Accounting for Variability and Uncertainty in Signal and Power Integrity Modeling

Andreas Cangellaris & Prasad Sumant
Electromagnetics Lab, ECE Department
University of Illinois
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Agenda

• Are we using the might of EM CAD wisely?
  – Uncertainty and Variability (UV) in Signal/Power Integrity Modeling

• Accounting for UV in SI/PI modeling and simulation
  – Interconnect electrical modeling
  – Model order reduction in the presence of UV

• Closing Remarks
Are we using the might of EM CAD for signal integrity-aware design wisely?
Electromagnetic modeling/simulation pervasive in physical design and sign-off analysis

Input file for EDA tools
Timing/power library
Interconnect library

Floor planning
Power planning
Placement
Clock planning
Routing
Physical Layout

Sign-Off analysis

Interconnect parasitic extraction
IR-drop analysis
EM analysis
Signal-integrity analysis
Timing analysis
Physical verification
DFM

Power Integrity

- Package impedance
  - Design/layout-dependent
  - $R$ impacted by manufacturing tolerances
- On-chip grid
  - $R$ variability due to CMP
- On-chip decoupling
  - Available $C$ dependent on operation, $V_T$, $T_{ox}$, ...
Impact of Package $R$ on IR drop(*)

Relative Impact on $\Delta V_{DD}$

Package has the largest relative impact on IR drop!

Statistics (m$\Omega$):
- $\mu = 263.4$
- $\sigma = 63.6$ (24%)
- Min = 157.5
- Max = 519.6

- A 10% tolerance on resistivity is insignificant compared to the systematic variations!

(*)Sani Nassif, IBM
Variability and Uncertainty

Stacked dies

Multiple layers

Source: Future-Fab International

Source: TNCSI

ECE ILLINOIS
Department of Electrical and Computer Engineering
Variability and Uncertainty
Variability and Uncertainty
Accounting for SI/PI Modeling and Simulation
Interconnect Cross-Sectional Geometry

• Parameters
  – Trace width
  – Trace thickness
  – Trace shape
  – Pitch
  – Height above ground
  – Surface roughness
  – Substrate permittivity
  – Metallization conductivity

• Derivative quantities
  – Transmission-Line Modeling
    • R, L, C, G (per-unit-length)
    • Characteristics Impedance
    • Phase constant
    • Attenuation constant
  – Full-wave modeling
    • Metallization surface impedance
Transmission-Line Parameter Extraction

mean geometry

random geometry

conductor

ground plane

\( \varepsilon(\vec{r}) \)

\( L(\vec{r}_o) \)

\( P(\vec{r}_o)(X,Y) \) → \( P'(\vec{r}_o)(x,y) \)
Mapping between random sample and mean geometry

Random sample

\[ \int_{L} \overrightarrow{E} \cdot d\overrightarrow{l}' = V_0 \quad \Rightarrow \quad \int_{L'} \frac{\overrightarrow{D}'(r') \cdot d\overrightarrow{l}'}{\epsilon(r')} = V_0 \quad \Rightarrow \quad |D_c| \int_{L'} \frac{d\overrightarrow{l}'}{\epsilon(r')} = V_0 \]

Mean geometry

Mapping relationship

\[ |\overrightarrow{D}_0'(r_0')| = Q |\overrightarrow{D}(r_0)| \]

\[ Q = \frac{\int_{L} \frac{d\overrightarrow{l}}{\epsilon(r)}}{\int_{L'} \frac{d\overrightarrow{l}'}{\epsilon(r')}} \]
Electric flux density computation using solution on mean geometry

Position-dependent flux length/gap on mean geometry:

\[
G(r_0) = \varepsilon(r_0) \frac{V_0}{D(r_0)} \quad \Rightarrow \quad \int_L \frac{1}{\varepsilon(r)} \, dl = \frac{G(r_0)}{\varepsilon(r_0)} \quad \text{(exact)}
\]

Flux length on random sample:

\[
L'(r_0) \approx L(r_0) - v(r_0)
\]

\[
\int_{L'} \frac{1}{\varepsilon(r')} \, dl' \approx \int_{L-v} \frac{1}{\varepsilon(r)} \, dl \quad \Rightarrow \quad \int_{L'} \frac{1}{\varepsilon(r')} \, dl' \approx \frac{G(r_0)}{\varepsilon(r_0)} - \frac{v(r_0)}{\varepsilon(r_0)}
\]

Electric flux density on random sample using mean geometry:

\[
\left| \vec{D}'(r_0) \right| = \left| \vec{D}(r_0) \right| \frac{G(r_0)}{G(r_0) - v(r_0)}
\]
Computation of capacitance of random sample

Capacitance on random sample:

\[ C' = \int_{C_{rs}} \left| \overrightarrow{D}'(\overrightarrow{r}') \right| dl' \]

\[ C' = \int_{C_{mean}} \frac{G(\overrightarrow{r}_0)}{G(\overrightarrow{r}_0) - \nu(\overrightarrow{r}_0)} \left| \overrightarrow{D}(\overrightarrow{r}_0) \right| |J| dl \]

Where \( J \) represents the Jacobian, the map between the random and mean geometry:
Representing uncertainty using polynomial chaos

Polynomial chaos expansion:

Orthogonal polynomials are random variables

\[ u(\vec{r}, \theta) = \sum_{i=0}^{\infty} \hat{a}_i(\vec{r})\Psi(\xi(\theta)) \]

Coefficients: functions of space

Type of polynomials depends on distribution of input random variable

- Gaussian distribution, Hermite polynomial chaos

\[ \Psi_0(\xi) = 1, \Psi_1(\xi) = \xi, \Psi_2(\xi) = \xi^2 - 1, \Psi_3(\xi) = \xi^3 - 3\xi, \Psi_4(\xi) = \xi^4 - 6\xi^2 + 3, \ldots \]

Truncated polynomial chaos expansion:

- Number of different input random variables: \( n \)
- Order of polynomials: \( p \)

\[ u(\vec{r}, \theta) = \sum_{i=0}^{N} \hat{a}_i(\vec{r})\Psi(\xi(\theta)) \]

\[ N + 1 = \frac{(n + p)!}{n! p!} \]
Computing stochastic capacitance

Displacement of random geometry from the mean geometry:

\[ \nu(\bar{r}, \theta) = \sum_{i=0}^{N} v_i(\bar{r}) \Psi(\xi(\theta)) \]

Use relationship between random and mean flux length:

\[ \tilde{G}(\bar{r}, \theta) = \bar{G}(\bar{r}) - \nu(\bar{r}, \theta) \]

Stochastic Electric Flux Density:

\[ |\tilde{D}'(\bar{r}')| \approx |\tilde{D}_0(\bar{r})| \frac{G(\bar{r})}{G(\bar{r}) - \nu(\bar{r})} \]

\[ |\tilde{D}'(\bar{r})| \approx \left(1 + \frac{\nu(\bar{r}, \theta)}{G(\bar{r})} + \frac{\nu^2(\bar{r}, \theta)}{G^2(\bar{r})} + \frac{\nu^3(\bar{r}, \theta)}{G^3(\bar{r})} + \ldots\right) |\tilde{D}_0(\bar{r})| \]

Stochastic Capacitance:

\[ \tilde{C} = \sum_{i=0}^{N} C_i(\bar{r}) \Psi(\xi(\theta)) \]

\[ \tilde{C} \approx \int_S \left(1 + \frac{\nu(\bar{r}, \theta)}{G(\bar{r})} + \frac{\nu^2(\bar{r}, \theta)}{G^2(\bar{r})} + \frac{\nu^3(\bar{r}, \theta)}{G^3(\bar{r})} + \ldots\right) |J| \tilde{D}_0 \cdot ds \]
The height of the conductor above the ground plane ‘H’ is uncertain.

Mean Geometry dimensions: L = 1 um, T = 0.1 um, H = 0.2 um

\[ H(\theta) = H_0 (1 - \nu \xi(\theta)) \]

- Where \( H_0 \) is the mean height
- \( \xi \) is a Gaussian random variable with mean 0 and variance 1
Single trace over ground plane

Displacement of random geometry from the mean geometry

\[ v(r, \theta) = \nu \xi H_0 \]

\[ \widetilde{C} \approx \int_S \left( 1 + \frac{\nu H_0}{G(\bar{r})} \xi + \frac{\nu^2 H_0^2}{G^2(\bar{r})} \xi^2 + \ldots \right) |J| |\widetilde{D}_0| dl \]

Second-order Hermite polynomial chaos for stochastic capacitance

\[ \widetilde{C} = C_0 + C_1 \xi + C_2 (\xi^2 - 1) \]

\[ C_0 = \int_S \left( 1 + \left( \frac{\nu H_0}{G(\bar{r})} \right)^2 \right) |J| |\widetilde{D}_0| dl \]

\[ C_1 = \int_S \left( \frac{\nu H_0}{G(\bar{r})} \right) |J| |\widetilde{D}_0| dl, \quad C_2 = \int_S \left( \frac{\nu H_0}{G(\bar{r})} \right)^2 |J| |\widetilde{D}_0| dl \]

Only one deterministic run needed to get \( |\widetilde{D}_0| \)
Single trace over ground plane

<table>
<thead>
<tr>
<th>%change in H</th>
<th>Monte Carlo</th>
<th>FEM based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std deviation</td>
</tr>
<tr>
<td>10%</td>
<td>329.7429</td>
<td>11.5365</td>
</tr>
<tr>
<td>20%</td>
<td>331.0031</td>
<td>23.6053</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Std deviation</td>
</tr>
<tr>
<td>10%</td>
<td>329.83</td>
<td>11.43</td>
</tr>
<tr>
<td>20%</td>
<td>331.48</td>
<td>23.31</td>
</tr>
</tbody>
</table>

Self-capacitance (pF/m)
Single trace over ground plane

Capacitance (pF/m)

<table>
<thead>
<tr>
<th>%change in L</th>
<th>Monte Carlo</th>
<th>FEM based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std deviation</td>
</tr>
<tr>
<td>10%</td>
<td>331.0028</td>
<td>7.26</td>
</tr>
<tr>
<td>20%</td>
<td>331.5143</td>
<td>14.92</td>
</tr>
</tbody>
</table>

Simulation time comparison
Time for 1 Capacitance extraction run ~1.2 s
- Monte Carlo: Time for 10000 runs ~ 12000 s
- Our approach ~ 2.0 s
Coupled symmetric microstrip

Mean Geometry dimensions: L = 1 um, S = 0.15 um, H = 0.2 um

\[ \varepsilon = 1 \]

\[ \varepsilon = 4 \]

\[ \varepsilon = 7 \]

% change in H and S

<table>
<thead>
<tr>
<th>% change in H and S</th>
<th>Monte Carlo</th>
<th>FEM based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std deviation</td>
</tr>
<tr>
<td>10%</td>
<td>368.7324</td>
<td>13.04</td>
</tr>
<tr>
<td>20%</td>
<td>370.2421</td>
<td>26.83</td>
</tr>
</tbody>
</table>

Self-capacitance (pF/m)
Microstrip with multi-dielectric substrate

\[ \varepsilon = 1 \]

\[ \varepsilon = 4 \]

\[ \varepsilon = 4 \]

\[ \varepsilon = 8 \]

\[ \varepsilon = 7 \]

Self-capacitance (pF/m)

<table>
<thead>
<tr>
<th>%change in each layer below Conductor</th>
<th>Monte Carlo (10000)</th>
<th>FEM based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std deviation</td>
</tr>
<tr>
<td>10%</td>
<td>268.94</td>
<td>6.76</td>
</tr>
<tr>
<td>20%</td>
<td>269.46</td>
<td>13.56</td>
</tr>
</tbody>
</table>
Remarks

• Expedient way for handling statistical variability in interconnect cross-sectional geometry
  – p.u.l. capacitance extraction 100x - 1000x faster than standard Monte Carlo

• Approach independent of the field solver used
Deterministic Model Order Reduction

\[
(Y_{\text{org}} + sZ_{\text{org}} + s^2 P_{\text{e org}})x = sB_{\text{org}} I
\]

\[
V = L_{\text{org}}^H x \quad \text{Order: } N
\]

Model Order Reduction

e.g. Krylov subspace based methods

Projection matrix \( F \)

\[
Y = F^H Y_{\text{org}} F
\]

\[
Z = F^H Z_{\text{org}} F
\]

\[
P_{\text{e}} = F^H P_{\text{e org}} F
\]

\[
(Y + sZ + s^2 P_{\text{e}})x = sB I
\]

\[
V = L^H x \quad \text{Order } n \ll N
\]

Generalized Multiport Impedance Matrix using reduced model:

\[
Z_G(s) = sL^H (Y + sZ + s^2 P_{\text{e}})^{-1} B
\]
Model order reduction under uncertainty

Deterministic Reduced Order Model

\[(Y + sZ + s^2 Pe) x = sBI\]
\[y = L^H x\]

Stochastic Reduced Order Model

\[(\tilde{Y} + s\tilde{Z} + s^2 \tilde{Pe}) \tilde{x} = sBI\]
\[\tilde{V} = L^H \tilde{x}\]

\[\tilde{Y} = \tilde{F}^H \tilde{Y}_{org} \tilde{F}\]
\[\tilde{Z} = \tilde{F}^H \tilde{Z}_{org} \tilde{F}\]
\[\tilde{Pe} = \tilde{F}^H \tilde{Pe}_{org} \tilde{F}\]

Represent stochastic system matrices using polynomial chaos expansion:

\[\tilde{Y}_{org} = Y_0 + Y_1 \xi_1 + Y_2 \xi_2, \quad \tilde{Z}_{org} = Z_0 + Z_1 \xi_1 + Z_2 \xi_2\]
\[\tilde{Pe}_{org} = P_0 + P_1 \xi_1 + P_2 \xi_2, \quad \tilde{F} = F_0 + F_1 \xi_1 + F_2 \xi_2\]
Augmented Stochastic Reduced Order Model

\[(Y_0 + Y_1\xi_1 + Y_2\xi_2) + s(Z_0 + Z_1\xi_1 + Z_2\xi_2) + s^2(Pe_0 + Pe_1\xi_1 + Pe_2\xi_2)](x_0 + x_1\xi_1 + x_2\xi_2) = s(B_0 + B_1\xi_1 + B_2\xi_2)I\]

Augmented reduced order model

\[(Y_{aug} + sZ_{aug} + s^2Pe_{aug})x_{aug} = sB_{aug}I\]

\[V_{aug} = L_{aug}^H x_{aug}\]

\[Z_{aug} = sL_{aug}^H (Y_{aug} + sZ_{aug} + s^2Pe_{aug})^{-1} B_{aug}\]

Order: 3n << N
Computing polynomial chaos coefficients

Coefficient matrices in polynomial chaos expansion:

- Integrate over the random space and use orthogonality of the polynomials

\[ \iint \tilde{Y}_{\text{org}} \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 = \iint (Y_0 + Y_1 \xi_1 + Y_2 \xi_2) \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 \]

\[ Y_0 = \iint \tilde{Y}_{\text{org}} \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 \]

\[ \iint \tilde{Y}_{\text{org}} \xi_1 \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 = \iint (Y_0 + Y_1 \xi_1 + Y_2 \xi_2) \xi_1 \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 \]

\[ Y_1 = \iint \tilde{Y}_{\text{org}} \xi_1 \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 \]
Smolyak Sparse Grid Integration

\[ I(f) = \int \int \tilde{Y}_{\text{org}} \rho_1(\xi_1) \rho_2(\xi_2) d\xi_1 d\xi_2 \approx \sum w_j f(u_j) \]

1-d integration rule: e.g. using chebyshev polynomial extrema

\[ I(f) = \sum_{j=1}^{q} f(u_j) w^j \quad u^j = -\frac{\cos \pi (j-1)}{q-1}, \quad j = 1...q \]

The case of multiple random variables:

**Deterministic Cartesian Product (DCP) rule:**

\[ I^Q[f] \equiv \left( I_{i_1}^{q_1} \otimes \ldots \otimes I_{i_N}^{q_N} \right)[f] \]

\[ = \sum_{i_1=1}^{q_1} \ldots \sum_{i_N=1}^{q_N} f(u_1^{i_1}, \ldots, u_N^{i_N}).(w_1^{j_1} \otimes \ldots \otimes w_N^{j_N}) \]

Number of calculations \( \propto q^N \)

**Smolyak Sparse Grid Algorithm**

Idea: Not all points are equally important; hence, discard the least important ones

\[ I^Q(f) \equiv A(J, N) = \]

\[ = \sum_{J-N+1 \leq |i| \leq J} (-1)^{J-|i|} \binom{N-1}{J-|i|} (I_{i_1} \otimes \ldots \otimes I_{i_N}) \]

Number of calculations \( \propto q \left( \log q \right)^{N-1} \)
Comparison of grids generated using Tensor Product and Smolyak Algorithm

- Tensor product grid (81 points)
- Smolyak Sparse grid (29 points)

\[ Q \propto q^N \]

\[ Q \propto N^p \]

- \( q \): number of points in 1-d rule
- \( N \): number of random dimensions
- \( p \): level in Smolyak algorithm
Algorithm for Stochastic MOR

- Represent uncertainty in the original system matrices through polynomial chaos expansion
- Generate sparse grid points and their corresponding weights using Smolyak Sparse grid algorithm
- Compute the transformation matrix through MOR of individual systems corresponding to Smolyak sparse grid points
- Compute the stochastic transform matrix
- Define the stochastic reduced order model

\[ \tilde{Y}_{org} = Y_0 + Y_1 \xi_1 + Y_2 \xi_2 \]

\[ \theta_M = \{\xi_i\}, w_M = \{w_i\} \]

\[ (Y_{org} + sZ_{org} + s^2 P_{org})x = sB_{org} I \]

\[ x = F_i z \]

\[ (Y + sZ + s^2 P)e)x = sBI \]

\[ \tilde{F} = F_0 + F_1 \xi_1 + F_2 \xi_2 \]

\[ (Y_{aug} + sZ_{aug} + s^2 P_{aug})x_{aug} = sB_{aug} I \]
Example Study: Terminated coaxial cable

- Air-filled coaxial cable, terminated at a resistive load:
  - L=1m, inner radius=5mm, outer radius=10mm
  - FEM system (Y,Z,Pe) of order 36840
  - Reduced order system of order 20.

- Randomness in two inputs
  - Permittivity: uniform random variable in [3.4-4.4]
  - Load resistance: uniform random variable in [25-35] ohms

- Monte Carlo: 10201 simulations
- Smolyak: 29 points
\[ \text{Re}\{Z_{\text{in}}(f)\} \]

- Mean of real part of input impedance
- Standard deviation of real part of input impedance
Corner simulations vs. stochastic simulations

• Standard practice to simulate corners for accounting for variability
• Corner simulations can be ‘conservative’
• Coaxial cable example – consider corner values for random input parameters $(\varepsilon, R_L)$
  – (3.6,25)
  – (4.0,30)
  – (4.4,35)
• Compare with information generated using stochastic MOR
Corner vs. stochastic simulation

- Corner simulation appears very conservative
- Mean parameter solution is not accurate compared to the mean of stochastic simulation
Remarks

• Very good accuracy obtained with 100x to 1000x improvement in computation time compared to standard Monte Carlo.
  – Stochastic MOR model appropriate for time-domain simulations

• Stochastic MOR can have advantages over traditional corner based simulations

• Approach independent of the deterministic MOR method
• Variability and uncertainty is not a curse
  – It is an essential part of the dynamo of our evolution toward the next, more advanced state
  “Chaos is the score upon which reality is written” – Henry Miller

• We should embrace uncertainty as an opportunity for tackling complexity
  – It will make our design tools more agile, more useful, and more conducive to complex system design flow

• It is critical for universities to pave the way down this path
  – Our future will not be built by deterministically-minded technologists and innovators