國立交通大學

Transceiver Designs for Multicarrier Transmission

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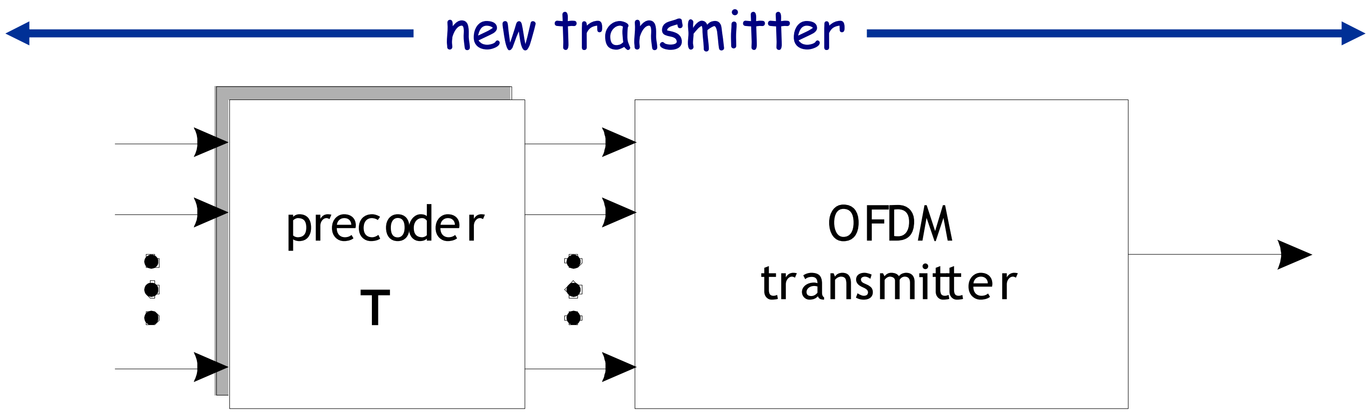
OFDM systems with precoders

transmitter of a general block transceiver by precoding:



(orthogonal frequency division multiplexing)

Aim: Design the transceiver (channel independent transmitter for minimum BER (bit error rate)

Earlier works

Block transceivers optimal in various senses have been investigated

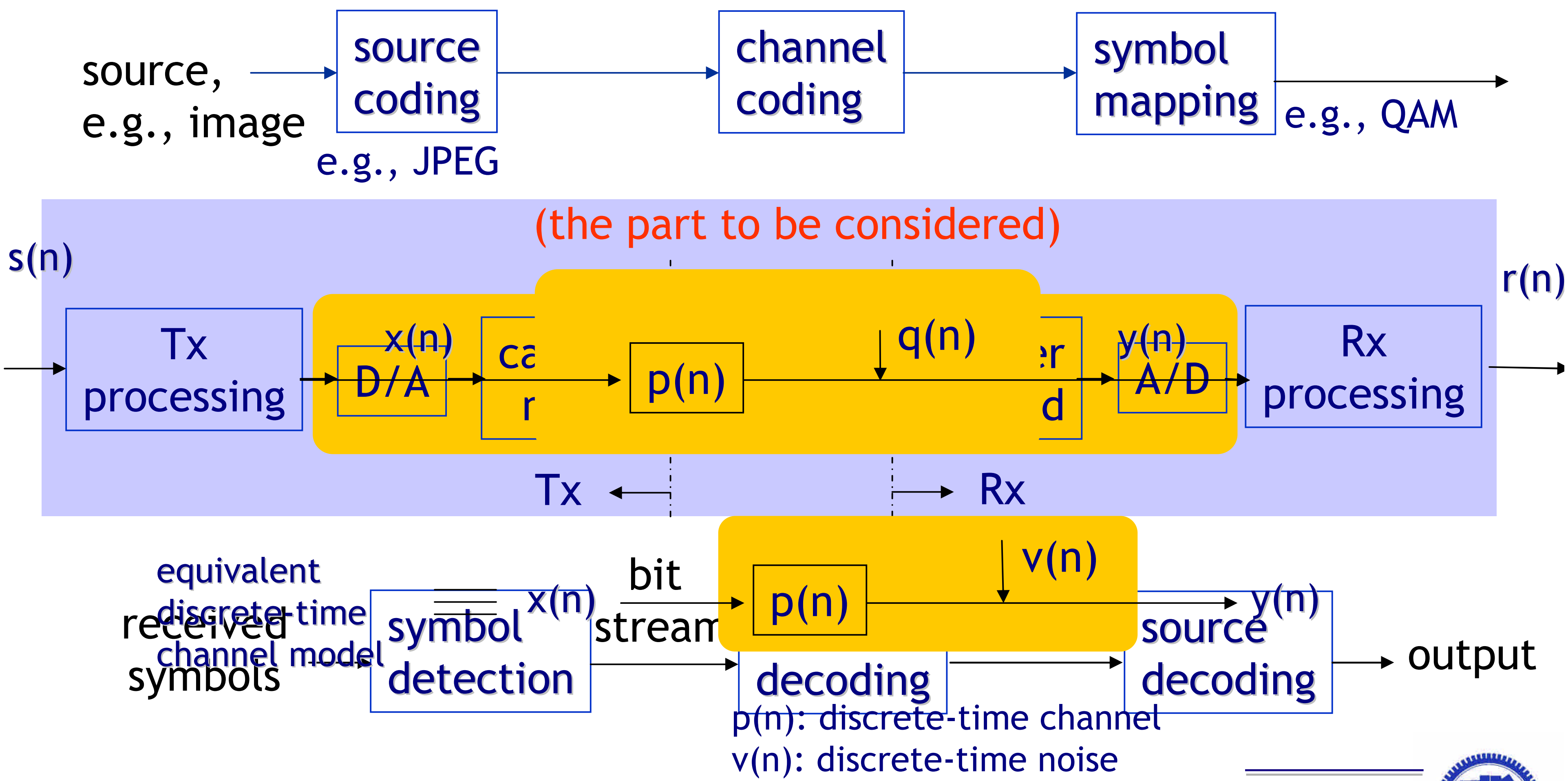
- minimum mean square error
  Scaglione, Giannakis, Barbarossa, 1999,
  Palomar, Cioffi, Lagunas, 2003
- maximum information rate
  Al-Dhahir and Cioffi, 1996, Scaglione, Giannakis, Barbarossa, 1999
- maximum bit rate,
  Yasotharan, 2006
- minimum transmission power,
  Lin, Phoong, 2001
- minimum bit error rate
  Ding, Davidson, Luo, Wong, 2003, Lin, Phoong, 2003

** In most earlier works, the transmitters are channel dependent.

Outline

- Introduction and preliminaries
  - channel model
  - guard interval, block transmission
- OFDM systems
- SC-CP systems
- Precoded OFDM systems
- Zero-forcing precoded OFDM systems
- MMSE precoded OFDM systems
- Examples and conclusion

Modern Communication System



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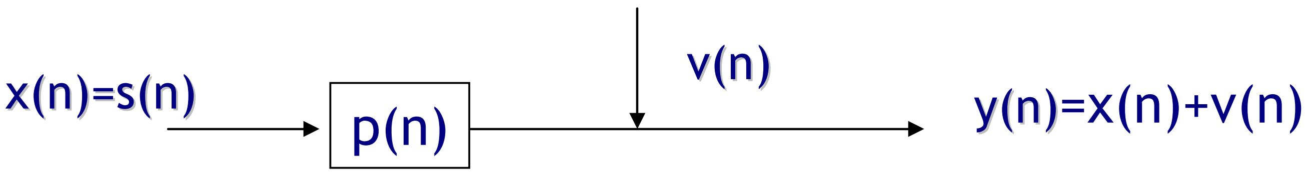
Ex. AWGN channel, QPSK

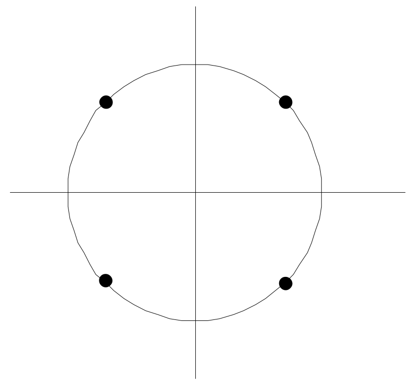
AWGN: $P(z)=1$
v(n): white Gaussian noise, variance N_0

-- no Tx processing, no Rx processing



-- s(n): QPSK, variance Es

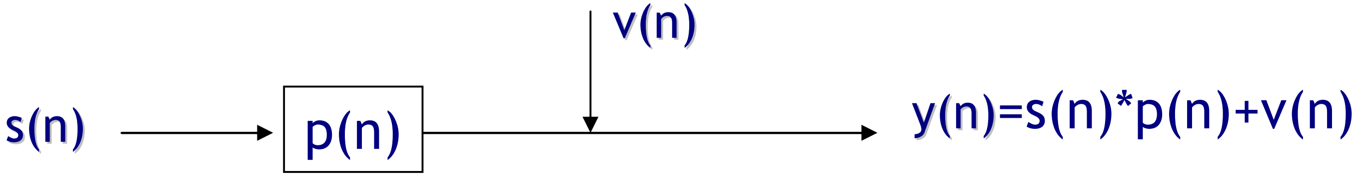
$$s(n) = \sqrt{\frac{E_s}{2}} \pm j\sqrt{\frac{E_s}{2}}, \quad |s(n)| = E_s$$



BER (bit error rate)

$$\mathcal{P} = Q\left(\sqrt{\gamma}\right), \quad \text{where} \quad \gamma = E_s/N_0$$

ISI channel



suppose P(z) is FIR, order L: $P(z) = p(0) + p(1)z^{-1} + \cdots + p(L)z^{-L}$

$$y(n) = \sum_{k=0}^{L} p(k)s(n-k) + v(n)$$
$$= p(0)s(n) + \Big[p(1)s(n-1) + \cdots + p(L)s(n-L)\Big] + v(n)$$

L ISI (inter-symbol interference) terms

- before symbol-by-symbol detection, ISI terms need to be removed—channel equalization, which can be done using Tx/Rx processing

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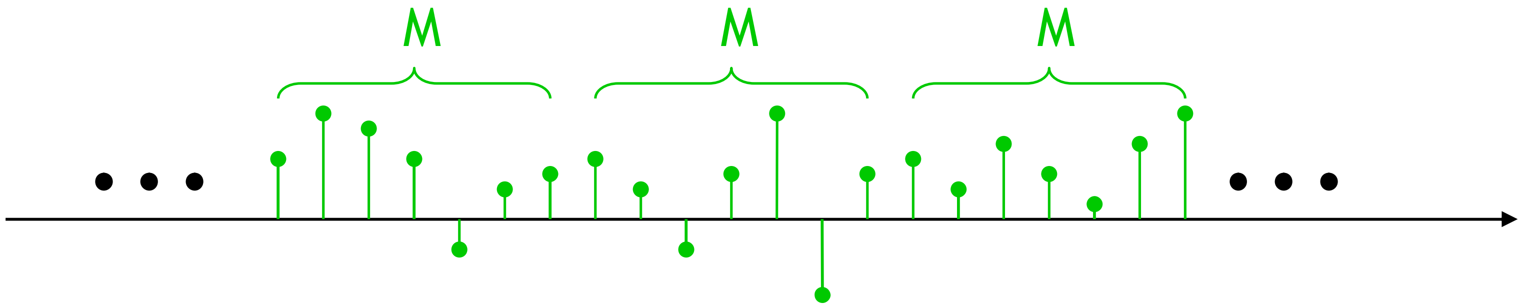
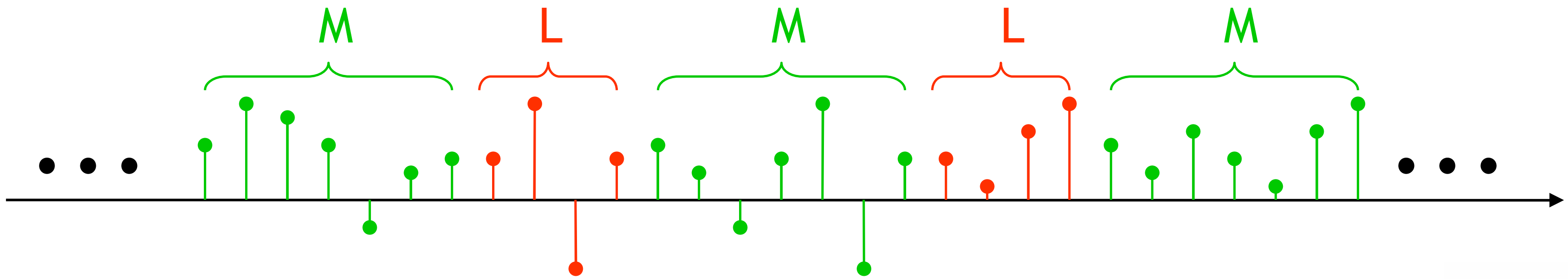
Using Guard Intervals for ISI Control

* partition into blocks of size M



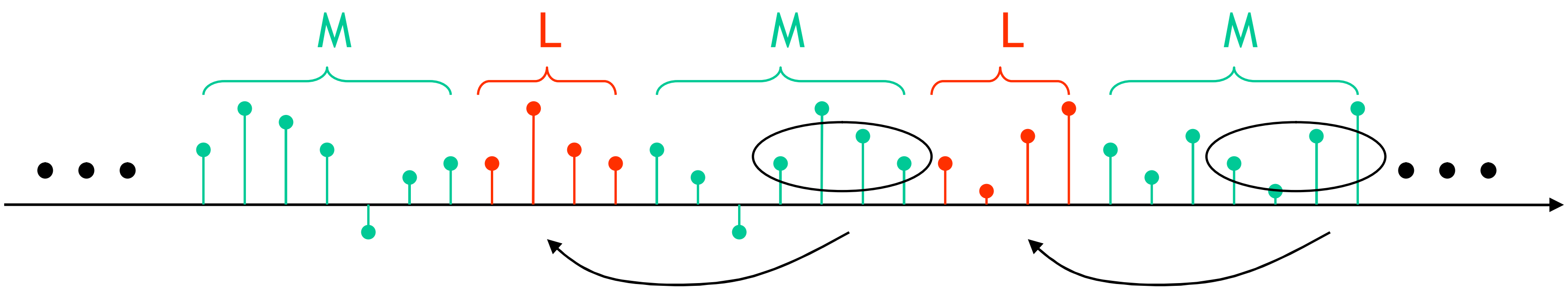
* insert guard interval of length L between every 2 blocks,
  samples in guard interval usually depends on the following block



A commonly used guard interval

★ **cyclic prefixing:**

the prefix is obtained by copying the last L samples from each block



IBI Free Property

- guard interval acts as a buffer between adjacent blocks
- no need of considering inter-block interference (IBI), i.e., IBI free
- only need to consider the transmission of a single block—one shot block transmission, though samples are transmitted consecutively
- ISI from only the same block

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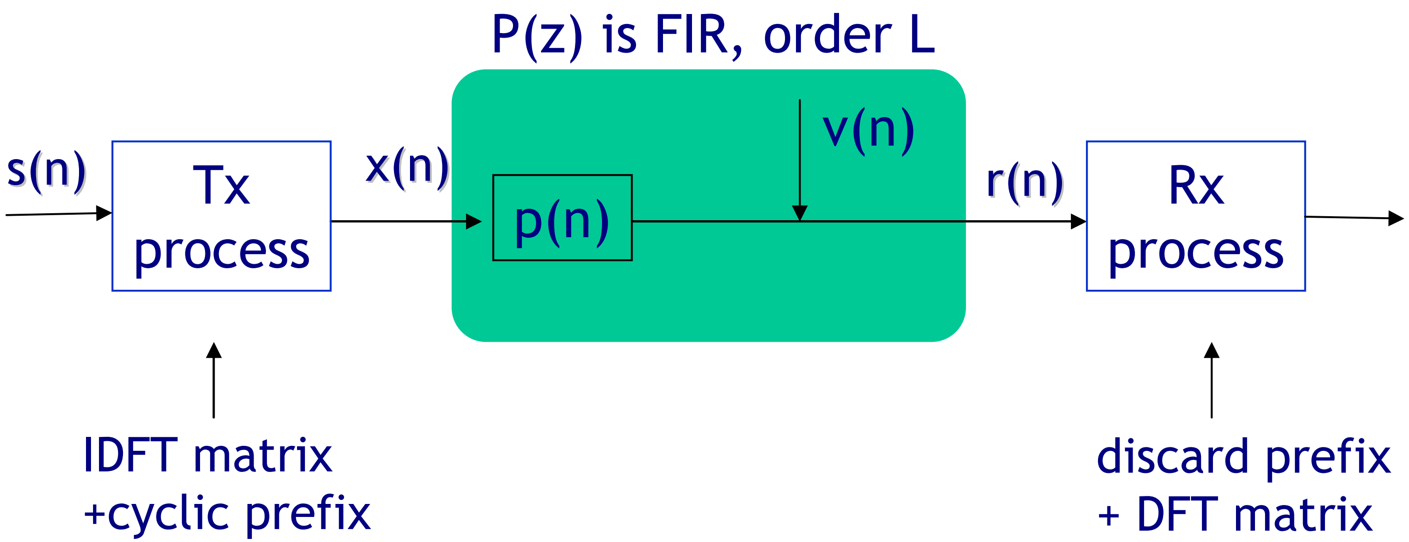
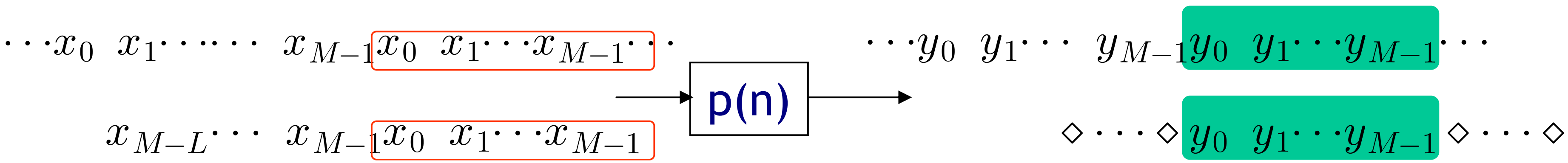
OFDM: DFT based Transceivers with Cyclic Prefix

- using IDFT matrix for Tx processing and DFT matrix for Rx processing

- overall system converted to M parallel AGN (additive Gaussian noise) subchannels



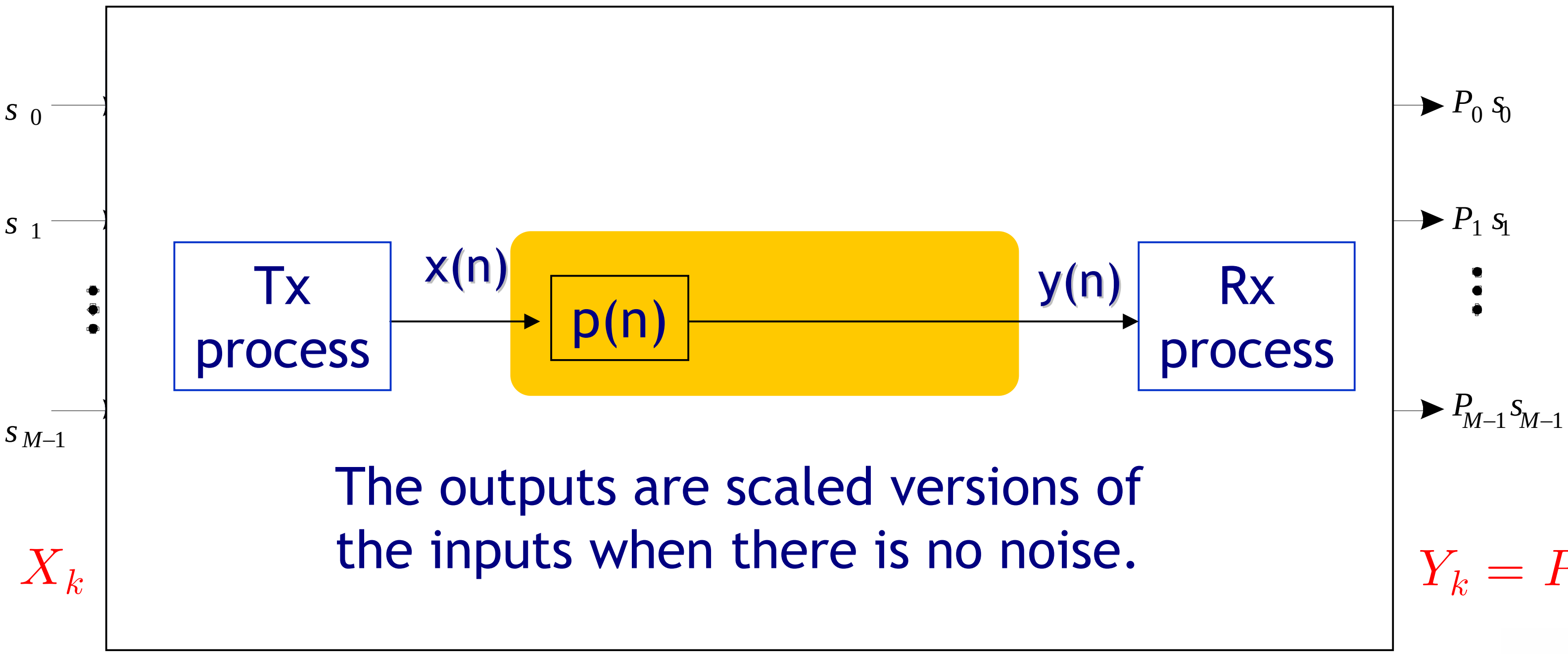
OFDM: DFT based Transceivers with Cyclic Prefix



when there is no noise

the same M points

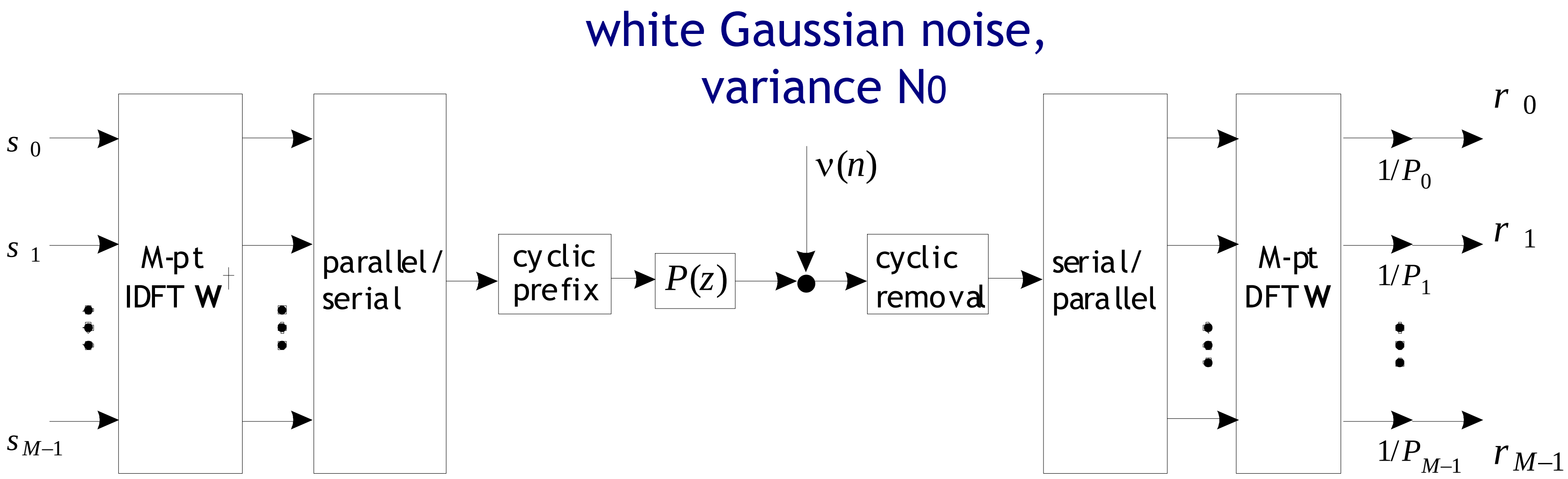
$s_k = X_k$

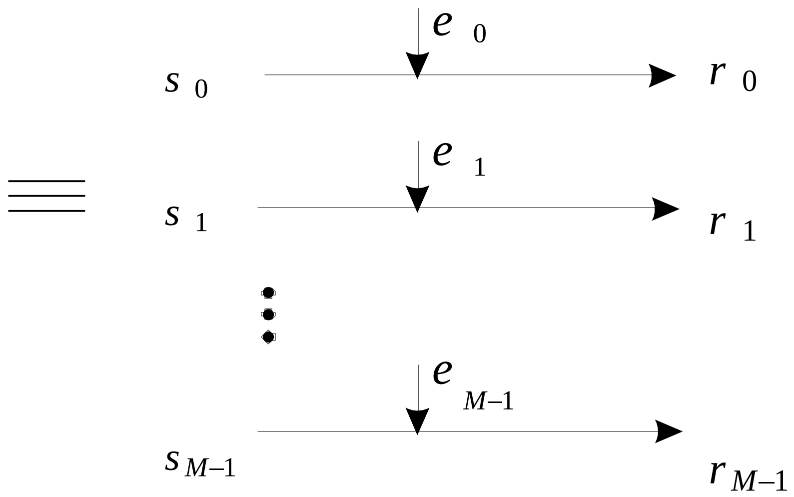


The outputs are scaled versions of the inputs when there is no noise.

$Y_k = P_k X_k$

OFDM System





. ISI free

. $r_i=s_i$, in the absence of channel noise v(n)

. e_i: Gaussian noise variance $N_0/|P_i|^2$ as DFT is unitary

BER of OFDM System

channel noise $\nu(n):$ AWGN, zero mean, variance $= N_0$

input symbols $s_k:$ QPSK symbol, $E[|s_k|^2] = E_s$

$$\text{SNR} \quad \gamma = E_s/N_0$$

subchannel noise: $\sigma^2_{ofdm}(i) = \dfrac{N_0}{|P_i|^2}$

average noise: $E_{rr} = \dfrac{N_0}{M} \sum_{i=0}^{M-1} \dfrac{1}{|P_i|^2}$

subchannel SNR: $\beta_{ofdm}(i) = \gamma |P_i|^2$

BER: $\mathcal{P}_{ofdm} = \dfrac{1}{M} \sum_{i=0}^{M-1} Q\left(\sqrt{\gamma |P_i|^2}\right)$

Advantages of OFDM Systems

- Channel diagonalization.

  ISI channels converted to parallel additive noise channels.

- Low complexity.

  IDFT at the transmitter and DFT at the receiver.

- Channel independent transmitter.

  No need of sending back channel profile to determine the transmitter. A property vital for wireless communications where the channel varies rapidly or for broadcast applications where there is on transmitter and many receivers.

- Low channel dependence receiver.

  Only a set of M scalars are channel dependent.

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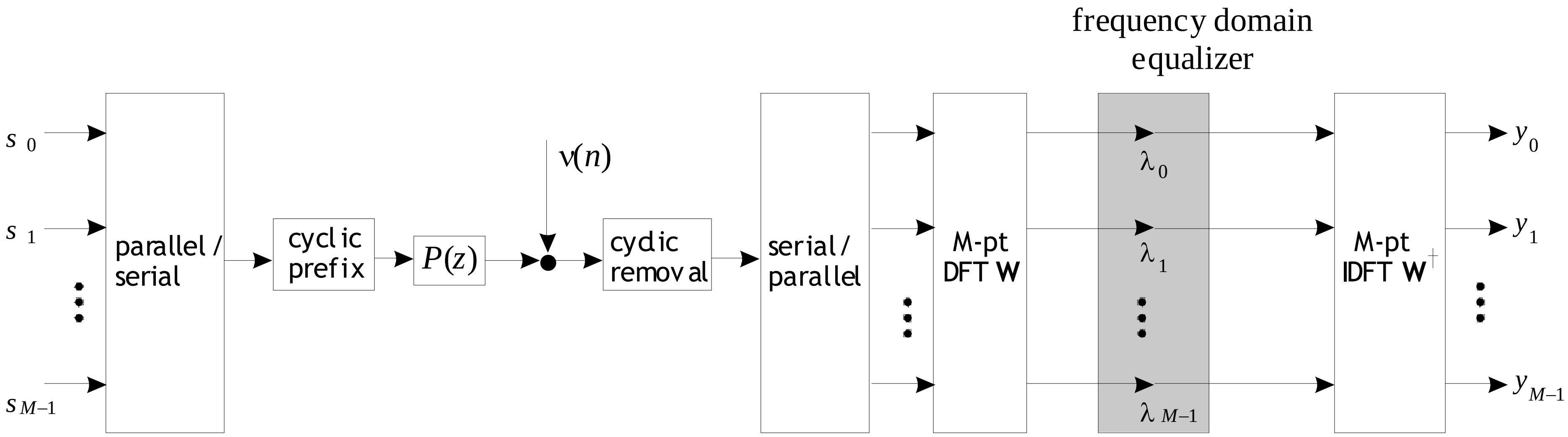
SC-CP (Cyclic-Prefixed Single Carrier) Systems

-- Also known as SC-FDE
(single carrier with frequency domain equalizer)



Connection of SC-CP to OFDM Systems

- can be obtained from OFDM by moving IDFT at transmitter to the end of receiver
- same overall complexity as OFDM, 2 DFT + FDE
- low PAPR (peak to average power ratio) as symbols are sent directly

  (In OFDM system, symbols are passed though an IDFT matrix before transmission. As a result, transmitted signals have Gaussian like distribution, and a large PAPR.

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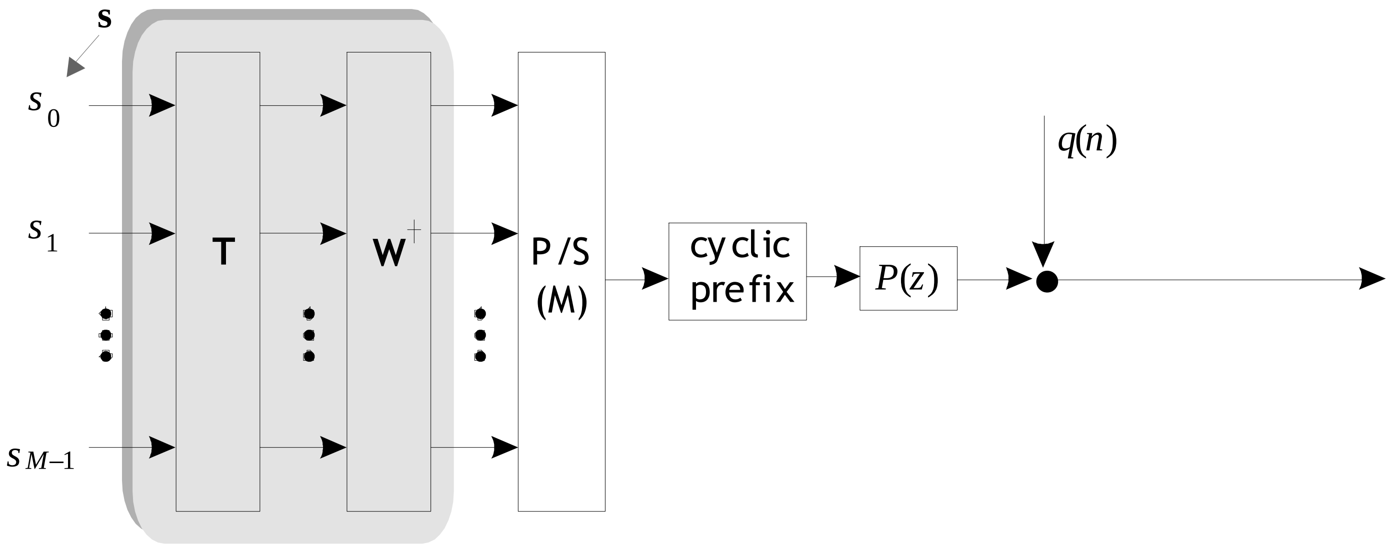
Problem of optimal precoders

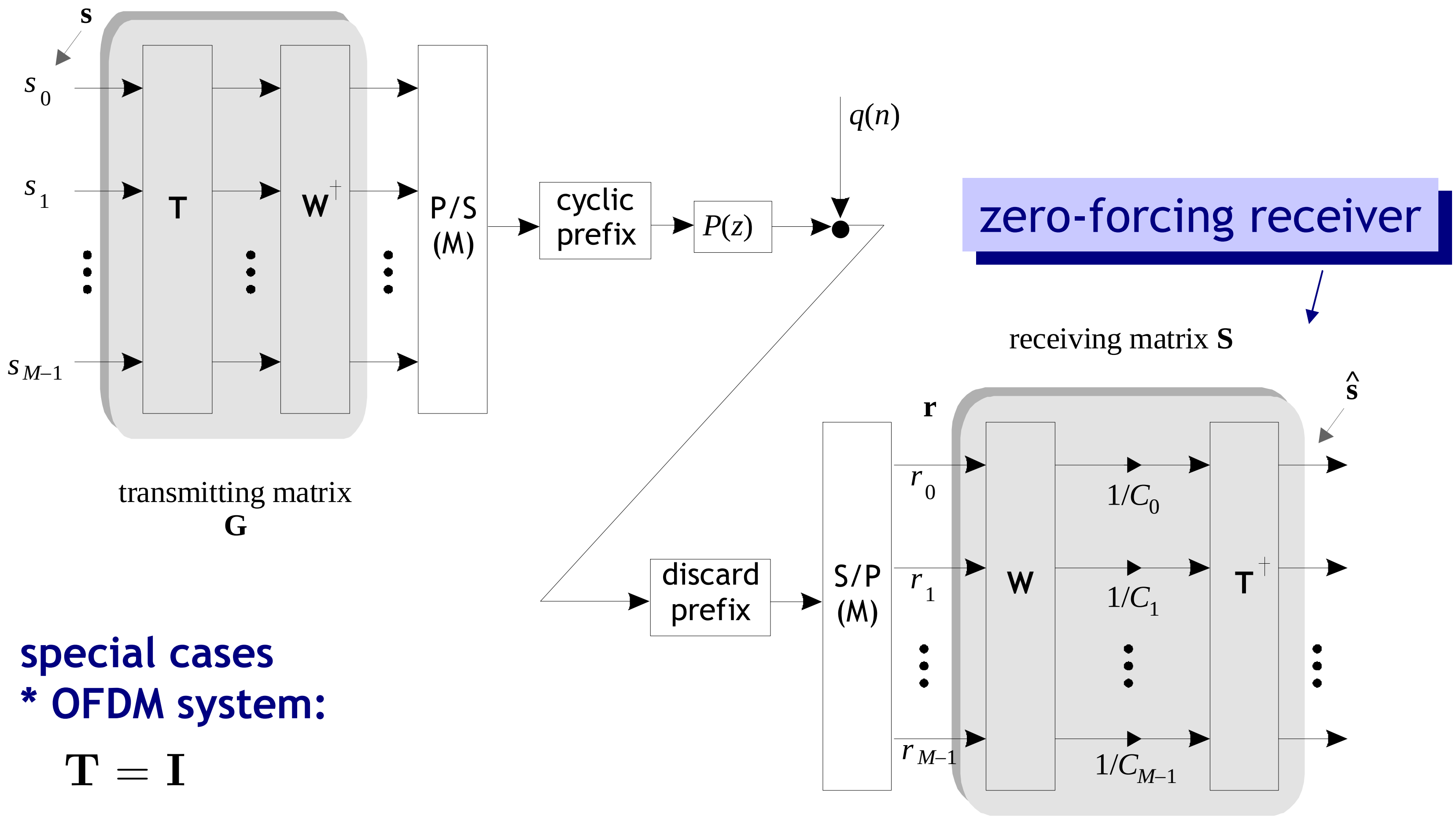


transmitting matrix
G

Q: How to design unitaryT such that BER is minimized for QPSK?
Can we have a channel independent T?

* For multicarrier/block transmission
system, MMSE does not necessarily minimize BER

Special Cases of precoder T



special cases

* OFDM system:

$$\mathbf{T} = \mathbf{I}$$

- SC-CP system

$$\mathbf{T} = \mathbf{W} \quad \text{(DFT matrix)}$$

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BER of Zero-forcing Precoded OFDM

(1) $\mathcal{P}_{ofdm} \leq \mathcal{P}_T \leq \mathcal{P}_{sc-cp}, \quad \text{for} \;\; \gamma \leq \gamma_0 = \min_i \frac{3}{|P_i|^2}$

(*)

(2) $\mathcal{P}_{ofdm} \geq \mathcal{P}_T \geq \mathcal{P}_{sc-cp}, \quad \text{for} \;\; \gamma \geq \gamma_1 = \max_i \frac{3}{|P_i|^2}$

(*)

* In each case, the 2nd inequality becomes an equality if and only if all subchannels have same SNR

- OFDM is the optimal solution for low SNR range $\gamma \leq \gamma_0$
- SC-CP is the optimal solution for high SNR range $\gamma \geq \gamma_1$
- OFDM is better for low SNR range and SC-CP is better for high SNR range.
- precoder OFDM has performance sandwiched between OFDM and SC-CP in either SNR range, optimal precoder is SNR dependent

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MMSE receivers

Lemma: The optimal receiving matrix S that minimize the average mean square error

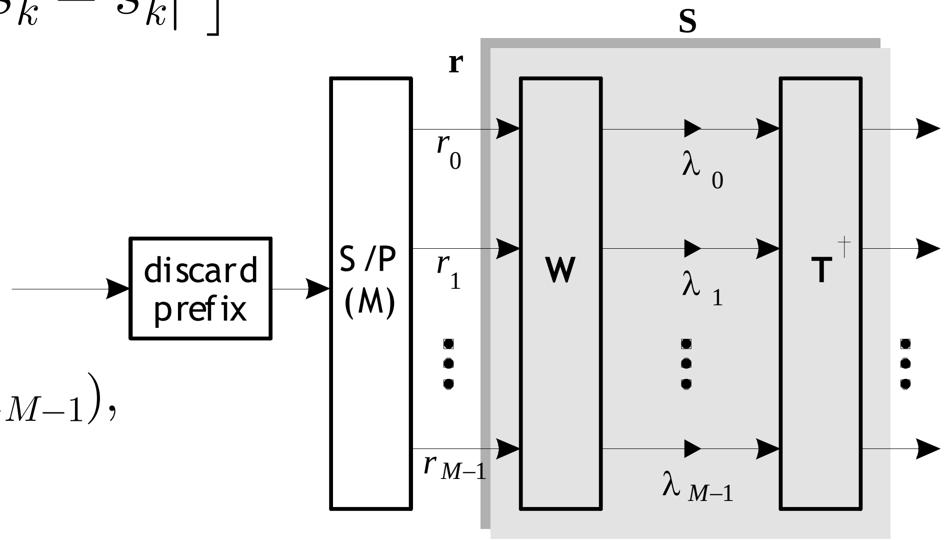
$$E_{rr} = \frac{1}{M}\sum_{k=0}^{M-1} E\left[|\hat{s}_k - s_k|^2\right]$$

is given by

$$\mathbf{S} = \mathbf{T}^{\dagger}\mathbf{\Lambda}\mathbf{W}$$

$$\mathbf{\Lambda} = \mathrm{diag}(\lambda_0 \;\; \lambda_1 \;\; \cdots \lambda_{M-1}),$$

where $\lambda_i = \dfrac{\gamma P_i^*}{(1+\gamma|P_i|^2)}$



* same structure as a zero-forcing receiver

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BER with MMSE receiver

* subchannel errors: interference + channel noise
* SINR (signal-to-interference-noise ratio):

$$\beta(i) = \left. \sum_{k=0}^{M-1} \frac{|t_{k,i}|^2 \gamma |P_k|^2}{1+\gamma|P_k|^2}\right) \Big/ \left. \sum_{k=0}^{M-1} \frac{|t_{k,i}|^2}{1+\gamma|P_k|^2}\right)$$

* BER:

Gaussian distribution, a good assumption for subchannel error [*] although subchannel errors is a mixture of interference and channel noise

$$\mathcal{P}_{T,mmse} = \frac{1}{M}\sum_{i=0}^{M-1} Q(\sqrt{\beta(i)})$$

[*] H. Vincent Poor, and Sergio Verdu, ``Probability of Error in MMSE Multiuser Detection", IEEE Trans. Information Theory, May 1997.

BER of MMSE Precoded OFDM

$$\mathcal{P}_{sc-cp,mmse} \leq \mathcal{P}_{T,mmse} \leq \mathcal{P}_{ofdm}, \quad \text{for all SNR } \gamma.$$

* The first inequality becomes an equality if and only if subchannel SINRs β(i) are equalized.

- The SC-CP system has the smallest BER, and the conventional OFDM system is the worst solution
- The precoder achieves the smallest error rate, if and only if SINR β(i) are equalized.
- SINR's can be equalized by choosing a precoder that has the equal magnitude property.

$$|t_{m,n}| = \frac{1}{\sqrt{M}}, \quad 0 \leq m, n \leq M-1.$$

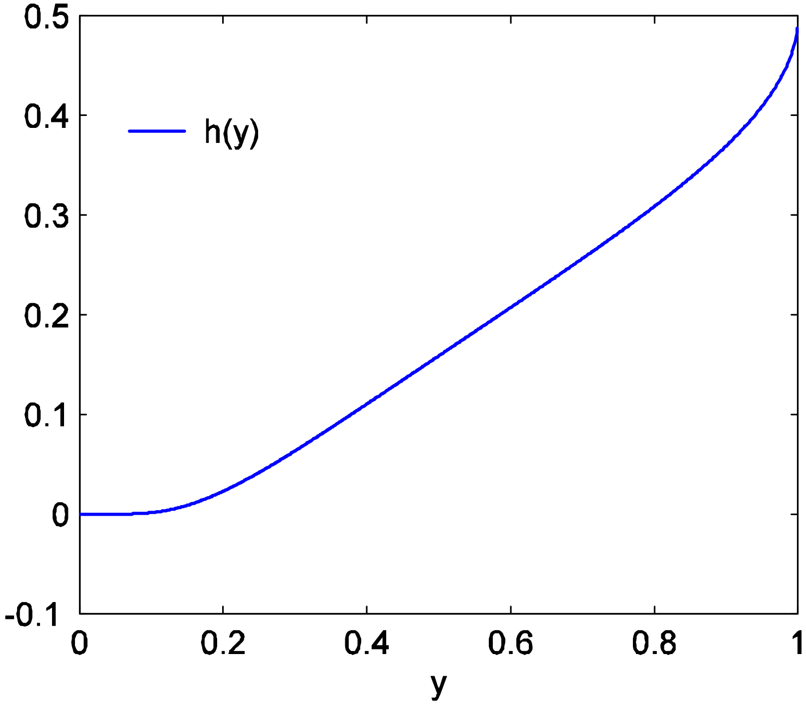
for example, Hadamard matrix and DFT matrix (channel independent)

BER of MMSE SC-CP, OFDM systems

Def: $h(y) \stackrel{\triangle}{=} Q(\sqrt{y^{-1}-1})$

$h(y) = Q(\sqrt{y^{-1}-1})$ is convex

$h'(y) > 0, \;\; h''(y) \geq 0, \;\; 0 \leq y \leq 1$



BER in terms of h(.):

$$\mathcal{P}_{sc-cp,mmse} = h\left(\frac{1}{M}\sum_{k=0}^{M-1}\alpha_k\right)$$

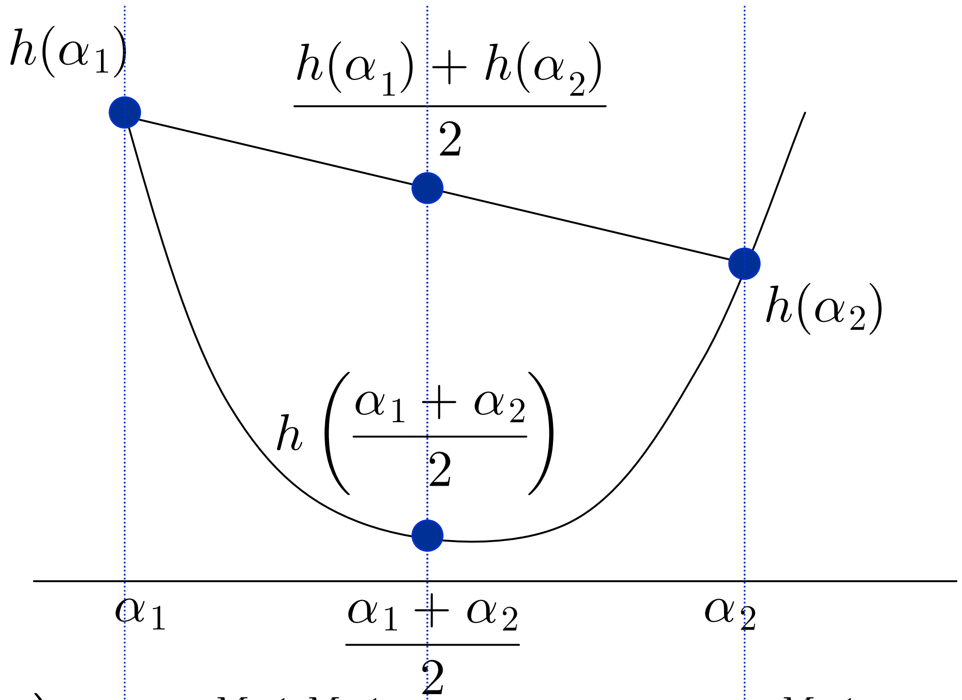
$$\mathcal{P}_{T,mmse} = \frac{1}{M}\sum_{i=0}^{M-1} h\left(\sum_{k=0}^{M-1}|t_{k,i}|^2\alpha_k\right)$$

$$\mathcal{P}_{ofdm,mmse} = \mathcal{P}_{ofdm} = \frac{1}{M}\sum_{k=0}^{M-1} h(\alpha_k)$$

where $\alpha_k = \dfrac{1}{1+\gamma|P_k|^2}$

Proof:

using convexity of h(.)



$$\frac{h(\alpha_1)+h(\alpha_2)}{2} \geq h\left(\frac{\alpha_1+\alpha_2}{2}\right)$$

$$h\left(\frac{1}{M}\sum_{k=0}^{M-1}\alpha_k\right) \leq \frac{1}{M}\sum_{i=0}^{M-1} h\left(\sum_{k=0}^{M-1}|t_{k,i}|^2\alpha_k\right) \leq \frac{1}{M}\sum_{i=0}^{M-1}\sum_{k=0}^{M-1}|t_{k,i}|^2 h(\alpha_k) = \frac{1}{M}\sum_{k=0}^{M-1} h(\alpha_k)$$

$$\left(\sum_{i=0}^{M-1}|t_{k,i}|^2 = 1\right)$$

$$\mathcal{P}_{sc-cp,mmse} \leq \mathcal{P}_{T,mmse} \leq \mathcal{P}_{ofdm}$$

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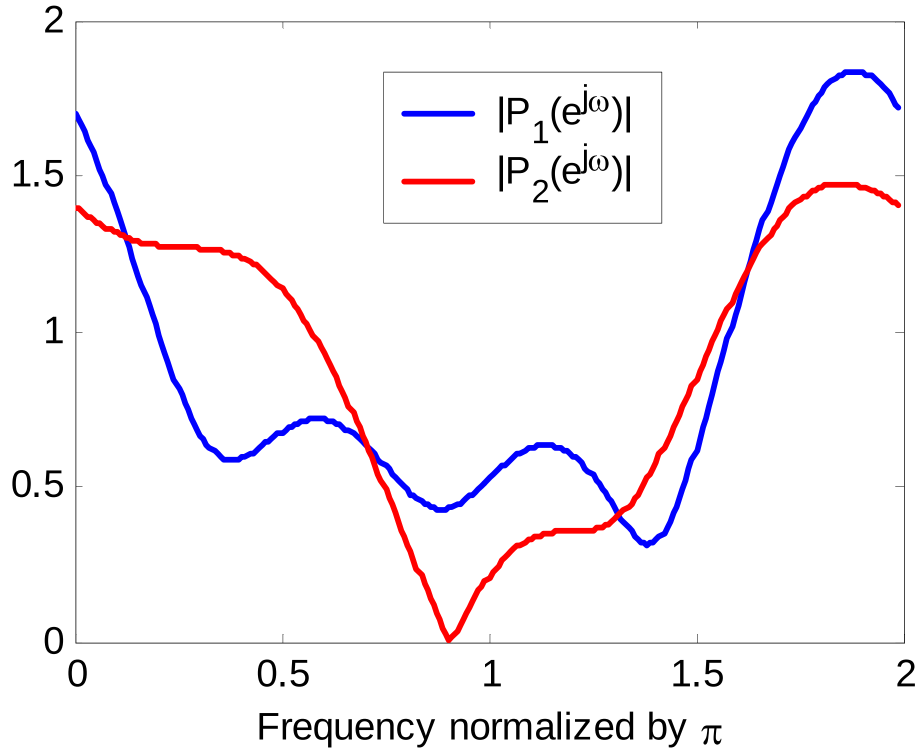
Example 1

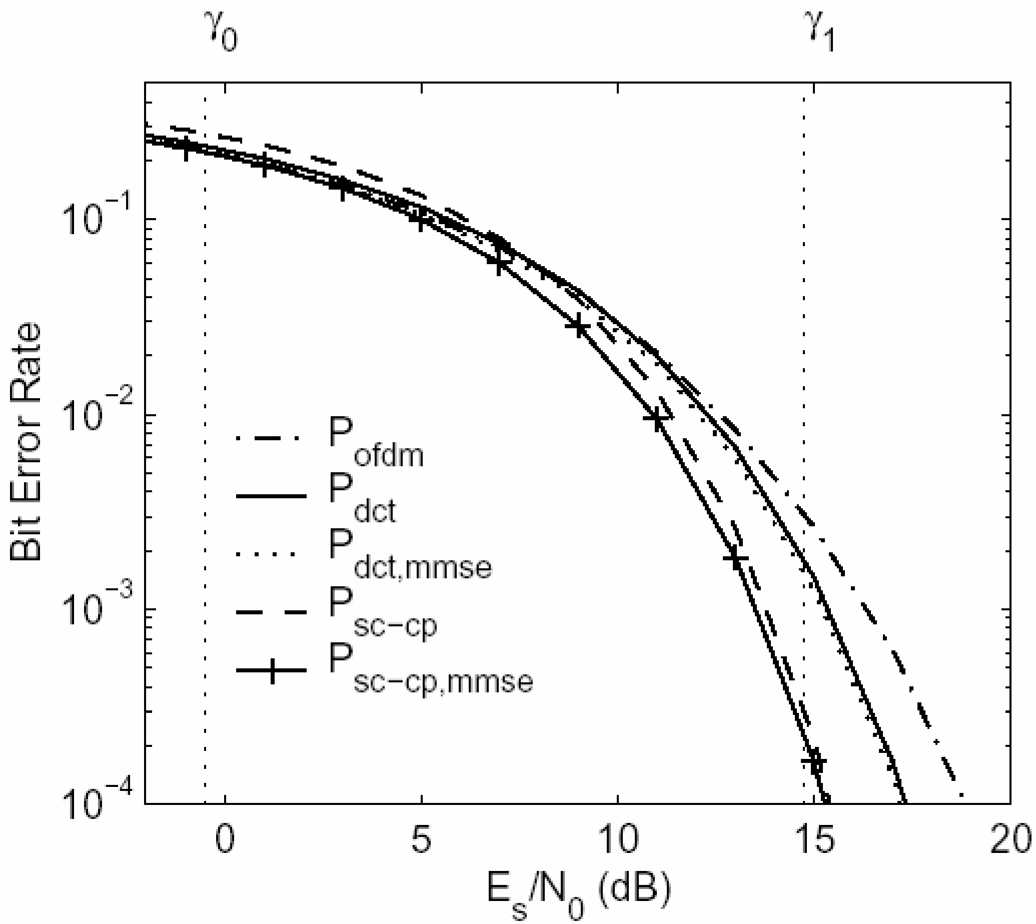
channel (4 coeffcients), M=64, L=3

p1(n): 0.3903 +j0.1049, 0.6050+j0.1422, 0.4402+j0.0368, 0.0714+j0.5002
p2(n): -0.3699-j0.5782, -0.4053-j0.5750, -0.0834-j0.0406, 0.1587-j0.0156



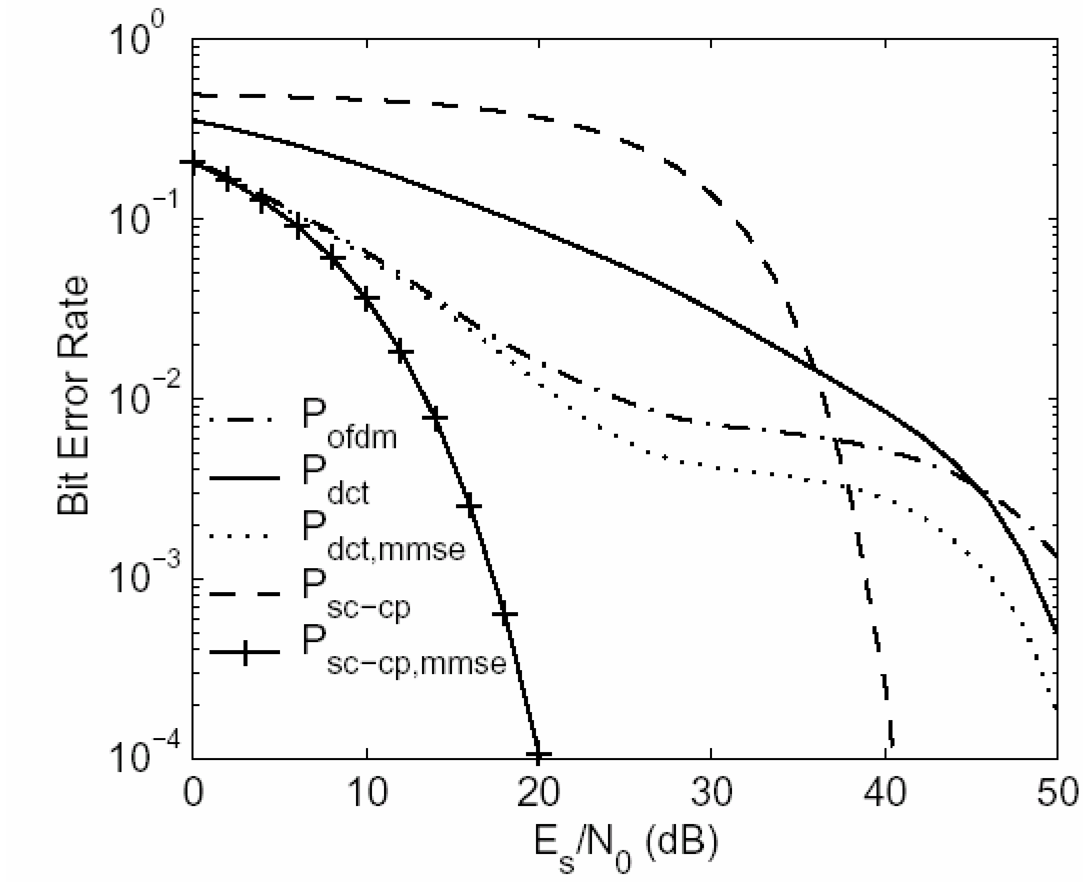
channel 2 has a zero around 0.9π

Example 1, channel 1



- MMSE SC-CP, DCT below OFDM for all SNR

Example 1, channel 2

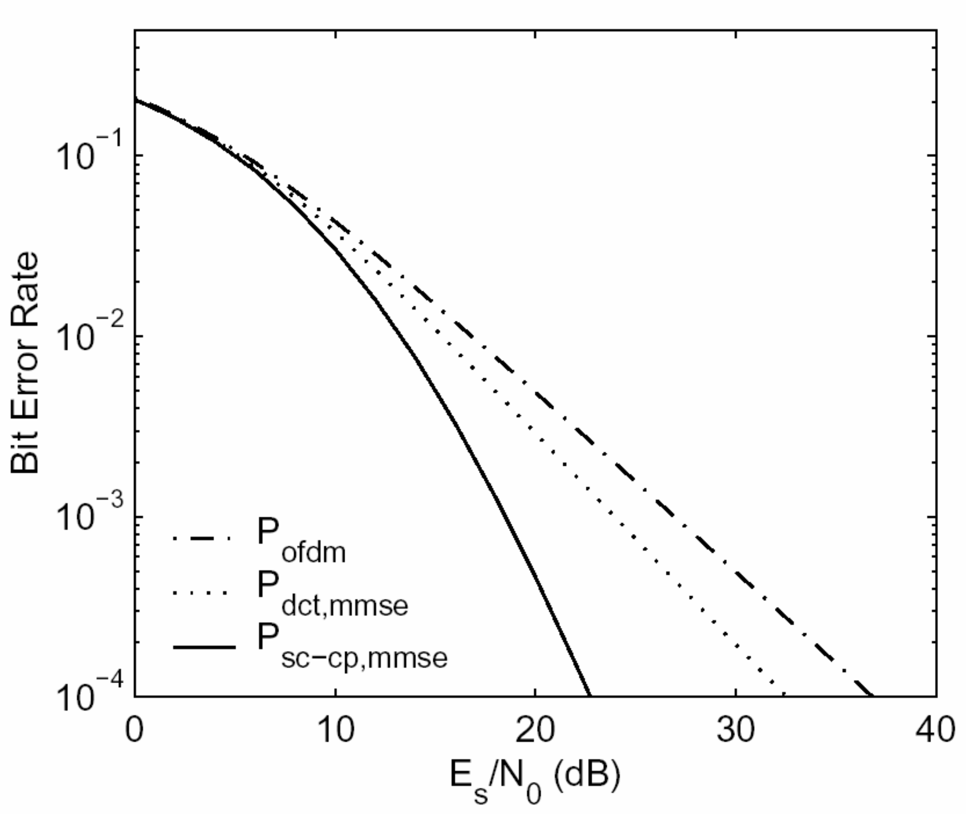


- for channel 2, the zero around 0.9π significantly degraded performances of SC-CP and OFDM systems, but not MMSE SC-CP

Example 2

multipath fading channel, 4 coefficients

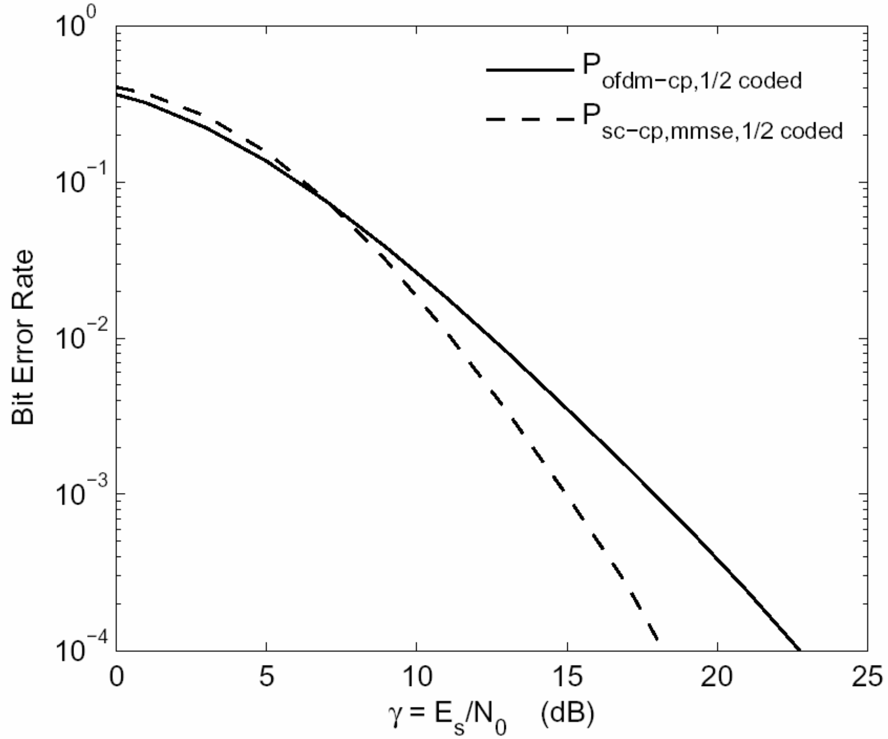
coefficients : independent complex Gaussian, zero mean, variances 8/15, 4/15, 2/15, 1/15

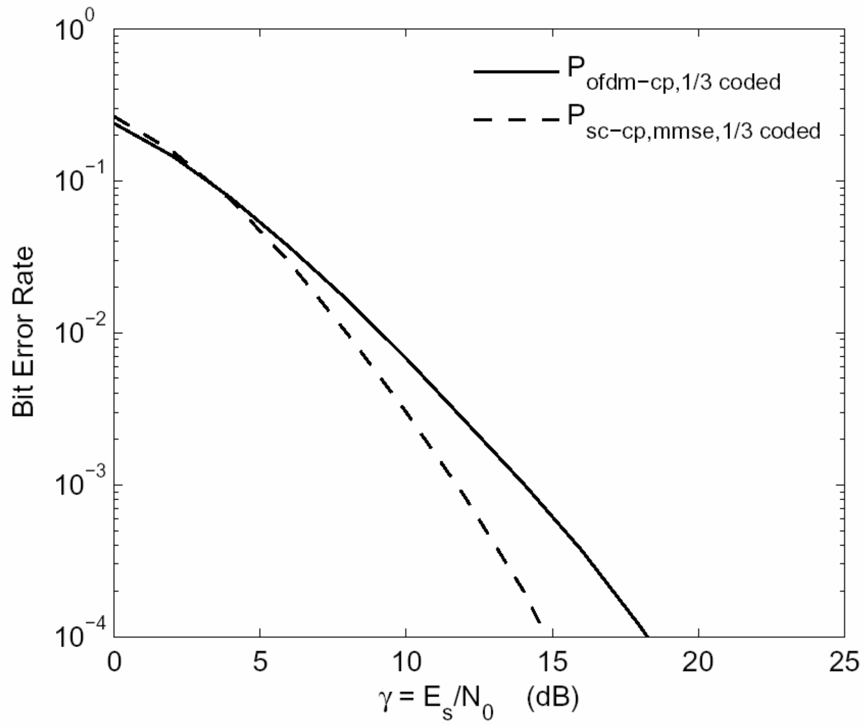


for BER=1.e-4, the SNR gap between MMSE SC-CP and OFDM is around 15dB

Example 2

same multipath fading channel,
the input symbols are coded using convolutional codes

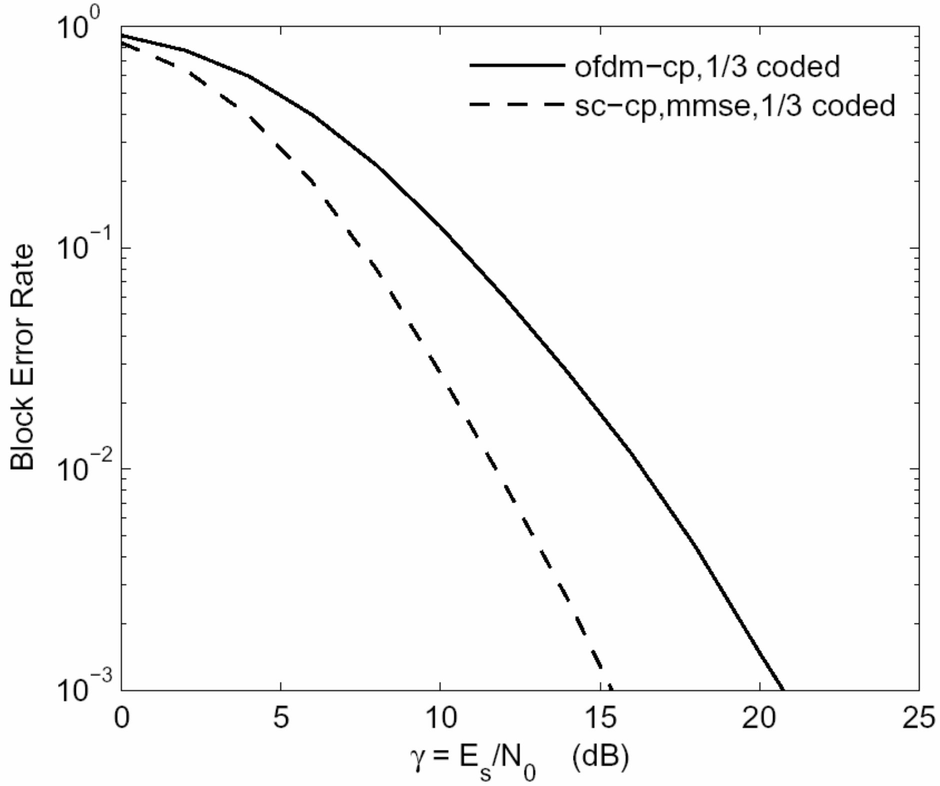




for BER=1.e-4,
the SNR gap narrows to 3dB for 1/3 coded case

Example 2, block error rate, coded symbols

block error rate, sometimes a more relevant measure because an uncorrected bit error usually means the whole block needs to be re-transmitted.



Concluding Remarks

For uncoded QPSK symbols,

- Optimal zero-forcing precoder depends on the SNR
- BER of an MMSE precoded OFDM systems between SC-CP and OFDM

For coded QPSK symbols:

- BER: whether OFDM or MMSE SC-CP is better depends on the coding rate and SNR.
- block error rate: for the same BER, MMSE SC-CP usually better as errors distributed more unevenly

Concluding Remarks

- Larger constellation QAM symbols:
  for both zero-forcing and MMSE cases
  the optimal precoder depends on the SNR γ,
  like the zero-forcing precoded system for
  QPSK symbols.

  For a practical BER range, the SC-CP MMSE
  system is better.