Inverted Pendulum

This paper presents the practical implementation of the paper, "Robust Controller for Nonlinear & Unstable Systems: Inverted Pendulum[1]*", by Mr. Ashab Mirza & Capt. Dr. Sarfraz Hussain in which a simple yet effective solution to the problem is presented. This assignment was carried-out as the Final Year Project of BE (IE) studies.*

This project was proposed by SUPARCO. It simulates the dynamics of rocket vehicle in Pitch, (or in Yaw) plane during vertical flight phase. Rocket vehicle behaves very much like an ***Inverted Pendulum*** during its vertical liftoff.

The Project was awarded ***FIRST PRIZE*** in the IEEEP:[2] 15th All Pakistan Students Seminar, held on 27th December 1997, at NED UET, Karachi.

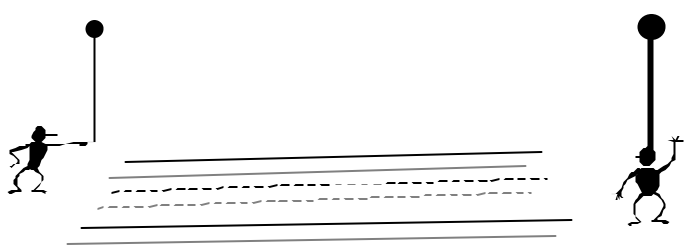
Two groups of the IIEE students carried out this assignment as their final project.

Group I: (Batch 1992-93)
Mr. Ali Irfan
Mr. Jameel Ashraf
Mr. Waheed Ahmed
Mr. Suleman Ayub K.

These students designed & developed mechanical hardware, selected the components, and gave an experimental model of the system and that of servomechanism. Finally, they verified the design by doing simulations on a software package, the MATLAB/Simulink.

Group II: (Batch 1993-94)
Mr. Imran Ali
Mr. Zia Khalid
Ms. Ratna Kumari
Ms. Iram Mahboob

This group made the project **functional**. They **redesigned** and **tuned** all the control and servo electronic circuitry. Now they are working on the implementation of the improved ***Analog*** and ***Digital*** design.



[1] Published in the Journal of AMSE France, (Dec., 2000-Vol. 55, N° 3,4) [www.amse-modeling.org]
[2] This paper was presented at the student session of **16th MULTI-TOPIC INTERNATIONAL SYMPOSIUM, OF IEEEP**, March 7, 1998. The paper was awarded **Gold Medal** for its **1st Position** in the student seminar.

INVERTED PENDULUM

Project Advisor:
ASHAB MIRZA, Member IEEE, Inc., USA
MS (Aerospace Eng.), ENSAE, France. (Dec. 1987)
BE (Electronics), DCET / NED, Karachi, Pakistan. (Mar. 1985)

ASSISTANT PROFESSOR (Chief Instructor),
Institute of Industrial Electronics Engineering, (**IIEE/PCSIR**).
ST-22/C, Block-6, Gulshan-e-Iqbal, Karachi-75300, Pakistan.
Phone: (92-21) 496 6274, 498 2353 Fax: (92-21) 498 2353
Email: **ASHABMIRZA@HOTMAIL.COM**

Research interest of the advisor is control system design for nonlinear and time-variant systems. He is working in this area since 1988, after securing his MS degree from ENSAE (Sup'Aéro), Toulouse, France.

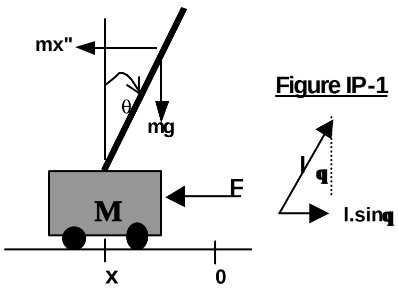


Figure IP-1

ABSTRACT

Being an unstable system, Inverted Pendulum is very common control problem being assigned to a student of Control System Engineering, (from Bachelor to postgraduate level) to control it's dynamics.

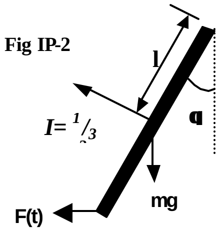
SUPARCO suggested this project because; the dynamics of the inverted pendulum is analogous to the dynamics of Pitch and Yaw motion of a rocket. As we have studied linear-control-system, the deviation of a ***rigid-body inverted-pendulum*** is curtailed within **±30°** to avoid large non-linearity, and to keep the problem simpler.

To control a rocket, the servomechanism, following a suitable control law, applies ***moments*** on the rocket. Thence it controls the flight dynamics of the rocket. The inverted pendulum (which is simulating the rocket, here) is mounted on a moving ***cart***. A servomotor is controlling the translation motion of the cart, through a belt/pulley mechanism. The motor is derived by ***servo*** electronics, which also contains ***controller-circuits***. A ***rotary-potentiometer*** is used to ***feedback*** the angular motion of the pendulum to servo electronics to generate ***actuating-signal***.

Controller circuits process the error signal, which then drives the cart through the servomotor and driving pulley/belt mechanism. To-or/and-fro motion of the cart applies moments on the inverted pendulum and thus it keeps the pendulum ***upright***.

SYSTEM DEFINITION

The inverted pendulum is mounted on a moving cart (as shown below); the cart is coupled with a servo dc-motor through pulley and belt mechanism. A rotary potentiometer is used for the position-feedback.

Fig IP-2


Referring the diagram shown above, the dynamic-equation of the system for its angular position **θ**, about the vertical axis, *(for b » 0) is* given as:

$$F.L\,COS\,(\theta) + mg.L.SIN(\theta) = I.D^2.\theta \quad \dots A.1$$

Linearized transfer function for very small θ is:

$$\frac{\theta(t)}{F(t)} = \frac{Kp}{D^2 / Ap^2 - 1} \quad \dots A.2$$

Where: $Kp = 1/mg$, $Ap = (mgL/I)^{1/2}$
& $F = (M+m).r.D\omega = -N$, $(I = L^2 \cdot m/3)$

PULLEY, BELT & CART

Load-Inertia to the motor consists of pulley (of radius ***r***) and masses of cart and pendulum. The load-torque, to be delivered by the motor is given as:

$$T_L = (M+m).r^2.D\omega \quad \dots A.3$$

Note: $T_L \propto r^2$, and $F \propto r$

MOTOR

Experimental transfer function for the armature-controlled servo dc-motor is given as:

$$\omega(rad/\sec) = Km \frac{E\,(volt)}{\tau D + 1} \quad \dots A.4$$

Where **τ** is the time constant, and depends upon the load drive. That is, heavier the load higher will be the value of **τ**. Km (radians/second/volts) is steady-state gain.

ANALYSIS

The pole position of the linearized model given in the equation ***A.2*** also shows that system is unstable, as one of the poles of the transfer function lies on the RHS of the ***s***-plane. Thus it concludes that the system is unstable.

Motor RPM with different Pulleys

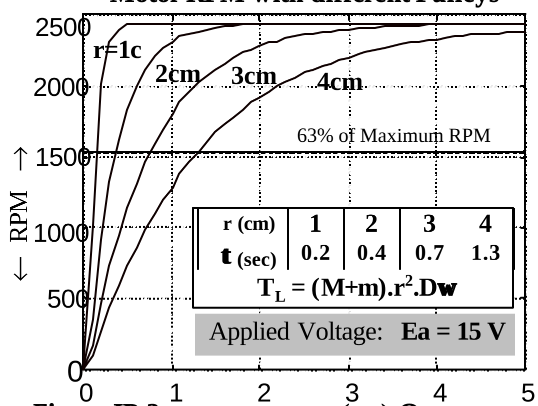
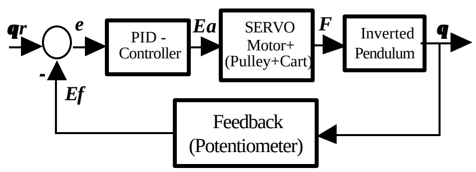


Figure IP-3

SOLUTION SEARCH

If the system is continuous linearized models for a non-linear system can be achieved. Using classical control theories, a controller can be designed for the linearized model of the system, obtained for its dynamics around equilibrium position. If control law gives very low sensitivity to the variation of the parameter of linearized model then this would be the solution to the problem of given non-linear control system.

Following is the over all block-diagram for the feedback control system:



Open loop and linearized transfer-function of the uncontrolled system is given as:

$$\frac{q(s)}{E(s)} = \frac{K.s}{(t.s + 1)(s^2 / Ap^2 - 1)} \quad \text{... A.5}$$

Where: ***K = Kp · Km · r · M***

E(s) = Error voltage

And, θ(s) = Angular position of the pendulum.

PD (proportional plus derivative) controller is simply enough to control the dynamics of inverted pendulum. However, the cart-pulley mechanism has inserted a zero at the origin, which makes the unity feedback response unstable, for any value of ***K***, (refer the root-locus plot, shown in figure IP-4). On the other hand, the dc-motor has a pole on the LHS of ***s***-plane within the bandwidth of system. Above all, the system behaves non-linearly for higher degrees of θ.

PID CONTROLLER

Solution to the problem explained above is a PID controller. Integral controller will nullify the effect of zero at origin, due to cart-pulley mechanism, whereas for a suitable gain, PD control will stabilize the transient response of the pendulum. However, if it is needed to have steady-error free response to an input command, it is required to have another integral-control action. Thus it is imperative to accommodate a PI control (or compensation to the servomechanism). In this way the ***disturbance rejection and sensitivity reduction*** benefits are achieved from the structure of the controller that includes an effective integrator.

Un-compensated Root-Locus

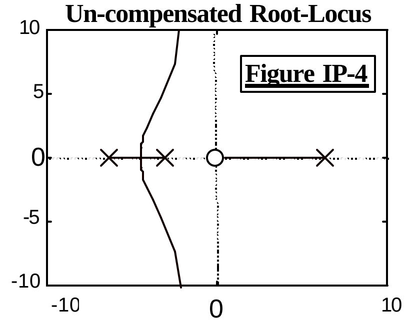


Figure IP-4

PID compensated Root-Locus

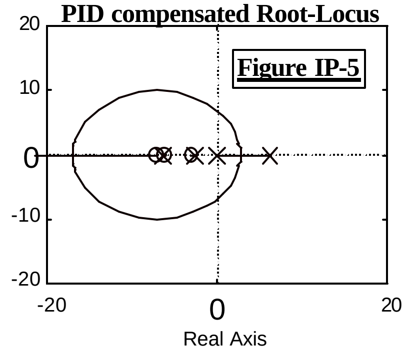


Figure IP-5

CONTROLLER DESIGN

The compensation for the servomechanism:

$$G_{C1}(s) = \frac{(t_1 s + 1)}{s} \quad \text{... A.6}$$

Where $t_1 » t$.

Now a PID controller can be designed by placing one ***pole*** at origin and two ***zeros*** on the either side of ***–A***, i.e. around ***s = –6.5***. Refer figure IP-5.

$$G_{C2}(s) = Kc \cdot (K_1 + \frac{K_2}{s} + K_3 \cdot s) \quad \text{... A.7}$$

$$G_C(s) = G_{C1}(s) \bullet G_{C2}(s) \quad \text{... A.8}$$

For the following data of our system;

Radius of Pulley,	***r = 2cm***
Time Constant of motor,	***τ = 0.23 second***
Mass of the Cart	***M = 925 gm***
Mass of the Pendulum	***m = 95 gm***
Length of Center of Gravity	***L = 23.565 cm***
Moment of Inertia (Pendulum)	***I = 5.25 gm-m²***
Gain of Motor	***Km = 17 rad/sec/V***
Length of rail	L_{rail} = 1meter
Thus:	***Ap = 6.5***
	Kp = 1.0741
	K = 0.0301

For the given data, the PID controller is found to be:

PID Controller Design

Kc	K_1	K_2	K_3
150	10/21	1	1/21

SIMULATIONS

A simulation model was tested on the ***Simulink in MATLAB*** environment. Following is the loop model used for the simulation.

The loop consists of blocks of **Controller, Motor, Feedback,** and **Cart & Pendulum.** Each block

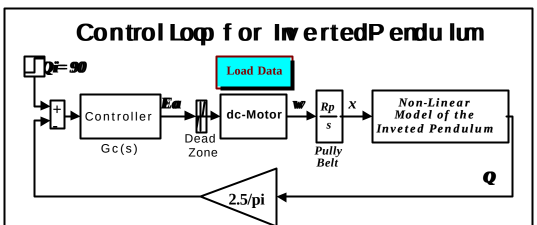


Figure IP-6

contains the mathematical model as described above.

NONLINEARITIES

Besides the nonlinear character of the system itself, following nonlinear sub-systems are further added in the loop-model of the control system, so that true simulation results could be achieved and the design could be directly implemented practically.

1. ***Saturation of Motor Supply (at ±15 Volts)***
2. ***Saturation of OP-AMP (at ±15 Volts)***
3. ***Dead Zone for Motor Driver Circuit (±0.5 Volts)***

The design was tuned by performance of repeated simulations and nearly an optimum response was obtained.

CONTROLLED TRANSIENT RESPONSE

Performance of the controller shows very low sensitivity to the variation in the Θ_{IC}, refer figure IP-7. Response of the non-linear system shows better damping characteristics then the linear one. Controller have firm hold on the dynamics of the Pendulum up to a limit of Θ_{IC}= 70°, and it controls the θ within ONE second, which can be considered as excellent performance.

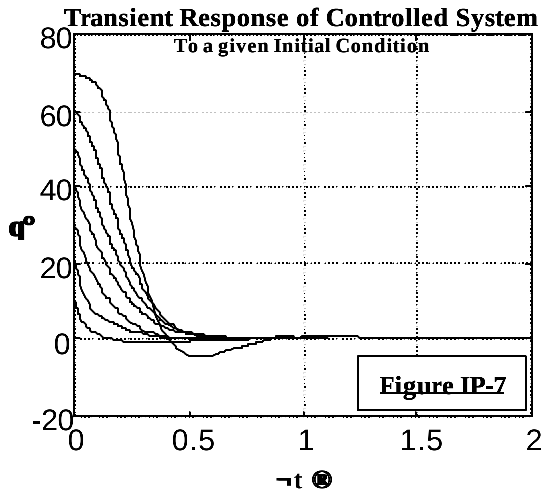


Figure IP-7

CONTROLLER IMPLEMENTATION

The PID controller was implemented in two ways:

- By Analog Circuitry, and
- By Digital Computer

Analog Controller was realized by combination of PI and PD Op-Amp based electronics circuits. Following are the networks used for PI, and PD circuits:

Besides control-electronics, there are Error-detector, Servo-Amplifier, and a Feedback-Sensor (Linear Rotary Potentiometer) in the control loop.

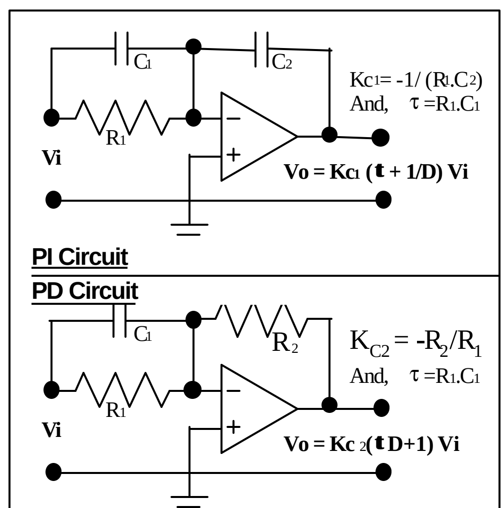


Figure IP-8

Digital Controller was realized by SBC 8051. The SBC has following characteristics:

- Clock 12 MHz, 8-bit Data bus, 4-ports
- RAM & EPROM (8K by 8)
- Peripheral Interface (8255)

- RS232 Interface

We can use it for:

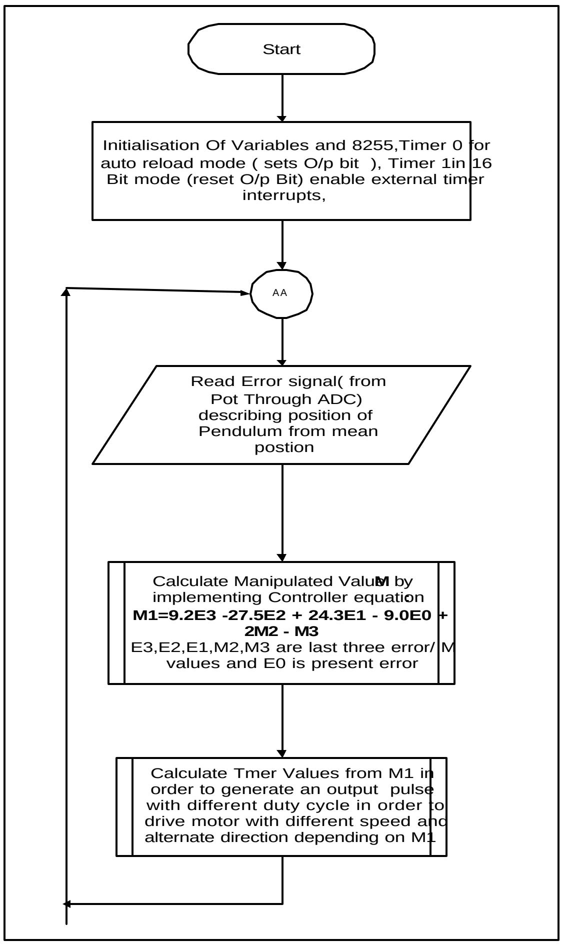
- Downloading a program from a host computer.
- Interrupt generation & Response
- Instrumentation & Control Application.

Digital Control Law was obtained by ***z***-transformation of the $G_C(s)$ given as A.8, for a given *sampling time*:

$$M1 = 9.3\,E3 - 27.5\,E2 + 24.3\,E1 - 9\,E0 + 2\,M2 - M3$$

Where ***M's*** are the present and previous values of output, to servo, and ***E's*** are the present and previous values of the input of the controller.

Following is the flow chart of the program:



FUTURE RECOMMENDATION

Implementation of digital control law was not achieved successfully as loop-processing time came out to be much higher then it was anticipated. Time limitation forced us to leave the remaining portion to be completed by the next batch (which already has started his job). It would be required to re-estimation ***gains*** for the ***digital-control-law,*** for the new sampling rate, or to, se PC or SBC having **Math-Coprocessor**, for the implementation of the same control law.

Further Development: The project may be enhanced by incorporation of cart-position control loop, which is analogous to the navigational-control of the rocket. Thus the project will become a true simulation of the ***flight dynamics*** of a rocket.

CONCLUSION

This project provided us with excellent opportunity to conceptualize, design and **successful** *implementation (of analog design, till now) of a control system problem. This exercise gave us a feel that how engineering principles can (and should) be applied to solve a real-world problem.*

We are indebted to concerned authority of SUPARCO for sponsoring our project. This project may be considered as an example of model coordination between a research organization (SUPARCO) with an engineering teaching institute (IIEE). We desire that our endeavor go a long way for these sorts of co-ordinations for an exemplary scientific future of our motherland "Pakistan"

ACKNOWLEDGMENTS

We would like to acknowledge **Mr. Ashab Mirza** for his invaluable, dedicated and painstaking guideline. Without his keen interest and his efforts, both, achieving sponsorship from SUPARCO and **successful** completion of the project was almost impossible.

APPENDEX-A

Data file for MATLAB Simulations

```
% Servo Motor & and System Data
% Experimented by: Ali Iran & Associates
% Controlled Bandwidth = 40rad/sec~ 6.4Hz

ICp = pi/6      % Initial Condition of The Pendulum  (30 degrees)
Ts = 5          % Simulation Time
Vcc = 15        % Supply Voltage
Ddz = 0.5       % Dead zone of Motor Drive
Kc = 200        % Cascade Gain [50 to 200]

% PULLEY
Rp = [8 4 2 1]*e-2/2;          % Different radii of Pulleys

% MOTOR DAT
Ra = 142.5;                    % OHMS of Armature
La = 3.5*10^(-3);              % Inductance of
Ta = La/Ra;                    % Tau in seconds
Ki = 3.1541;    % N.m/Ampere
Kb = 0.0588;    % Estimated V/rad/sec
Lcg= 23.565e-2;                % Length of CG from Bottom
Lr = 0.3;                      % Length (m) of RodJm = 7e-6;
                % Approx. (in Kg.m^2)

% CART/PENDULUM Data
Mc = 0.925;     % Mass of Cart in Kg(s)
Mh = 0.040;     % Mass of Head
Mr = 0.055;     % Mass of Rod
Mp = 0.095;     % Mass of Pendulum
Mcp = Mc + Mp; % Total Mass

% Moment of Inertia (Kg.m^2):
Ih = Lr^2 * Mh ;                          % I(Head)      Ir = (1/3) * Lr^2 * Mr;
                % I(Rod)      Ip = Ir + Ih;                  % I(Pendulum)

% MODEL of Pendulum + Cart {Gp(D)}:
Ap = (9.8*Mp*Lcg/Ip)^0.5;
Kcp = Mp * 9.8 \ Mcp;

% Motor
Xm  = Jm + Mcp * Rp^2;
Km  = Ki/Ra;
Nm = [Km];
Dm = [Xm*Ta Xm Km*Kb];
%                  End of the DATA
```