Transmission Lines with Reactive Terminations

J. L. Drewniak and R. E. DuBroff
EMC Laboratory, University of Missouri-Rolla

Objectives:

1. Study reflection from reactive terminations.
2. Investigate differences in typical ``matching" schemes.

Equipment:

1. Tektronix 11801B Mainframe with SD-24 TDR/Sampling Head
2. Capacitive and inductive loads: C, L, series RL and RC, and parallel RL and RC.

The reflection coefficient concept in the time-domain is valid only for resistive loads. For reactive elements, enforcing Kirchhoff's voltage and current laws at the load leads to a differential equation that must be solved for the reflected wave V^-. The solutions for V^- and the total voltage at the load for C, L, series RC and RL, and parallel RC and RL terminations are given below. A schematic representation of the transmission-line system is shown in Figure 1. The voltage of the step generator is V_g, and R_o = 50 Ω is the characteristic impedance of the transmission line of length ℓ. The delay time of the line is $t_d = \dfrac{\ell}{v_p}$.

- The solution for a capacitive load C is

$$V^-(\ell,t) = V_g \frac{R_o}{R_o + R_g}(1 - 2e^{-(t-t_d)/\tau})$$

$$V_L = 2V_g \frac{R_o}{R_o + R_g}(1 - e^{-(t-t_d)/\tau})$$

for $t_d < \mathrm{t} < 3t_d$, where $\tau = R_o C$ is the time-constant. When $R_g = R_o$

$$V_L = V_g(1 - e^{-(t-t_d)/\tau}) \qquad t > t_d$$

- The solution for a parallel $R_L C$ load is

$$V^-(\ell,t) = V_g \frac{R_o}{R_o + R_g}\left(\frac{R_L - R_o}{R_L + R_o} - 2\frac{R_L}{R_L + R_o} e^{-(t-t_d)/\tau}\right)$$
$$V_L = 2V_g \frac{R_o}{R_o + R_g}\frac{R_L}{R_L + R_o}(1 - e^{-(t-t_d)/\tau})$$

for $t_d < t < 3t_d$, where $\tau = (R_o \| R_L)C = \frac{C}{G_L + G_o}$ is the time-constant, and $G = \frac{1}{R}$. When $R_L = R_g = R_o$

$$V_L = \frac{1}{2}V_g(1 - e^{-(t-t_d)/\tau}) \qquad t > t_d$$

and $\tau = \frac{1}{2}R_o C$.

- The solution for a series $R_L C$ load is

$$V^-(\ell,t) = V_g \frac{R_o}{R_o + R_g}\left(1 - 2\frac{R_o}{R_o + R_L} e^{-(t-t_d)/\tau}\right)$$
$$V_L = 2V_g \frac{R_o}{R_o + R_g}\left(1 - \frac{R_o}{R_L + R_o} e^{-(t-t_d)/\tau}\right)$$

for $t_d < t < 3t_d$, where $\tau = (R_o + R_L)C$ is the time-constant. When $R_L = R_g = R_o$

$$V_L = V_g\left(1 - \frac{1}{2}e^{-(t-t_d)/\tau}\right) \qquad t > t_d$$

and $\tau = 2R_o C$.

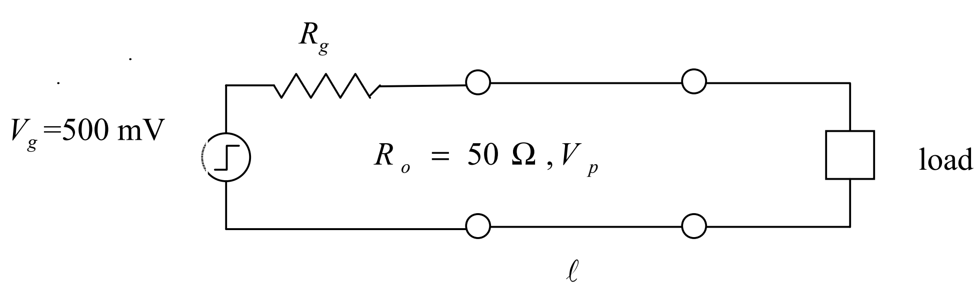

Figure 1: Schematic representation of a reactively loaded transmission line of length ℓ.

- The solution for an inductive load L is

$$V^{-}(\ell,t) = 2V_g \frac{R_o}{R_o + R_g}(e^{-(t-t_d)/\tau} - \frac{1}{2})$$

$$V_L = 2V_g \frac{R_o}{R_o + R_g} e^{-(t-t_d)/\tau}$$

for t_d < t < 3 t_d, where $\tau = \frac{L}{R_o}$ is the time-constant. When $R_g = R_o$

$$V_L = V_g e^{-(t-t_d)/\tau} \qquad t > t_d$$

- The solution for a series $R_L L$ load is

$$V^{-}(\ell,t) = V_g \frac{R_o}{R_o + R_g}(2\frac{R_o}{R_o + R_L} e^{-(t-t_d)/\tau} + \frac{R_L - R_o}{R_L + R_o})$$

$$V_L = 2V_g \frac{R_o}{R_o + R_g}(\frac{R_L}{R_L + R_o} + \frac{R_o}{R_L + R_o} e^{-(t-t_d)/\tau})$$

for t_d < t < 3 t_d, where $\tau = \frac{L}{R_o + R_L}$ is the time-constant. When $R_L = R_g = R_o$

$$V_L = \frac{1}{2} V_g (1 + e^{-(t-t_d)/\tau}) \qquad t > t_d$$

and $\tau = \frac{1}{2}\frac{L}{R_o}$.

- The solution for a parallel $R_L L$ load is

$$V^{-}(\ell,t) = V_g \frac{R_o}{R_o + R_g}(2\frac{R_L}{R_o + R_L} e^{-(t-t_d)/\tau} - 1)$$

$$V_L = 2V_g \frac{R_o}{R_o + R_g}\frac{R_L}{R_L + R_o} e^{-(t-t_d)/\tau}$$

for t_d < t < 3 t_d, where $\tau = \frac{L}{R_L \parallel R_o} = L(G_L + G_o)$ is the time-constant. When $R_L = R_g = R_o$

$$V_L = \frac{1}{2} V_g (1 + e^{-(t-t_d)/\tau}) \qquad t > t_d$$

and $\tau = \frac{1}{2}\frac{L}{R_o}$.

When the generator is matched to the characteristic impedance of the line, i.e., $R_g = R_o$, as in the case of the test configuration, $V_g \frac{R_o}{R_o + R_g} = \frac{1}{2} V_g$, and all of the above solutions are valid for $t > t_d$.

The primary difference between the solution sets in the case of either a capacitive or inductive load is the behavior at $t = t_d$, the time constant, and the final voltage value. TDR traces for the three different termination cases, C, parallel $R_L C$, and series $R_L C$ are shown in Figure 2. In these cases, $R_L = R_g = R_o$, and $V_g = 500mV$. The voltage at the load is the same as the TDR trace after $53ns$, but shifted in time by t_d.

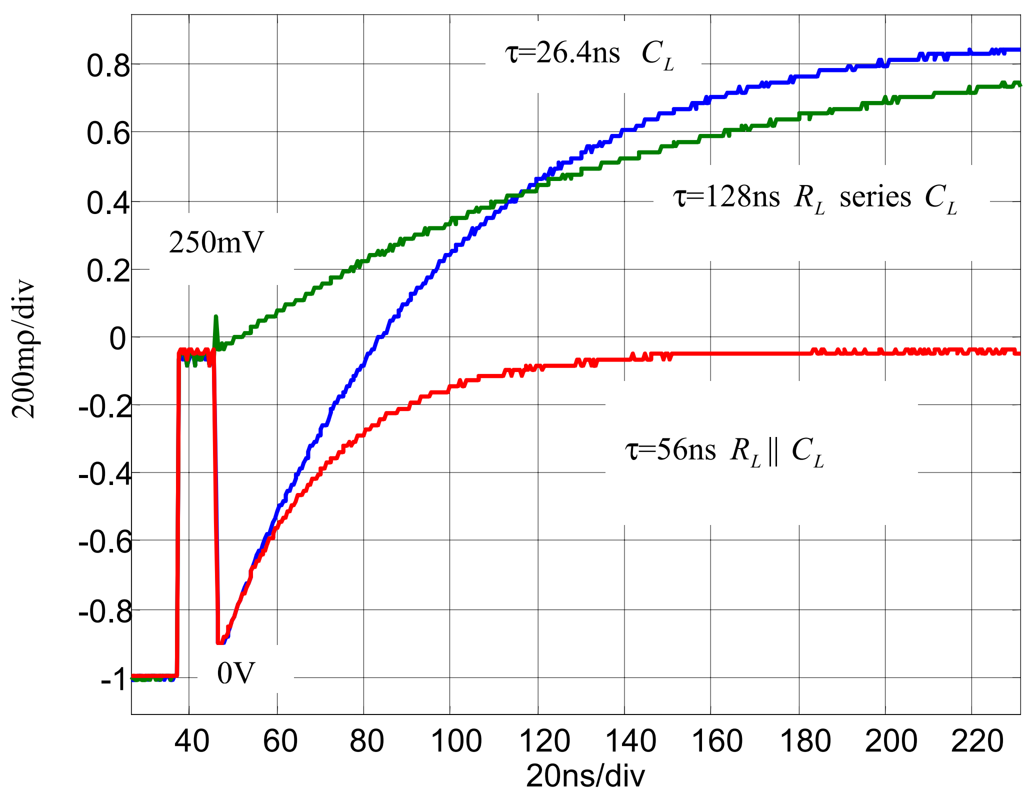

Figure 2:TDR traces for a single C, parallel $R_L C$, and series $R_L C$ terminations on a 50Ω transmission line.

For a simple capacitive load C, the voltage at the load at $t = t_d$ is zero, since the voltage across C cannot change instantaneously. Consequently, there is a jump to zero volts in the TDR trace when the incident voltage wave reaches the load. The case is similar for a parallel $R_L C$ load. The voltage at the load for a series $R_L C$ termination (across both elements) by contrast is not initially zero. At $t = t_d$ the capacitor is a short, since the voltage across it cannot change instantaneously, and the line appears matched. The final value of the voltage then increases to $V_g = 500mV$. A significant difference between the series and parallel-terminated cases is the DC power draw [1]. The time constant in the three cases C, parallel $R_L C$, and series $R_L C$ were $26.4ns$, $56ns$, and $128ns$, respectively, for the three different nominally $1000pF$ capacitors used.

The TDR traces for an inductive load L, and parallel and series $R_L L$ loads are shown in Figure 3. The final value of the voltage For the L and parallel $R_L L$ cases is $0V$.

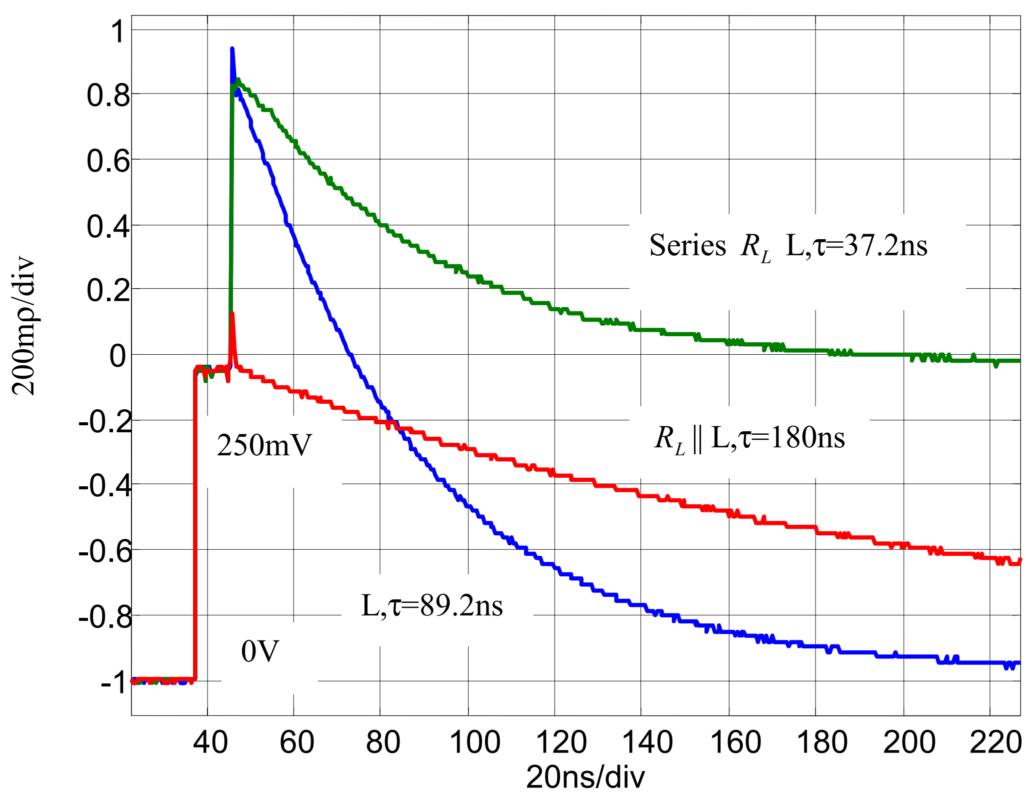

Figure 3: TDR traces for a single L, a parallel $R_L L$, and series $R_L L$ terminations on a 50Ω transmission line.

By contrast, the final value of the voltage for the series $R_L L$ termination is $250mV$. The time constants for the three cases of L, a parallel $R_L L$, and series $R_L L$ loads are $\tau = 89.2ns$, $180ns$, and $37.2ns$ for the three different nominally $4.6\mu H$ inductors.

References

[1]. H. W. Johnson and M. Graham, *High-Speed Digital Design: A Handbook of Black Magic*, Prentice-Hall, Englewood Cliffs, NJ, 1993.

[2]. N. N. Rao, *Elements of Engineering Electromagnetics, 4th ed.*, Prentice-Hall, Englewood Cliffs, NJ, 1994.