Enhancing Image Fidelity through Spatio-Spectral Design for Color Image Acquisition, Reconstruction, and Display

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Joint work with Xiao-Li Meng (Harvard Statistics) and Patrick Wolfe (Harvard SEAS)
Outline

1. Introduction
2. Wavelet-Based Image Processing with Missing Data
3. Spatio-Spectral Sampling for Acquisition
4. Spatio-Spectral Sampling for Display
5. Summary
1. Introduction

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5. Summary
Image Processing

natural scene statistics  digital camera & hardware  signal & image processing  display device, human vision
Image Processing

natural scene statistics

digital camera & hardware

signal & image processing

display device, human vision

data generating model

discretization, noise

analysis, estimation, processing

subjective analysis
natural scene statistics

digital camera & hardware

signal & image processing

display device, human vision

DATA LOST HERE!!

↓

impose limits on DSP

DATA LOST HERE!!

↓

impose limits on vision
Avenues for Improved Color Imaging

- **Color Image Acquisition**
  - quantitative analysis of the information loss
  - fundamental limitations to DSP imposed by current hardware
  - new hardware designs that minimize these losses and limitations
  - new hardware designs that enable fast algorithms

- **Color Image Display**
  - quantitative analysis of the visual information loss
  - fundamental limitations to vision imposed by current hardware
  - new hardware designs that minimize these losses and limitations

- **Spatio-Spectro Sampling**
  - the loss of data comes from hardware noise and from representing the image signal with discrete samples of pixels and colors.
  - we argue that there exist logical tradeoffs between spatial and spectral information.
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Wavelet Transform with Missing Data?

$\mathbf{f}$

$\mathbf{f}_{\text{obs}}$

$d = W\mathbf{f}$

$d = \text{?}$
Types of Estimation Problems

\[ y_{\text{obs}} = f_{\text{obs}} \]

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\[ y_{\text{obs}} = f_{\text{obs}} \]

\[ y_{\text{obs}} = f_{\text{obs}} \]

\[ y_{\text{obs}} = f + e \]

\[ \hat{f} = E[f | y_{\text{obs}}, \theta] \]

our estimate

where \( \theta \) is an estimate of hyper-parameter and nuisance parameter.
Interpolation + Denoising Problem

\[ f \]
desired image

\[ y_{\text{com}} = f + e \]
noisy image

\[ y_{\text{obs}} = f_{\text{obs}} + e_{\text{obs}} \]
observed data

\[ \hat{f} = E[f | y_{\text{obs}}, \theta] \]
our estimate

- Attempt to preserve sharpness in image also amplifies noise.
- Noise patterns form false edge structures.
- Interpolation adds structure to noise.
- Denoising before interpolation results in blurry output images.
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\[ f \quad \text{desired image} \]

\[ y_{\text{com}} = f + e \quad \text{noisy image} \]

\[ y_{\text{obs}} = f_{\text{obs}} + e_{\text{obs}} \quad \text{observed data} \]

\[ \hat{f} = E[f | y_{\text{obs}}, \theta] \quad \text{our estimate} \]
Empirical Partial Bayes Strategy (EPB)

- It is EPB because $\theta$ contains both hyper-parameter and nuisance parameter (e.g. Noise Covariance Matrix).

- The *Chicken-and-Egg* problem:
  1. estimate $d = Wf$ from $\theta$ and $y_{obs}$ using posterior mean.
  2. estimate $\theta$ from $d = Wf$ using maximum likelihood.

- Use *Expectation-Maximization (EM)* algorithm to iterate between these two steps!
Histogram of Wavelet Coefficients
Comparison of Models

log-histogram and models

QQ plot
### Wavelet and Sampling Models

#### Wavelet Model (prior)
- \( d = Wf \)
- \( d_k \mid q_k \sim \mathcal{N}(0, \Sigma_d / q_k) \)
- \( q_k \sim \chi^2 / \nu \)

#### Noise Model (likelihood)
- \( y = f + e \)
- \( w = Wy \)
- \( w_k \mid d_k \sim \mathcal{N}(d_k, \Sigma_w) \)

#### Marginal Likelihood Conditioned On \( q_k \)
- \( w_k \mid q_k \sim \mathcal{N}(0, \Sigma_d / q_k + \Sigma_w) \)

#### Sampling
- \( y = \begin{bmatrix} y_{\text{obs}} \\ y_{\text{mis}} \end{bmatrix} \)

#### Parameters
- \( \theta = \{ \Sigma_d, \Sigma_w, \nu \} \)

#### Complete Data
- \( x = \{ d, w, q \} \)
- \( q = \{ q_1, \ldots, q_K \} \)
Empirical Bayes & EM Algorithm

E-Step

- impute sufficient statistics

M-Step

- updated \( \theta \)

\[
E[w_k|y_{obs}, \theta], \\
E[d_k|y_{obs}, \theta], \\
etc...
\]

- impute \( x \) from \( y_{obs} \) and \( \theta \) using posterior mean.
- estimate \( \theta \) from \( y_{obs} \) (and \( x \)) using maximum likelihood.

\[
\log p(y_{obs}|\theta^{new}) \geq \log p(y_{obs}|\theta^{prev})
\]

log likelihood of \( \theta^{new} \)
EM Algorithm: M-Step

\[ \theta_{\text{new}} = \arg \max_\theta \, E \left[ \log p(x|\theta) \mid y_{\text{obs}}, \theta^{\text{prev}} \right] \]

Assuming that \( \nu \) is known...

\[ Q(\theta; \theta^{\text{prev}}) = -\frac{1}{2} \sum_k E \left[ \log |\Sigma_w| + (w_k - d_k)^T \Sigma_w^{-1} (w_k - d_k) \right. \]

\[ \left. + \log |\Sigma_d| + q_k d_k^T \Sigma_d^{-1} d_k \mid y_{\text{obs}}, \theta^{\text{prev}} \right] + \text{constant} \]

Hyper- and Nuisance Parameters

\[ \Sigma_w^{\text{new}} = K^{-1} \sum_k E \left[ \begin{bmatrix} q_k & d_k^T \end{bmatrix} \mid y_{\text{obs}}, \theta^{\text{prev}} \right] \]

\[ \Sigma_d^{\text{new}} = K^{-1} \sum_k E \left[ \begin{bmatrix} w_k & w_k^T - w_k d_k^T \end{bmatrix} \mid y_{\text{obs}}, \theta^{\text{prev}} \right] \]
### EM Algorithm: E-Step

**Noisy wavelet coefficients**

\[
\hat{w}_k|q = E[w_k|q, y_{obs}, \theta] = W_k E[y_{com}|q, y_{obs}, \theta]
\]

\[
\hat{w}_k = E[w_k|y_{obs}, \theta] = E[\hat{w}_k|q|y_{obs}, \theta] = \int_0^\infty \hat{w}_k|q \rho(q|y_{obs}, \nu) dq
\]

**Ideal wavelet coefficients**

\[
\hat{d}_k|q = E[d_k|q, y_{obs}, \theta] = E[E[d_k|w, q, y_{obs}, \theta]|q, y_{obs}, \theta]
\]

\[
= (\Sigma_w^{-1} + \Sigma_d^{-1} q_k)^{-1} \Sigma_w^{-1} \hat{w}_k|q
\]

\[
\hat{d}_k = E[d_k|y_{obs}, \theta] = E[\hat{d}_k|q|y_{obs}, \theta] = \int_0^\infty \hat{d}_k|q \rho(q|y_{obs}, \nu) dq
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Experimental Results: Interpolation + 10% noise

original

noisy CFA image

Gunturk ‘02 (PSNR=21.21)

Gunturk ‘02+Portilla ‘03 (PSNR=25.04)

Hirakawa ’06 (PSNR=25.25)

proposed (PSNR=26.06)
Experimental Results: Interpolation + 10% noise

Original

Noisy CFA image

Gunturk '02 (PSNR=20.68)

Gunturk '02 + Portilla '03 (PSNR=24.79)

Hirakawa '06 (PSNR=23.58)

Proposed (PSNR=25.17)
**Summary**

- Combine sophisticated Wavelet Models with the Missing Data treatment.
- Empirical Partial Bayes using EM Algorithm.
  - Posterior Mean Estimation of Noisy and Clean Wavelet Coefficients.
  - Maximum Likelihood Estimation of Hyper- and Nuisance Parameters.
- Experimental Results:
  - Image quality better than treating denoising and interpolation independently.
  - Visible improvement over previous methods.
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What are the fundamental limitations to DSP imposed by the acquisition hardware?

- Types of data losses?
  - Spatial resolution (e.g. 6 megapixel)
  - Spectral resolution (e.g. red, green, blue)
  - Quantization (e.g. 24-bit color)
  - Temporal resolution (e.g. frame rate)
  - Noise (e.g. shot noise)

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- Can we design hardware that minimizes information loss?
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Color Image Acquisition

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Digital Camera Image Processing Pipeline

Observations:

- CFA represents one of the very first steps in acquisition.
- Subsequent steps process sensor data acquired through CFA.
- We see diminishing return in image quality for additional complexity in algorithm.

Goal:

- Design a new CFA pattern that preserves the integrity of the signal.
- ... should yield better computation-quality trade-offs.
- ... should enhance the performance bounds.
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\[ y(n) = c_g(n)g(n) + c_r(n)r(n) + c_b(n)b(n) \]
\[ = g(n) + c_r(n)\alpha(n) + c_b(n)\beta(n) \]

\( y(n) \) = CFA Image & Difference Image

\( y(n) \) = Intro, Missing Data, Acquisition, Display, Summary

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Spatio-Spectral Color Imaging
CFA Image & Difference Image—Fourier Transform

\[ y(n) = g(n) + c_r(n)\alpha(n) + c_b(n)\beta(n) \]

\[ \mathcal{F}\{y\} = \mathcal{F}\{g\} + \mathcal{F}\{c_r\alpha\} + \mathcal{F}\{c_b\beta\} \]

- No global solution to recovering the image signal
- Need additional assumptions about the signal
- Motivates nonlinear processing driven by local statistics
- Nonlinearity affects noise characterization

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Analysis of Data Loss in Image Acquisition

Bayer Pattern\textsuperscript{a}

4 Color Pattern\textsuperscript{b}

Hexagon\textsuperscript{c}

Octagon/Quincunx\textsuperscript{d}

Aliased (overlapped) regions mean lost data!


\textsuperscript{b}www.sony.net/sonyinfo/news/press_archive/200307/03-029E

\textsuperscript{c}R.M. Mersereau, “The processing of hexagonally sampled two-dimensional signals,” Proceedings of the IEEE Vol.67, No.6, 1979

\textsuperscript{d}home.fujifilm.com/pma2000/sprccd.html
Theorem (Hirakawa & Wolfe 2008)

No choice of pure-color CFA (Bayer, Hexagonal, Octagonal, etc.) will admit the maximal spectral radius at baseband.

Proof follows from the theory of (sampling) lattices.

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\(^b\)www.sony.net/sonyinfo/news/press_archive/200307/03-029E
\(^d\)home.fujifilm.com/pma2000/sprccd.html
Amplitude modulation (AM) radio uses modulation of signal $x(n)$ by carrier frequency $c(n)$ via multiplication in the time domain:

$$y(n) = x(n)c(n).$$

The partitioning in the frequency domain allows transmission of multiply speech/music signals to be carried over the same media.
Sensor Data Model

\[ y(n) = c_g(n)g(n) + c_r(n)r(n) + c_b(n)b(n) \]

Simplification

Impose convex combination constraint:
\[ c_g(n) + c_r(n) + c_b(n) = 1. \]

Then
\[
\begin{align*}
y(n) &= \left(1 - c_r(n) - c_b(n)\right)g(n) + c_r(n)r(n) + c_b(n)b(n) \\
&= g(n) + c_r(n)(r(n) - g(n)) + c_b(n)(b(n) - g(n)) \\
&= g(n) + \underbrace{c_r(n)\alpha(n)}_{\text{amplitude modulation}} + \underbrace{c_b(n)\beta(n)}_{\text{amplitude modulation}}
\end{align*}
\]
### Sensor Data Model

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= g(n) + \underbrace{c_r(n)\alpha(n)}_{\text{amplitude modulation}} + \underbrace{c_b(n)\beta(n)}_{\text{amplitude modulation}}
\]
It’s a **Sphere Packing** problem!

- Recall spectral support of the green image.
- Green image spectrum does not occupy frequency regions far away from the origin.
- **Main Idea:** Use $c_r$ and $c_b$ to modulate $\alpha(n)$ and $\beta(n)$ away from origin!
Design of Color Filter Array

We design $c_r$ and $c_b$ in the 2D Fourier domain:

1. Pick carrier frequencies $\{\tau_k \in \mathbb{R}^2 : \|\tau_k\|_\infty = \pi\}$.
2. Pick corresponding weights $\{s_i, t_i \in \mathbb{C}\}$.
3. Set $C_r(\omega) = s_0 + \sum k s_k \delta(\omega - \tau_k) + \bar{s}_k \delta(\omega + \tau_k)$.
4. Set $C_b(\omega) = t_0 + \sum k t_k \delta(\omega - \tau_k) + \bar{t}_k \delta(\omega + \tau_k)$.
5. Take inverse Fourier transform: $c_r = \mathcal{F}^{-1}\{C_r\}$, $c_b = \mathcal{F}^{-1}\{C_b\}$.
6. $c_g = 1 - c_r - c_b$.

\[
\mathcal{F}\{c_r \cdot \alpha\}(\omega) = s_0 \mathcal{F}\{\alpha\}(\omega) + \sum_k s_k \mathcal{F}\{\alpha\}(\omega - \tau_k) + \bar{s}_k \mathcal{F}\{\alpha\}(\omega + \tau_k)
\]

\[
\mathcal{F}\{c_b \cdot \beta\}(\omega) = t_0 \mathcal{F}\{\beta\}(\omega) + \sum_k t_k \mathcal{F}\{\beta\}(\omega - \tau_k) + \bar{t}_k \mathcal{F}\{\beta\}(\omega + \tau_k)
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\]
CFA Image & Difference Image—Fourier Transform

Bayer  Yamanaka  Lukac  Kodak  Hirakawa
Disadvantages to traditional CFA design

- **aliasing** occurs when one signal “contaminates” another signal.
- **anti-aliasing** reduces resolution.
- “un-doing” aliasing is an **ill-posed** problem ⇒ additional assumption and complexity! (e.g. directionality).

Benefits to spatio-spectral CFA design

- minimize data loss ⇒ improved image quality
- not sensitive to directions ⇒ completely linear fast reconstruction method
- low-complexity, low-power, low-memory
- improvements for noise and video (panchromatic)

Main idea: with sufficient partitioning in Fourier domain, a very crude filter will suffice for reconstruction.
Minimizing Data Loss in Image Acquisition

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Efficient Linear Demosaicking for Spatio-Spectral CFA

Demosaicking as AM Demodulation

\[ \hat{x}(y) = \begin{bmatrix} 1-c_r(n) & 1 & -c_b(n) \\ -c_r(n) & 1 & -c_b(n) \\ -c_r(n) & 1 & 1-c_b(n) \end{bmatrix} \begin{bmatrix} h \ast \{\theta\alpha y\} \\ h \ast \{\theta\beta y\} \end{bmatrix} \]

- **Matrix operator** combined with color correction offline \( \Rightarrow \) comes for free!
- **Spatial processing** is the only demosaicking cost.

\[
\begin{align*}
    h \ast \{\theta\alpha y\} &= h'(n_2) \ast \{\theta'\alpha(n_2)y'(n)\} \\
    h \ast \{\theta\beta y\} &= h'(n_2) \ast \{\theta'\beta(n_2)y'(n)\} \\
    y' &= h'(n_1) \ast \{(-1)^{n_1}y\}
\end{align*}
\]
Efficient Linear Demosaicking for Spatio-Spectral CFA

- with sufficient partitioning in Fourier domain, a very crude filter will suffice for reconstruction.
- so, can we use cheap filters? YES!
- we rival state-of-the-art demosaicking with
  - only 10 add operations per full-pixel reconstruction!
  - no nonlinear elements such as if-then or greater-than.
  - no multiplier (except $\theta \in \{0, \pm 1\}$)

Triangle Filter $h'$

4 adds, $2N + 1$ delay lines

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Efficient Linear Demosaicking for Spatio-Spectral CFA

\[ (-1)^{n_1} \]

\[ y \rightarrow \times \rightarrow Z_1^{-N} \rightarrow + \rightarrow Z_1^{-1} \rightarrow + \rightarrow Z_1^{-(N-1)} \rightarrow + \rightarrow Z_1^{-1} \rightarrow y' \]

\[ y' \rightarrow \times \rightarrow \theta'_\alpha \rightarrow Z_2^{-N} \rightarrow \times \rightarrow \theta'_\beta \rightarrow Z_2^{-N} \rightarrow \times \rightarrow \theta'_\alpha \rightarrow Z_2^{-1} \rightarrow + \rightarrow Z_2^{-(N-1)} \rightarrow + \rightarrow Z_2^{-1} \rightarrow \times \rightarrow Z_2^{-1} \rightarrow h_{\alpha} \ast \{\theta_\alpha y\} \]

\[ y' \rightarrow \times \rightarrow \theta'_\alpha \rightarrow Z_2^{-N} \rightarrow \times \rightarrow \theta'_\beta \rightarrow Z_2^{-N} \rightarrow \times \rightarrow \theta'_\alpha \rightarrow Z_2^{-1} \rightarrow + \rightarrow Z_2^{-(N-1)} \rightarrow + \rightarrow Z_2^{-1} \rightarrow \times \rightarrow Z_2^{-1} \rightarrow h_{\alpha} \ast \{\theta_\beta y\} \]

- \( \theta'_\alpha, \theta'_\beta \in \{0, \pm 1\} \Rightarrow \) savings with \( \theta'_\alpha y' = 0 \) and \( \theta'_\beta y' = 0 \)
- \( Z_1 \) line buffers, \( Z_2 \) registers (ASIC)
- 10 adds, \( 2N + 1 \) line buffers, \( 3N + 2 \) registers
Example of Data Acquisition and Reconstruction

Bayer Pattern\textsuperscript{a}

Pattern A

Pattern B

Pattern C

Example of Data Acquisition and Reconstruction

- Bayer Pattern
- Pattern A
- Pattern B
- Pattern C

Example of Data Acquisition and Reconstruction

Bayer Pattern\textsuperscript{a}

Pattern A

Pattern B

Pattern C

Example of Noisy Sensor

Spatio-Spectral Linear Demosaicking

Bayer Nonlinear Demosaicking
Outline

1. Introduction
2. Wavelet-Based Image Processing with Missing Data
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4. Spatio-Spectral Sampling for Display
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Motivation

What limitations does hardware impose on visual perception?

- Types of data losses? Resolutions in Spatial, Spectral, Quantization, Temporal... 
- Given human visual system models and what we know about the signal, can we quantify information loss?
- Can we design a hardware that minimizes information loss?
Color Filter Array & Display Device

Observations:
- CFA represents one of the very last steps in display device.
- Human visual system processes image data displayed via CFA.

Goal:
- Design a new CFA pattern that preserves the integrity of the signal.
- ... should yield better resolution-quality trade-offs.
- ... should enhance the performance bounds.

Keigo Hirakawa

Spatio-Spectral Color Imaging
Quick Review: Luminance-Chrominance (L-C)

Visual spatial processing organized as parallel channels (components) in the nervous system.

- **luminance** ("intensity") & **chrominance** ("red-green" and "blue-yellow")
- The contrast sensitivity functions (CSF) reveal that the passband structure in vision.

\[
\mathcal{W}\{x\}_{\text{perceived}} = \begin{bmatrix} h_1(n) \ast y_1(n) \\ h_2(n) \ast y_2(n) \\ h_3(n) \ast y_3(n) \end{bmatrix} = \begin{bmatrix} h_1(n) \ast \\ h_2(n) \ast \\ h_3(n) \ast \end{bmatrix} \begin{bmatrix} M \\ x(n) \end{bmatrix}_{\text{CSF}}
\]

CSF (Wandell '99)
Stimulus $u(n)$ controls the intensity of color $c(n)$. 

\[
\begin{align*}
\mathbf{v}(n) &= \begin{bmatrix} v_r(n) \\ v_g(n) \\ v_b(n) \end{bmatrix} = \begin{bmatrix} c_r(n) \\ c_g(n) \\ c_b(n) \end{bmatrix} u(n) \\
&= \begin{bmatrix} c_r(n) \\ c_g(n) \\ c_b(n) \end{bmatrix} \begin{bmatrix} d_r(n) & d_g(n) & d_b(n) \end{bmatrix} \begin{bmatrix} x_r(n) \\ x_g(n) \\ x_b(n) \end{bmatrix} 
\end{align*}
\]
**Stimulus**

\[ u(n) = \begin{bmatrix} d_r(n) & d_g(n) & d_b(n) \end{bmatrix} M^{-1} M \begin{bmatrix} x_r(n) \\ x_g(n) \\ x_b(n) \end{bmatrix} \]

\[ \phi^T = \text{projection in L-C} \]

\[ y = \text{image in L-C} \]

\[ = \phi_1(n)y_1(n) + \phi_2(n)y_2(n) + \phi_3(n)y_3(n) \]

**Fourier Transform**

- Luminance \((\mathcal{F}y_1)\)
- Chrominance \((\mathcal{F}y_2)\)
- Chrominance \((\mathcal{F}y_3)\)
\[ u(n) = \phi_1(n)y_1(n) + \phi_2(n)y_2(n) + \phi_3(n)y_3(n) \]

Idea: When \( \phi_i \) is a sinusoid, \( \phi_i y_i \) is a modulation and \( \phi_i \) called carrier.

Idea: Spectral overlap is called aliasing \( \Rightarrow \) lost information!
Aliasing is eliminated with oversampling, but it increases pixel count.

**Vertical (2500 subpixels)**

**Oversampled by 3 (7500 subpixels)**
Quick Review: Amplitude Modulation

\[ u(n) = \sum_i \phi_i(n) y_i(n) \]

coded signal

carrier frequency signal

\[ \hat{y}_i(n) = h_i(n) \ast \{ \psi_i(n) u(n) \} \]

reconstruction
de-modulation

The partitioning in the frequency domain allows transmission of multiply speech/music signals to be carried over the same media.
Motivation

Ultimately, want $\mathcal{W}\{x\} \approx \mathcal{W}\{v\}$, where observed image is...

$$\mathcal{W}\{v\} = \begin{bmatrix} h_1^* \\ h_2^* \\ h_3^* \end{bmatrix} M \begin{bmatrix} c_r(n) \\ c_g(n) \\ c_b(n) \end{bmatrix} u(n) = \begin{bmatrix} h_1^* \\ h_2^* \\ h_3^* \end{bmatrix} \psi(n) \underbrace{d(n)M^{-1}}_{\text{de-modulation}} \phi(n)^T y(n)$$

- This is **Amplitude Modulation and De-Modulation!!!**
- **Idea 1**: Design $\phi_i$ and $\psi_i$ such that $\mathcal{W}\{v\}$ is an amplitude demodulation. We “borrow” the convolution filters from the observer’s eye.
- **Idea 2**: Parameterize $\phi_i$ and $\psi_i$ in Fourier domain explicitly such that we achieve partitioning (i.e. no aliasing).
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Fourier Domain CFA Design

We design the carrier frequencies $\phi_i(n) = k_i \psi_i(n)$ such that:

- $h_i(n) \ast \{\psi_i(n) \phi_j(n)\} = 1$ when $i = j$.
- $h_i(n) \ast \{\psi_i(n) \phi_j(n)\} = 0$ when $i \neq j$.
- $u = \phi_1 y_1 + \phi_2 y_2 + \phi_3 y_3$ is alias free.
- $u \geq 0$.

We choose $\phi(n)$ in the 2D Fourier domain:

1. Set $\phi_1(n) = 1$.
2. Pick carrier frequencies $\{\tau_k \in \mathbb{R}^2 : \|\tau_k\|_{\infty} = \pi\}$.
3. Pick corresponding weights $\{s_k, t_k \in \mathbb{C}^2\}$.
4. Set $\Phi_2(\omega) = \sum_k s_k \delta(\omega - \tau_k) + \bar{s}_k \delta(\omega + \tau_k)$.
5. Set $\Phi_3(\omega) = \sum_k t_k \delta(\omega - \tau_k) + \bar{t}_k \delta(\omega + \tau_k)$.
6. Take inverse FFT: $\phi_2 = \mathcal{F}^{-1}\{\Phi_2\}$, $\phi_3 = \mathcal{F}^{-1}\{\Phi_3\}$.

$\psi$ follows immediately from $\phi$. 
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Spatio-Spectral Color Imaging
Fourier Analysis of Stimuli

Fourier Transform of Stimuli

\[
\mathcal{F}\{u\} = \mathcal{F}\{\phi^T y\}
\]

\[
= \mathcal{F}\{y_1\}(\omega) + \sum_k \{s_i \mathcal{F}\{y_2\} + t_i \mathcal{F}\{y_3\}\}(\omega - \tau_k)
\]

\[
+ \sum_k \{\bar{s}_i \mathcal{F}\{y_2\} + \bar{t}_i \mathcal{F}\{y_3\}\}(\omega - \tau_k)
\]

By choosing the carriers \(\tau_k\) away from the baseband (high frequency),

- The chances of \(\phi_2 y_2\) and \(\phi_3 y_3\) overlapping with \(\phi_1 y_1\) is minimized.
- \(\phi_1 \psi_2, \phi_1 \psi_3, \phi_2 \psi_1, \phi_3 \psi_1\) fall outside of the passband for \(h_1, h_1, h_2, h_3\), respectively.
Fourier Analysis of Stimuli

Fourier Transform of Stimuli

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\[ = \mathcal{F}\{y_1\}(\omega) + \sum_k \{s_i\mathcal{F}\{y_2\} + t_i\mathcal{F}\{y_3\}\}(\omega - \tau_k) \]
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An Example of New CFA

- black=display device; green=RGB; red=real image data.
- Not unique to the above—offers much flexibility in design!
- Larger gamut, but does not cover all of RGB.
Display Example

Striped

Diagonal

Proposed

Keigo Hirakawa
Spatio-Spectral Color Imaging
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- **Color Image Acquisition**: avoidance of information loss.
  - Examined the aliasing inherent in CFA patterns.
  - Theorem: suboptimality of pure-color CFA patterns.
  - Designed a new way to capture color image data using CFA as a modulation operator.

- **Color Image Processing**: representation of sampled data in transform domain.
  - Combine sophisticated wavelet models with missing data treatment.
  - Empirical partial Bayes using EM Algorithm.
  - $L^2$ estimation of clean wavelet coefficient.
  - MLE of parameters

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Thank you!

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