
  - No time delay in estimated speed.
  - System stable if estimated speed used as feed-back.
- **Sensorless operation**
  - Estimation error even during steady-state operation.

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**EMD**

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Speed estimation 4 - Improved estimation (combined 2 & 3)

\[
\omega_h = \omega_d \frac{1}{Ts + 1} + \omega_e \frac{Ts}{Ts + 1} = \omega_d + (\omega_e - \omega_d) \frac{Ts}{Ts + 1}
\]

\[
= \omega_d + \omega_{dif} \frac{Ts}{Ts + 1} = \omega_d + \omega_{com}.
\]

† \(\omega_{dif}\): difference of estimations 2. & 3.; \(\omega_{com}\): compensation.

✓ Accurate speed estimation during steady-state \((s \to 0, \omega_h \to \omega_d)\).
✓ Fast dynamic response to speed changes \((s \to \infty, \omega_h \to \omega_e)\).
✓ System stable if estimated speed used as feed-back.
Comparison of estimated and actual speeds

**Sensored operation**

- Accurate speed estimation during steady-state \((s \rightarrow 0, \omega_h \rightarrow \omega_d)\).
- Fast dynamic response to speed changes \((s \rightarrow \infty, \omega_h \rightarrow \omega_e)\).
- System stable if estimated speed used as feed-back.

**Sensorless operation**

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(5) Based on Extended Kalman Filter

**EKF** - An optimal recursive estimation algorithm for nonlinear systems

**Applications** - High-accuracy estimates of non-linear system

- State variables (current, speed from measured terminal variables and machine model)
- Model parameters (influence of temperature on resistance and back-emf, or saturation)
- Eliminating measurement noise (combined state observer & filtering functions)

**Pros and cons**

**Pros**

- Non-linear system

**Cons:**

- Computation requirement
- Parameter sensitivity
- Initial conditions (particularly noise behaviour)
Extended Kalman filter (only for illustration)

Non-linear discrete models with white noise

\[ x(k + 1) = f(x(k), u(k)) + w(k) \]
\[ y(k) = h(x(k)) + v(k) \]

I. Prediction stage
- calculates states at time (k+1) from those at time k

(a) State estimation neglecting noise
\[ \hat{x}(k + 1 / k) = f(\hat{x}(k / k), u(k)) \]

(b) Estimation of an error covariance matrix
\[ P(k + 1 / k) = \Gamma(k)P(k / k)\Gamma^T(k) + Q \]

II. Correction stage (filtering stage)
- corrects estimation process in recursive manner based on deviation of estimated values from measured values

(c) Computation of a Kalman filter gain
\[ K(k + 1) = P(k + 1 / k)\Delta(k)^T[\Delta(k)P(k + 1 / k)\Delta(k)^T + R]^{-1} \]

(d) Update of an error covariance matrix
\[ P(k + 1 / k + 1) = [I - K(k + 1)\Delta(k)]P(k + 1 / k) \]

(e) State estimation
\[ \hat{x}(k + 1 / k + 1) = \hat{x}(k + 1 / k) + K(k + 1)[y(k + 1) - h(\hat{x}(k + 1 / k))] \]
Extended Kalman filter (only for illustration)

Linearization is required at each sampling interval

If

\[ f(x(k), u(k)) = \begin{bmatrix} f_1(x(k), u(k)) \\ f_2(x(k), u(k)) \\ \vdots \\ f_N(x(k), u(k)) \end{bmatrix} \]

Then, the Jacobian matrices are given by:

\[ \Gamma(k) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{bmatrix}_{x(k) = \hat{x}(k/k)} \]

\[ \Delta(k) = \begin{bmatrix} \frac{\partial h(x(k))}{\partial x_1} \\ \frac{\partial h(x(k))}{\partial x_2} \\ \vdots \\ \frac{\partial h(x(k))}{\partial x_N} \end{bmatrix}_{x(k) = \hat{x}(k+1/k)} \]
Surface-mounted PM motor (only for illustration)

Decoupled electrical and mechanical equations

\[
\begin{bmatrix}
\frac{d}{dt} i_\alpha \\
\frac{d}{dt} i_\beta \\
\frac{d}{dt} \omega \\
\frac{d}{dt} \theta
\end{bmatrix} =
\begin{bmatrix}
-\frac{R_s}{L_s} i_\alpha + \frac{\lambda_m}{L_s} \omega \sin \theta + \frac{u_\alpha}{L_s} \\
-\frac{R_s}{L_s} i_\beta - \frac{\lambda_m}{L_s} \omega \cos \theta + \frac{u_\beta}{L_s} \\
\frac{3}{2} n_p^2 \frac{\lambda_m}{J} (i_\beta \cos \theta - i_\alpha \sin \theta) - \frac{D}{J} \omega - \frac{n_p T_f}{J}
\end{bmatrix}
\]
Salient-pole PM motor (only for illustration)

\[
\begin{bmatrix}
    u_\alpha \\
    u_\beta
\end{bmatrix} =
\begin{bmatrix}
    R_s - 2\omega L_2 \sin 2\theta & 2\omega L_2 \cos 2\theta \\
    2\omega L_2 \cos 2\theta & R_s + 2\omega L_2 \sin 2\theta
\end{bmatrix}
\begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} +
\begin{bmatrix}
    L_0 + L_2 \cos 2\theta & L_2 \sin 2\theta \\
    L_2 \sin 2\theta & L_0 - L_2 \cos 2\theta
\end{bmatrix}
\begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} + \lambda_m \omega \begin{bmatrix}
    -\sin \theta \\
    \cos \theta
\end{bmatrix}
\]

\[
\begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} = \frac{1}{L_0^2 - L_2^2}
\begin{bmatrix}
    R_s L_0 - R_s L_2 \cos 2\theta - 2\omega L_2 L_0 \sin 2\theta \\
    - R_s L_2 \sin 2\theta + 2\omega L_2^2 + 2\omega L_2 L_0 \cos 2\theta
\end{bmatrix}
\begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix}
\]

where

\[L_\alpha = L_0 + L_2 \cos 2\theta\]
\[L_\beta = L_0 - L_2 \cos 2\theta\]
\[L_{\alpha\beta} = L_2 \sin 2\theta\]
\[L_0 = \frac{L_d + L_q}{2}\]
\[L_2 = \frac{L_d - L_q}{2}\]

where, \(L_d = d\) axis inductance \(L_q = q\) axis inductance
Comments on application to PM brushless machines

In addition to the state observer, such as position and speed

It can be used to estimate:

- Stator resistance and/or emf, for high temperature applications
- Winding inductances, for better modeling of magnetic saturation
- Load torque and/or rotor inertia, to improve dynamic speed control
- It is still far too complicated to implement the full-order EKF observer
- Hence, the reduced-order EKF is most desirable
(6) Example - Sensorless DTC based on simplified EKF

Stator voltage equation: \[ \vec{u}_s = R_s \vec{i}_s + \frac{d\vec{\psi}_s}{dt} \]

Stator flux linkage vector obtained from measured stator voltages and currents:

\[ \vec{\psi}_s = \int (\vec{u}_s - R_s \vec{i}_s) dt \]

This equation can be expressed in stationary reference frame:

\[ \psi_{s\alpha} = \int (u_{s\alpha} - R_{s\alpha} i_{s\alpha}) dt \]
\[ \psi_{s\beta} = \int (u_{s\beta} - R_{s\beta} i_{s\beta}) dt \]

Magnitude of stator flux linkage:

\[ \psi = \sqrt{\psi_{s\alpha}^2 + \psi_{s\beta}^2} \]

Electromagnetic toque equation:

\[ T = \frac{3}{2} p (\psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha}) \]
Control strategy of DTC

Block diagram of DTC for PM BLAC drive
Sensorless DTC

- Conventional approach:

  **Stator voltage equation:**
  \[ \bar{u}_s = R_s \bar{i}_s + \frac{d\bar{\psi}_s}{dt} \]

  **Stator flux linkage vector obtained from measured stator voltages and currents:**
  \[ \bar{\psi}_s = \int (\bar{u}_s - R_s \bar{i}_s) \, dt \]

  This equation can be expressed in stationary reference frame:
  \[ \psi_{s\alpha} = \int (u_{s\alpha} - Ri_{s\alpha}) \, dt \]
  \[ \psi_{s\beta} = \int (u_{s\beta} - Ri_{s\beta}) \, dt \]

  Estimated stator flux position
  \[ \theta_s = \arctan \frac{\psi_{s\alpha}}{\psi_{s\beta}} \]

  Estimated speed
  \[ \omega = \frac{d\theta}{dt} \approx \frac{\theta_s(k) - \theta_s(k-1)}{T_s} \]

  Need filters
Sensorless DTC based on simplified EKF

Output variables

\[
\begin{bmatrix}
y_1(k)
y_2(k)
\end{bmatrix} = \begin{bmatrix}
\psi_s \beta \\
\psi_s \alpha
\end{bmatrix}
\]

Input variables \( u(k) = 0 \)

For brushless ac drives, fundamentals of fluxes are sinusoidal

\[
\begin{bmatrix}
y_1(k)
y_2(k)
\end{bmatrix} = \begin{bmatrix}
\cos \theta(k) \\
\sin \theta(k)
\end{bmatrix} + \begin{bmatrix}
v_1(k) \\
v_2(k)
\end{bmatrix}
\]

State variables

\[
x = [\theta, \omega_r, w']^T
\]

State-space model

\[
x(k+1) = Fx(k) + w(k)
\]

\[
y(k) = h(x(k)) + v(k)
\]

Kalman filter gain can now be significantly simplified and is given by

\[
K = \begin{bmatrix}
0 & k_1 \\
0 & k_2 \\
0 & k_3
\end{bmatrix} \cdot \begin{bmatrix}
\cos \hat{\theta} & \sin \hat{\theta} \\
-\sin \hat{\theta} & \cos \hat{\theta}
\end{bmatrix}
\]

where \( k_1, k_2, \) and \( k_3 \) are tuning parameters, and can be pre-computed from simulations, by using, for example, the Matlab DLQE command for Kalman estimator design of discrete-time systems.
Sensorless DTC based on simplified EKF

- Simplified extended Kalman filter (EKF) based sensorless DTC:

  \[
  \begin{bmatrix}
  y_1(k) \\
  y_2(k)
  \end{bmatrix} =
  \begin{bmatrix}
  \psi_s \beta \\
  \psi_s \alpha
  \end{bmatrix}
  \]

  Recursive Algorithm:

  \[\varepsilon(k) = y_2(k) \cos \hat{\theta}(k) - y_1(k) \sin \hat{\theta}(k)\]

  \[\hat{\theta}(k + 1) = [\hat{\theta}(k) + T_s \hat{\omega}_r(k) + k_1 \varepsilon(k)]\]

  \[\hat{\omega}_r(k + 1) = \hat{\omega}_r(k) + w'(k) + k_2 \varepsilon(k)\]

  \[w'(k + 1) = w'(k) + k_3 \varepsilon(k)\]
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Phase current and stator flux linkage

(a) Phase current (simulation)

(b) Locus of stator flux linkage (simulation)

(c) Phase current (experiment)

(d) Locus of stator flux linkage (experiment)
Comparison of measured and estimated speed

(a) Using encoder for feedback, estimated speed derived from stator flux-linkage without speed filter

(b) Using estimated speed for feedback, speed derived from stator flux-linkage with speed filter

(c) Using estimated speed for feedback, speed derived from stator flux-linkage by simplified EKF
Comparison of measured and estimated rotor position

(a) Estimated directly from stator flux-linkage

(b) Estimated by using simplified EKF
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Speed and electromagnetic torque

(a) Speed (simulation)

(b) Electromagnetic torque (simulation)

(c) Speed (experiment)

(d) Electromagnetic torque (experiment)
Comparison of measured and estimated rotor position

- Position estimation:
  Since the electromagnetic torque can be estimated as:
  \[
  T = \frac{3}{2} p (\psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha})
  \]
  for a surface-mounted permanent magnet BLAC motor, \( L_d = L_q = L_s \)

  \[
  T = \frac{3p\psi_s}{4L_d L_q} [2\psi_r L_q \sin(\delta) - \psi_s (L_q - L_d) \sin 2(\delta)]
  \]
  \[
  = \frac{3p\psi_s}{2L_s} \psi_r \sin(\delta)
  \]
  Thus,
  \[
  \delta = \arctg \left( \frac{2L_s T}{3p\psi_s \psi_r} \right)
  \]
  The stator flux-linkage position is converted to the rotor position by subtracting the load angle, \( \delta \), that is
  \[
  \theta_r = \theta_s - \delta
  \]