



Math with your brain ...

THE SLIDE RULE

THE SLIDE RULE



DEDICATED TO:

Dr. Clifford L. Schrader, Ph. D. (1937-2001)



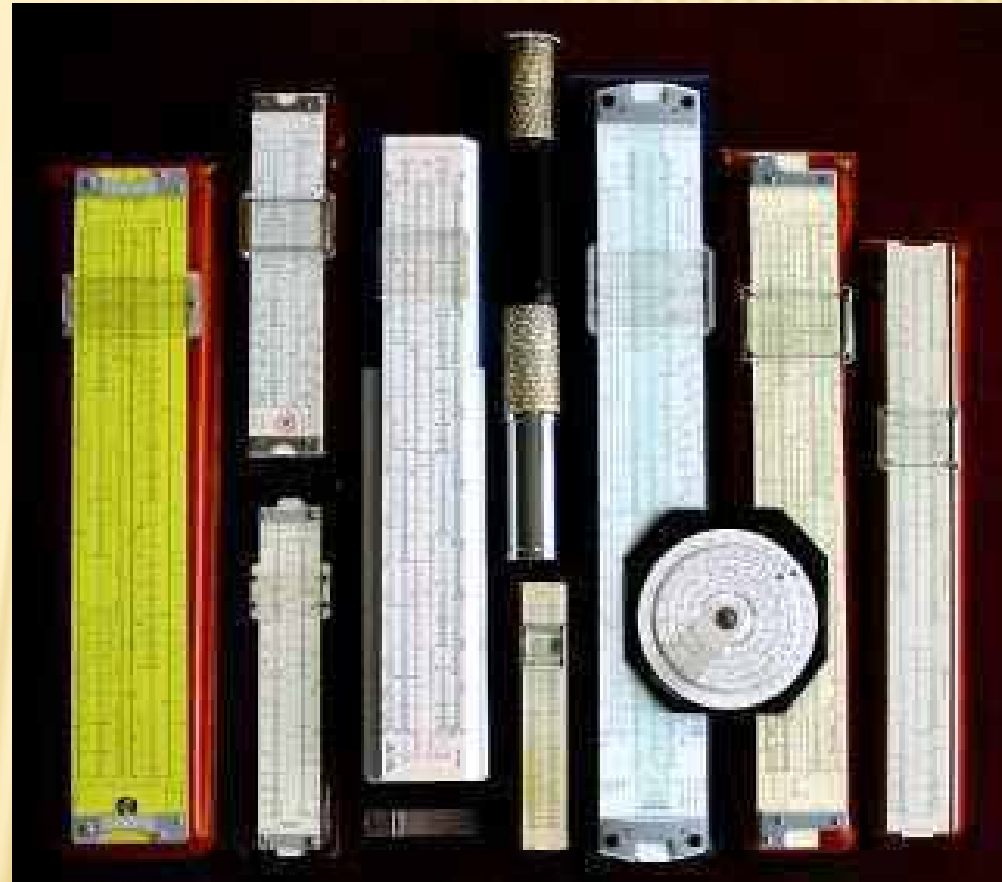
My high school chemistry teacher (1976) who would only let us use slide rules (and our brains) during the first semester.

Also, the creator of the "Universal Slide Rule League" a lunchtime 5-minute competition of 10 problems for the slide rule. *Harassment is allowed!*



WHAT IS A SLIDE RULE?

- ∅ A mechanical analog computer.
- ∅ Invented nearly 400 years ago.
- ∅ Straight, circular, spiral & even helical all use logarithms.
- ∅ The indispensable tool of an Engineer for generations!





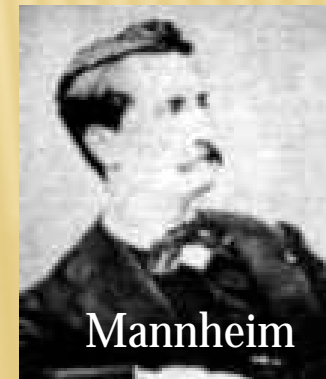
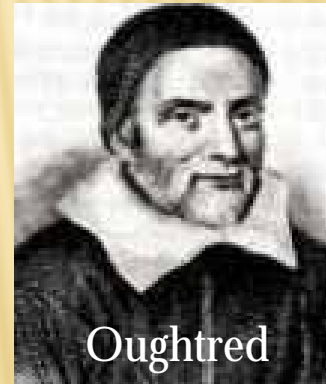
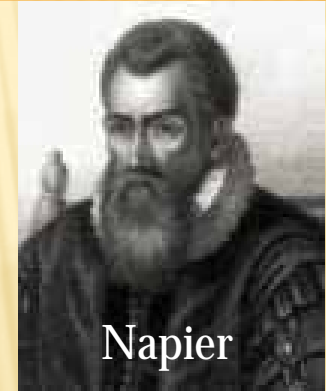
THE SLIDE RULE – TABLE OF CONTENTS

- Ø History: Where do they come from?
- Ø Principles: Why and How they Work
- Ø Nomenclature: Slide rule naming conventions
- Ø Precision, Accuracy & Significant Figures
- Ø How to use a slide rule
 - Ø Basic Operations
 - Ø Time Savers and Handy Shortcuts
 - Ø Advanced Operations
- Ø Applications
- Ø Why use a slide rule today?



SLIDE RULE HISTORY

- ∅ 1614 - Invention of logarithms by John Napier
 - ✓ Yes, logarithms were a deliberate invention!
- ∅ 1617 - Development of logarithms 'to base 10' by Henry Briggs, Oxford University.
- ∅ 1620 - Interpretation of logarithmic scale form by Edmund Gunter, London.
- ∅ 1630 - Invention of the slide rule by the Reverend William Oughtred, London.
- ∅ 1850 - Modern arrangement of scales. Amédée Mannheim, France
- ∅ 1972 – Invention of the Electronic 'Slide Rule' by Hewlett-Packard
- ∅ 1975 – K&E shut down their engraving machines



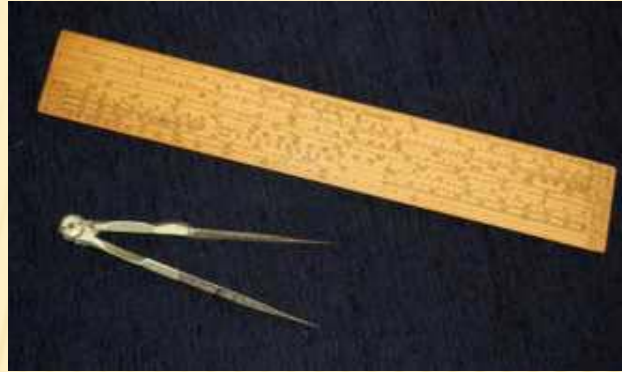


SLIDE RULE EVOLUTION

COMMON LOGARITHMS $\log_{10} x$

x	0	1	2	3	4	5	6	7	8	9	Δ_{10}	1 2 3
50	.6990	.6995	.7007	.7016	.7024	.7033	.7042	.7050	.7059	.7067	9	1 2 3
51	.7076	.7084	.7093	.7101	.7110	.7118	.7126	.7135	.7143	.7152	8	1 2 3
52	.7160	.7168	.7177	.7185	.7193	.7203	.7210	.7218	.7226	.7235	8	1 2 3
53	.7243	.7251	.7259	.7267	.7275	.7284	.7292	.7300	.7308	.7316	8	1 2 3
54	.7324	.7333	.7340	.7348	.7356	.7364	.7372	.7380	.7388	.7396	8	1 2 3
55	.7404	.7413	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474	8	1 2 3
56	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551	8	1 2 3
57	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627	8	1 2 3
58	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701	8	1 2 3
59	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774	7	1 2 3

Table of Logarithms



Gunter's Rule



Bissaker Slide Rule



King Pocket Calculator
(66" Scales)



Faber Castell 2/83n



Nestler 23R (Mannheim)



Circular Slide Rule



Thacher Calculator (30 Ft.)



Mannheim Slide Rule (1850)





LOGARITHMS – A QUICK REVIEW

Ø Invented by John Napier in 1594 for the purpose of making multiplication & division easier.

Ø $100 (10^2) \times 1000 (10^3) = 10,000 (10^5)$

similarly

8 =	2^3	or	$\text{Log}_2(8) =$	3
64 =	2^6	or	$\text{Log}_2(64) =$	6
512 =	2^9	or	$\text{Log}_2(512) =$	9

8	3
x 64	+ 6
<hr/>	
512	9

Ø Napier reasoned that he could develop an artificial number (later logarithm) that could, by means of a table, convert real numbers to their 'artificial number', which could be added together, and converted back to a real number which would be the product of the two real numbers.





LOGARITHMS – A QUICK REVIEW

- Ø Napier then spent the next 20 years of his life calculating

$$10^7 (1 - 10^{-7})^L \Big\{_{L=1}^{100} \text{ and } N = 10^7 (1 - 10^{-7})^L \Big\}_{N=5}^{10,000,000}$$

- Ø Although not actually e , Napier's base is very close



$$(1 - 10^{-7})^{10^7} \approx \frac{1}{e}$$

- Ø Napier published in 1614, and was popularized by many mathematicians and astronomers like Johannes Kepler.
- Ø In 1617, Henry Briggs converted Napier's numbers to Logarithms to the base 10 (Log_{10})
- Ø Logarithms to the base 10 make the tables smaller and easier because the characteristic (left of decimal) is easy to compute.





LOGARITHMIC IDENTITIES

Function	Formula
Product	$\log_b(xy) = \log_b(x) + \log_b(y)$
Quotient	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
Power	$\log_b(x^p) = p \log_b(x)$ 
Root	$\log_b \sqrt[p]{x} = \frac{\log_b(x)}{p}$
Change of Base	$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$ 



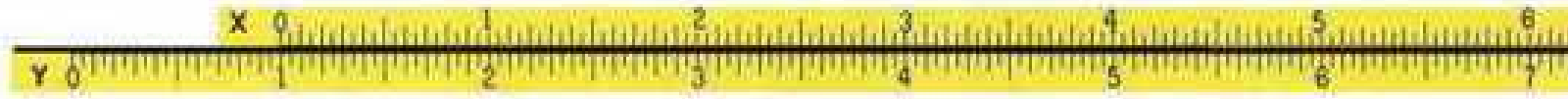
BASIC PRINCIPLES

- ∅ Multiplying & Dividing is Adding & Subtracting the logarithms of the numbers you're using!

Addition using linear scales

Figure B

Linear Scales

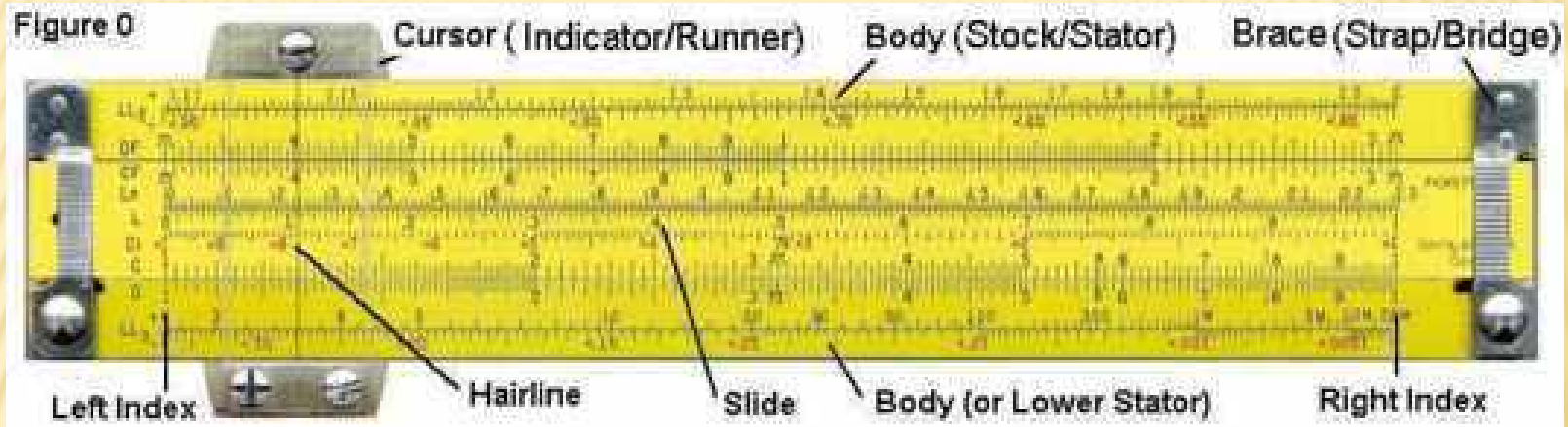


Logarithmic Scales



Multiplication using logarithmic scales

SLIDE RULE CONSTRUCTION



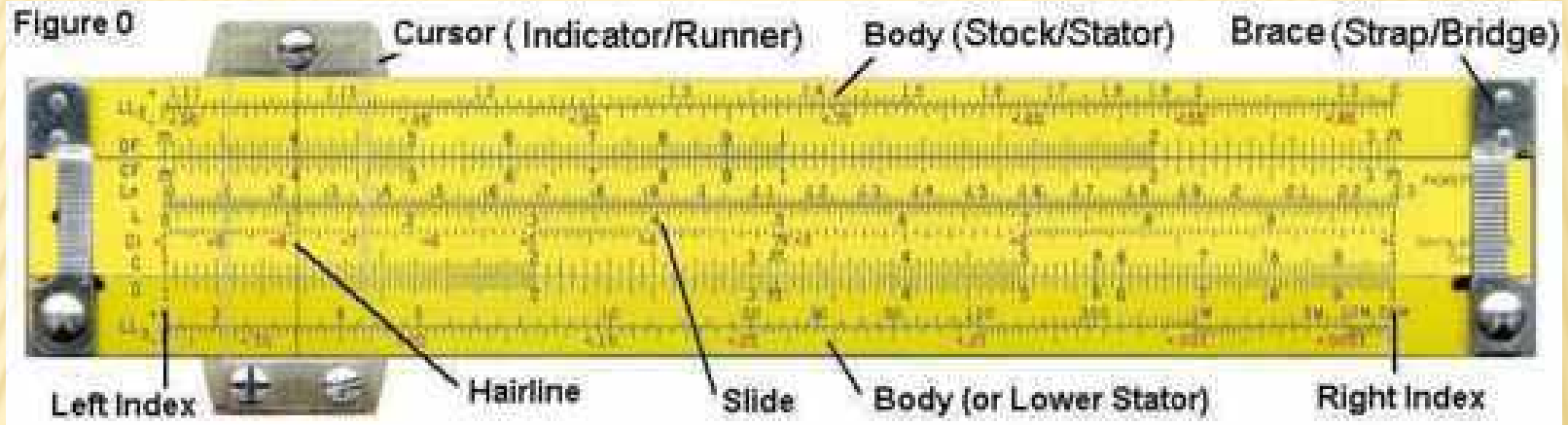
Hints for a Happy Slide Rule 'Experience'

- ∅ When moving the slide, hold the body (upper or lower) with your fingers front and back. Do NOT hold it top and bottom, thus squeezing the slide.
- ∅ If your slide is too loose, on most high quality slide rules it can be adjusted, but in the meantime, let the thumb and fingers of your holding hand add a little friction.
- ∅ Fine adjustments of the slide can be made with the thumb and finger of the 'moving' hand at the slide to body joint and twisting.





SLIDE RULE CONSTRUCTION



Hints for a Happy Slide Rule 'Experience'

- ∅ When adjusting the cursor, or flipping the slide rule over (for scales on the opposite side), then *gently* squeeze the top and bottom rails together to hold the slide in place.
- ∅ Adjust the cursor with fingers & thumbs on both sides gently rolling them on one side or the other to nudge the cursor into place.
- ∅ You can clean under the cursor / hairline by sliding a strip of paper under it and running it back and forth with moderate pressure.



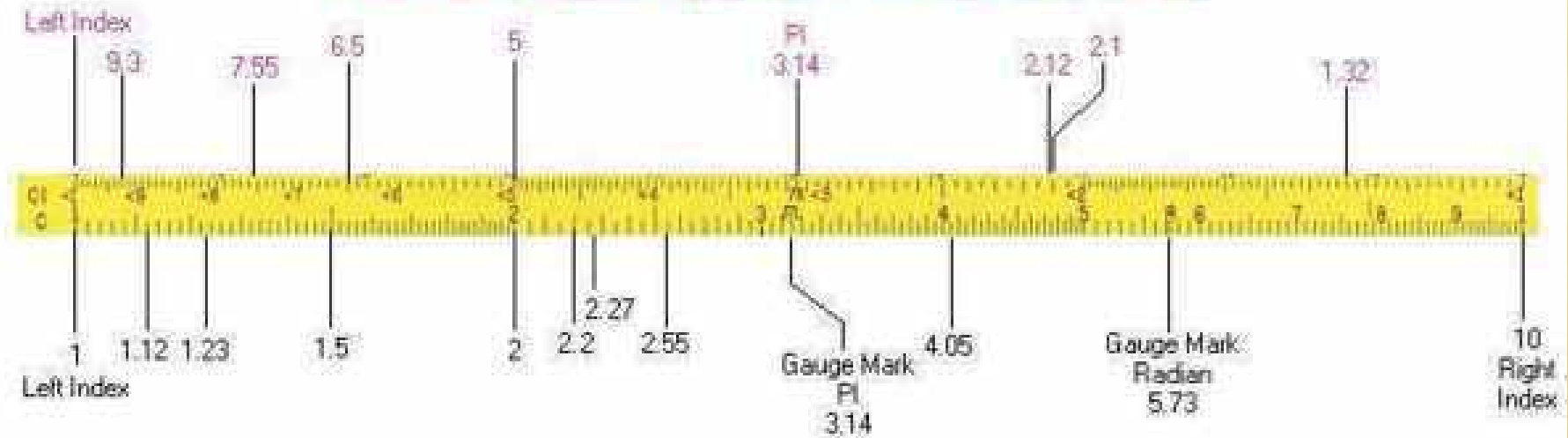


REGULAR AND INVERSE SCALES

- Ø C and D scales are single logarithmic (1-10) scales. Most scales begin with 1
- Ø The A and B scales are double logarithmic (1-10-100) having two cycles of 1-10,
- Ø The Primary divisions are whole numbers. The secondary divisions divide the Primary by 10, the Tertiary divisions divide the secondary by 5.

Figure C

Inverse Scales (CI, DI, -LL, Cos, etc)

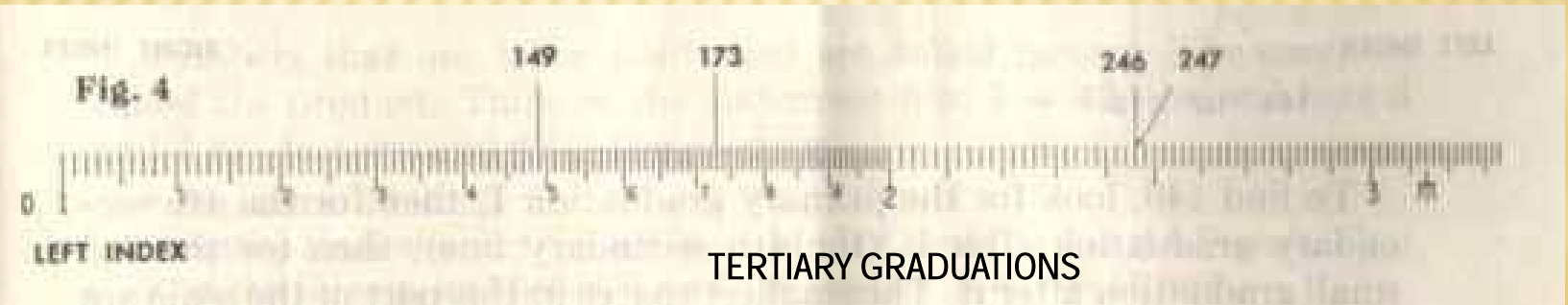
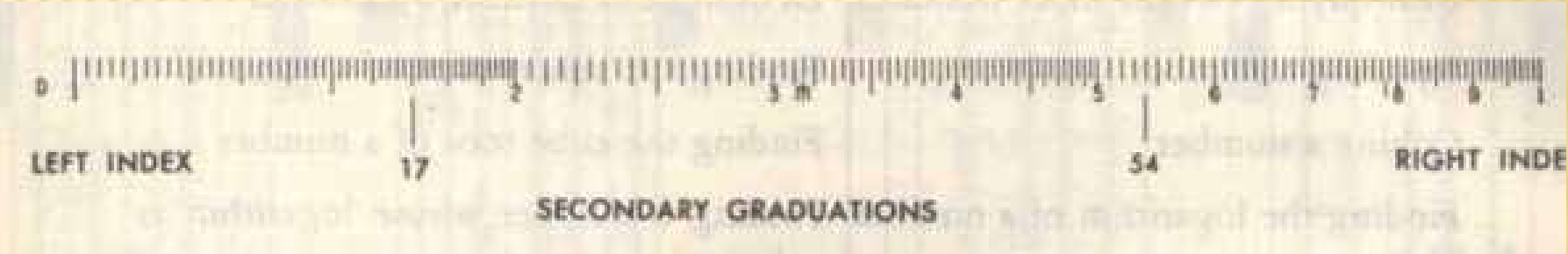
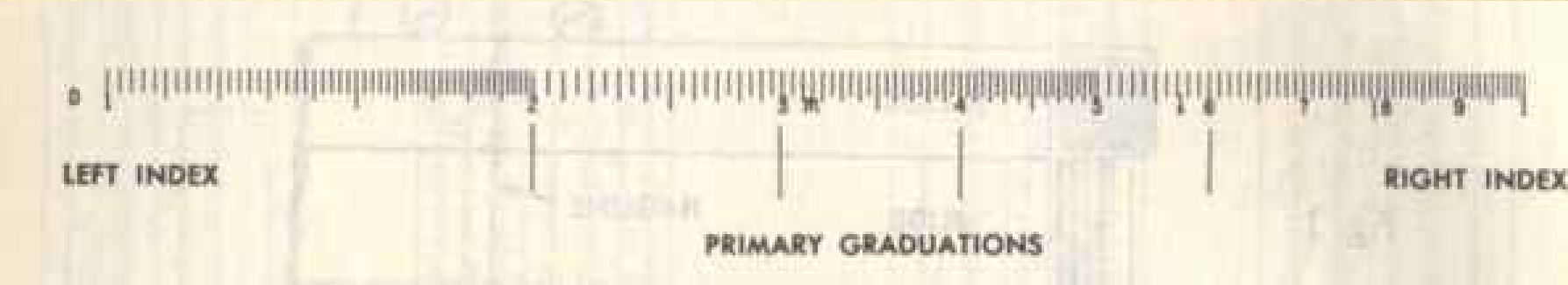


Single Logarithmic Scales (C.D)





READING A SLIDE RULE SCALE





READING A SLIDE RULE SCALE (PRACTICE)

On the scale below are some sample readings.

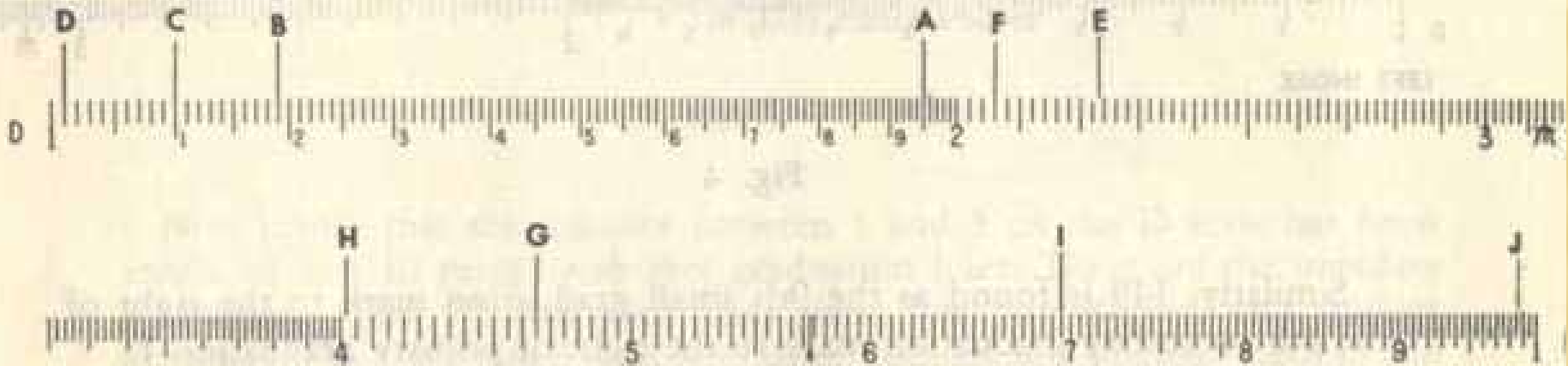


Fig. 6

A: 195	F: 206
B: 119	G: 465
C: 110	H: 402
D: 101	I: 694
E: 223	J: 987

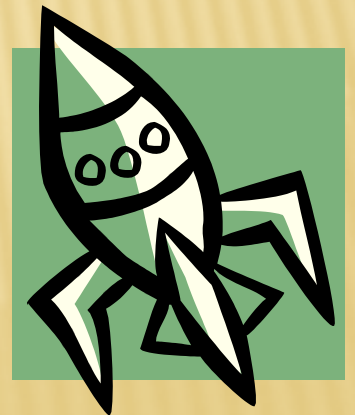


WHAT'S IN A NAME?

“Dad says that anyone who can't use a slide rule is a cultural illiterate and should not be allowed to vote. Mine is a beauty - a K&E 20-inch Log-log Duplex Decitrig”

-Have Space Suit - Will Travel, 1958.

by Robert A. Heinlein (1907-1988)





SLIDE RULE NOMENCLATURE

- Mannheim:** A standard single-face rule with scales to solve problems in multiplication, division, squares, square roots, reciprocals, trigonometry and logarithms.
- Polyphase:** Like a Mannheim but added a scale for cubes and cube roots and an inverted C scale (CI) to make certain problems easier to solve. (Some manufacturers used Mannheim and Polyphase interchangeably.)
- Duplex:** A double-faced rule. Typically added three folded scales (CF, CIF, DF) to those of the Polyphase rule to make many problems easier to solve.
- Trig:** A rule with scales for solving trigonometry problems (S, ST, T). [$^{\circ}$ ' "]
- Decitrig:** A rule with the trigonometric scales with decimal degrees (S, ST, T) [0.0 $^{\circ}$]
- Log Log:** A rule with scales for raising numbers to powers. (LL₀, LL₁, LL₀₁, etc.)
- Vector :** A rule with hyperbolic functions.
- Dual Base:** A rule with that read both common and natural logs.



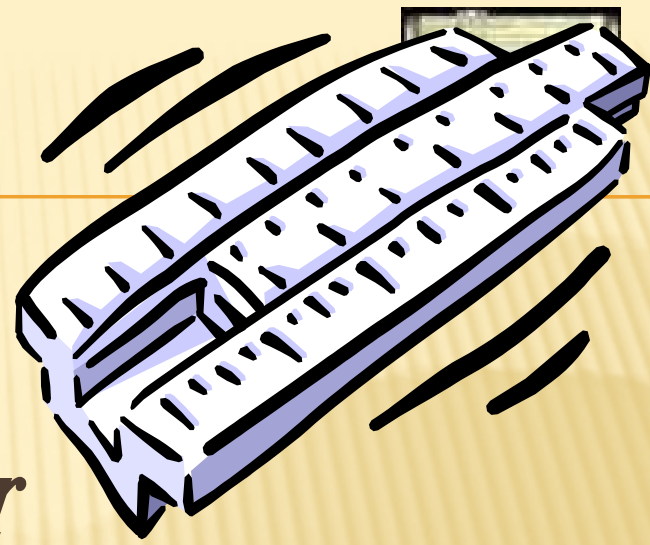


SLIDE RULE SCALES

Name	F(x)	Description
C, D	X	Fundamental rule scale, in most cases 'D' on body, 'C' on slide
A, B	X ²	Scale of Squares. 2 in sequence, compressed to 1/2 length of C & D
CI, DI	1/x	Reciprocal of C or D Scales. CI is common on polyphase, D less so
CF, DF	πX	Folded Scales, Begins at π and used to multiply / divide by π , Also CIF
K	X ³	Scale of Cubes, Compressed to 1/3 length of D Scale (on body)
R1, R2	\sqrt{x}	Opposite of A, B. Double length 1/2 on each scale for extra precision.
S, T, ST	Trig.	Sin(x) Double numbers for Cos(x) and X>45, Tan(x), ST (for $\leq 5.7^\circ$)
L	Log ₁₀	Linear scale used to determine the log (mantissa) of C & D values.
LL0,1,2	e ^{nx}	Log-log scales, LL0 usually e ^{0.001x} , LL1 e ^{0.01x} , LL2 e ^{0.1x} , LL3 e ^x
LL0n/0	e ^{-nx}	Log-log scales, LL00 or LL/0 e ^{-0.001x} continuing to LL03 or LL/3 e ^{-x}
CF/M	Log _e 10	Also DF/M Folded scale, but begins at 2.30 which is Log _e 10. Used on some rules where LL scales are to base 10, to convert to base e.
Sh, Th	Sinh	Hyperbolic Sines and Tangents



SLIDE RULE LIMRICK

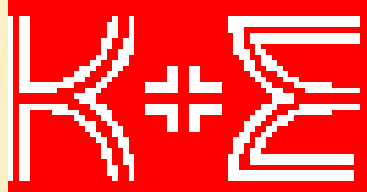


*There once was a randy professor
Who whipped out his Keuffel and Esser
To astound a young maid.
The result? I'm afraid
He completely failed to impress 'er.*



SLIDE RULE MANUFACTURERS

**PICKETT**



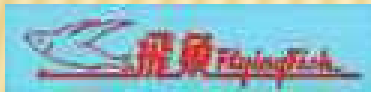
DIETZGEN

HEMMI

**STAEDTLER**

NESTLER

ARISTO





SLIDE RULE PRECISION, ACCURACY

⚠ DANGER

Calculator Gone Wild!

FLASH PROTECTION	SHOCK PROTECTION
Flash Hazard at 36 inches	Shock Hazard when 13800 VAC
Min. Arc Rating: 61773 cal/cm ²	Clear Approach 60 inches
Flash Protection Boundary: 2514857 inches	Limited Approach 36 inches
Glove Class: 2	Restricted Approach 25 inches
Clothing Category: Dangerous!	Prohibited Approach 7 inches
NO SAFE PPE EXISTS	

... AND SIGNIFICANT FIGURES





RELATIVE OR % ACCURACY OF THE SCALES

- Ø All regular 10" slide rules, regardless of where manufactured are really 25cm (9.84") scale length.
- Ø The position X (in in) of any number N is therefore given by: $X = 9.84 \text{Log}_{10} N$
- Ø Taking the derivative in terms of N : $\frac{dN}{N} = 2.3026 \frac{dx}{9.84}$
- Ø The relative or % error dN/N is independent of X .
- Ø This is true for all the logarithmic scales but the constant $(dx/9.84)$ is different for different scales.





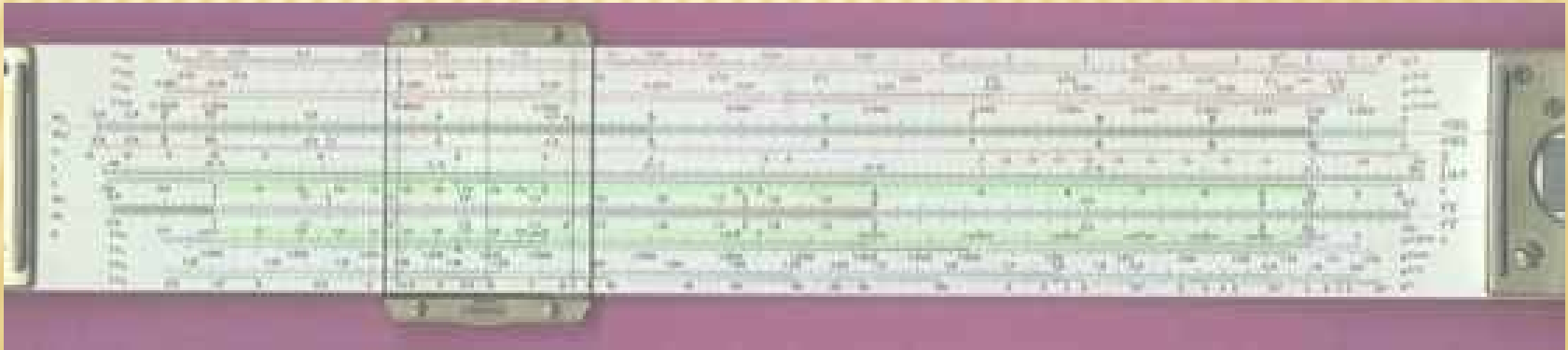
SLIDE RULE PRECISION, ACCURACY

- ∅ The engraving accuracy of the scales on a 20th century slide rule is quite high.
(K&E would serialize the slides and stators to be sure the same machine engraved each.)
- ∅ % accuracy in setting and reading the values is independent of the number set or read.
- ∅ From the scales, normally one can extract about 3 significant figures (4 for the 1. range).



PRECISION VS. ACCURACY

- ∅ Let's multiply $324 \times 520 \times 112$.
- ∅ Using a slide rule, I get 1.887×10^7
- ∅ My calculator tells me 18,869,760
- ∅ The calculator is certainly more precise.
But which is more accurate?





SIGNIFICANT FIGURES – THE RULES

- 1) ALL non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant.
 - 2) ALL zeroes between non-zero numbers are ALWAYS significant.
 - 3) ALL zeroes which are SIMULTANEOUSLY to the right of the decimal point AND at the end of the number are ALWAYS significant.
 - 4) ALL zeroes which are to the left of a written decimal point and are in a number ≥ 10 are ALWAYS significant.
 - 5) ZEROS to the left of an implied decimal point (e.g. 190) might be significant.
- ✓ A helpful way to check rules 3 and 4 and avoid 5 is to write the number in scientific notation. If you can/must get rid of the zeroes, then they are NOT significant





SIGNIFICANT FIGURES – THE RULES

Rules for mathematical operations

- 1) In addition and subtraction, the result is rounded off to the last common digit occurring furthest to the right in all components.
(i.e. depends on *position*)
 - 2) In multiplication and division, the result should be rounded off to have the same number of significant figures as in the component with the least number of significant figures.
- ✓ So, my I should have rounded my answer to 1.89×10^7
(Giving 520 the benefit of the doubt).
But I really wanted to show that I could get that 4th digit in the left end.
[A slide rule will at least help keep you from being embarrassingly wrong.]





HOW TO USE A SLIDE RULE





BASIC OPERATIONS

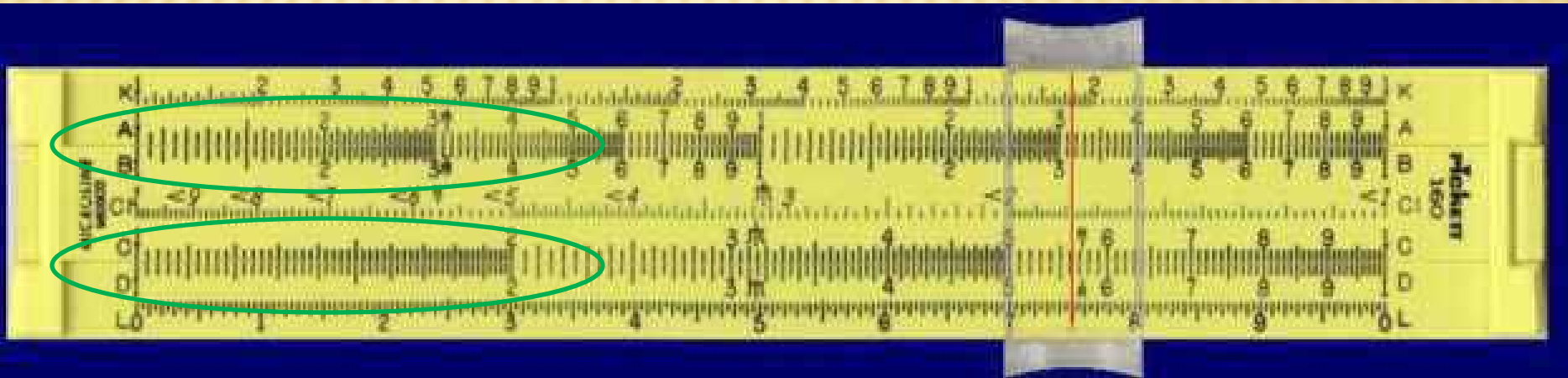
- ∅ Multiplication
- ∅ Division
- ∅ Squares & Square Roots
- ∅ Cubes & Cube Roots
- ∅ Logarithms
- ∅ Trigonometry

- ✓ Most examples and the explanatory text is courtesy the self-guided slide rule course on the International Slide Rule Museum website, images created from emulators designed by [Derek Ross](#). Initial examples based on Derek's Self Guided demo and expanded by Mike Konshak



MULTIPLICATION

- ∅ Multiplication normally uses the C & D scales.
- ∅ The A & B scales, used together, can also be used for multiplication.
- ∅ The CF/DF Pair, as well as CI/D or C/DI can also be used.



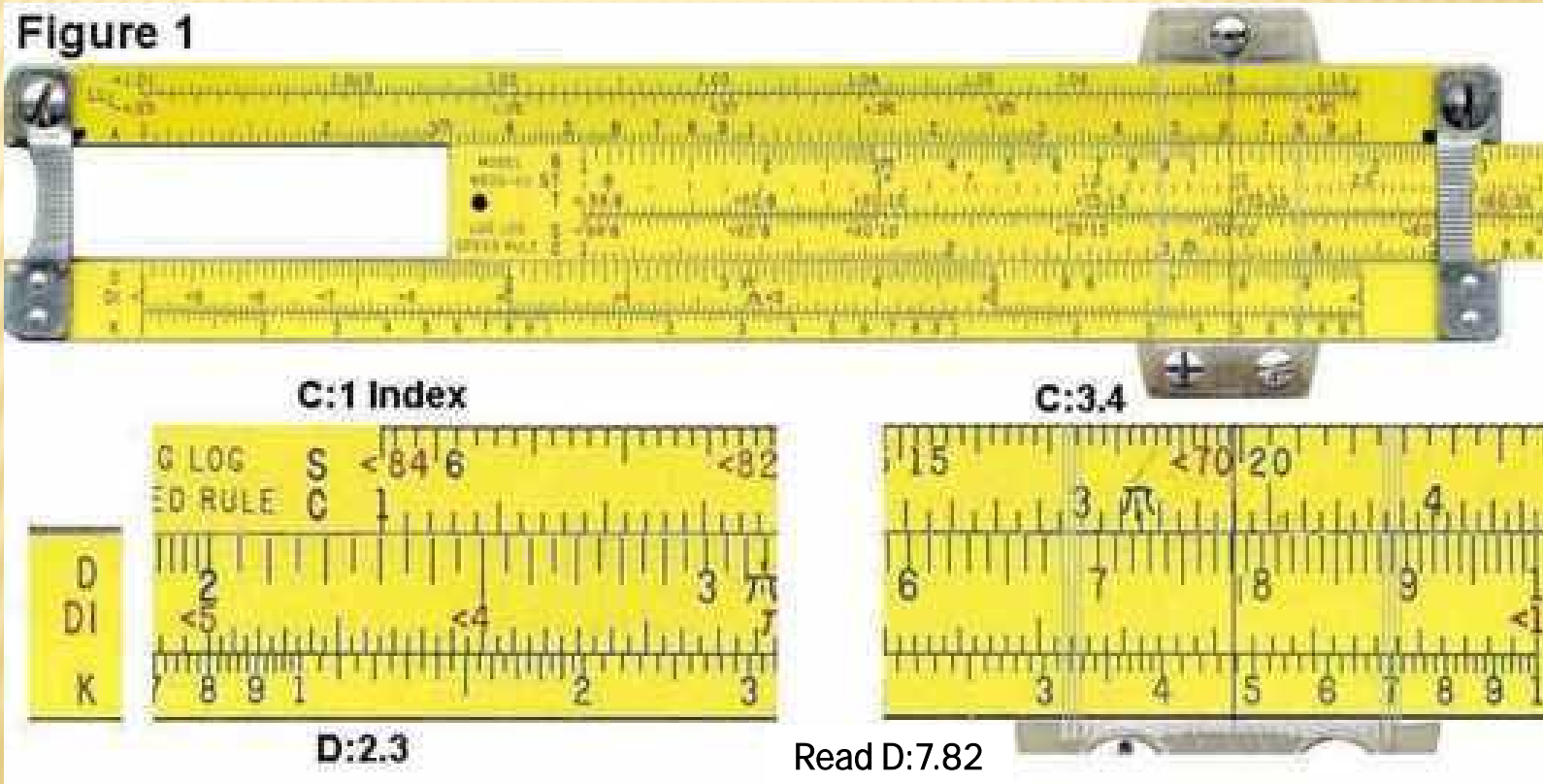


MULTIPLICATION

Multiply 2.3×3.4

- Slide the leftmost Index '1' on C over 2.3 on the D scale.
- Move the cursor to 3.4 on the C scale.
- The cursor is on the D scale just a bit over 7.8 or 7.82. This is the answer.

Figure 1



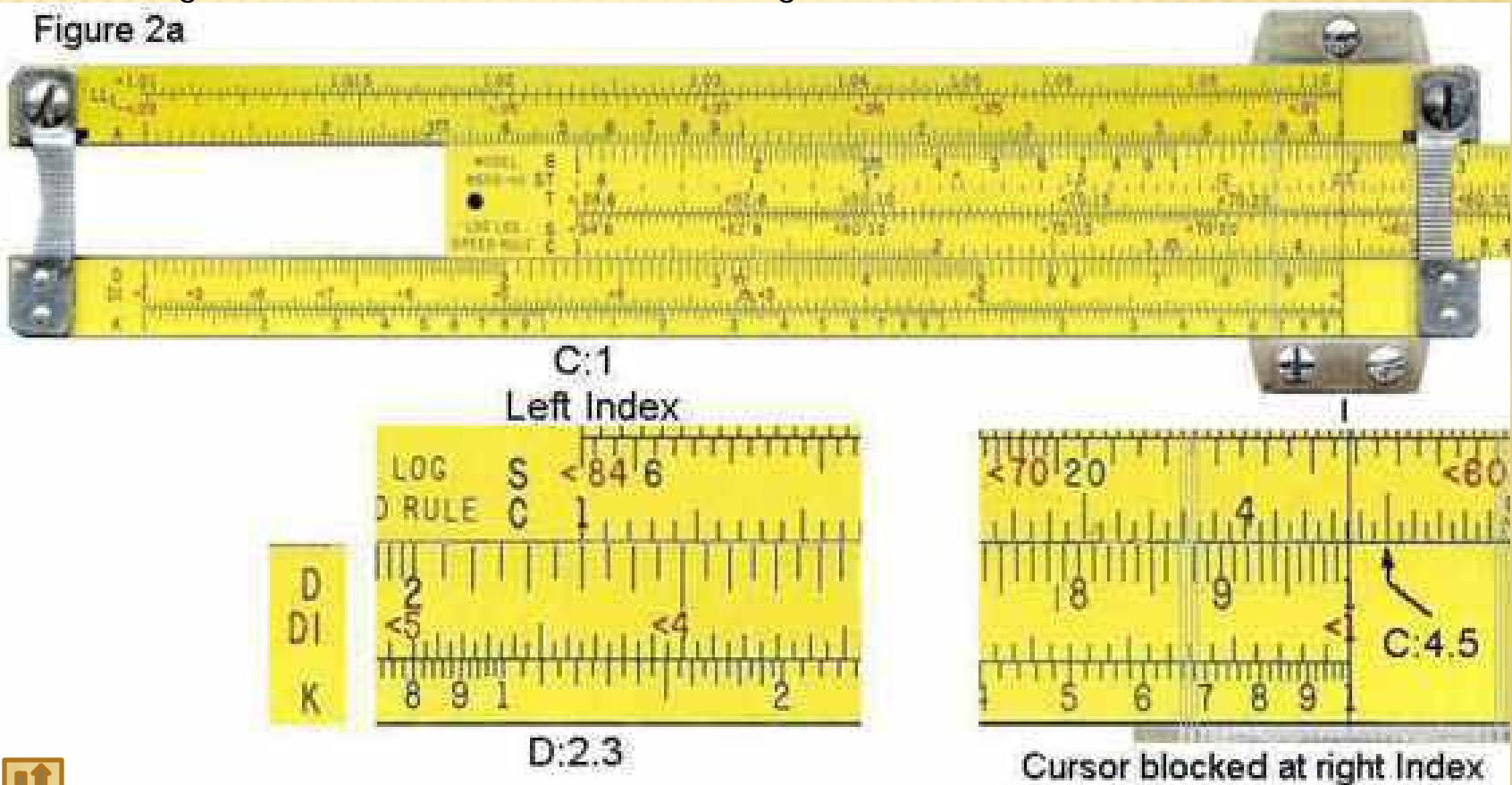


WRAP-AROUND MULTIPLICATION

Multiply 2.3×4.5

- Move the left Index on C to 2.3 on the D scale.
- Try to move the cursor to 4.5 on the C scale. The cursor is blocked by the brace.
- The target C:4.5 is off the D scale. The right Index must now be used.

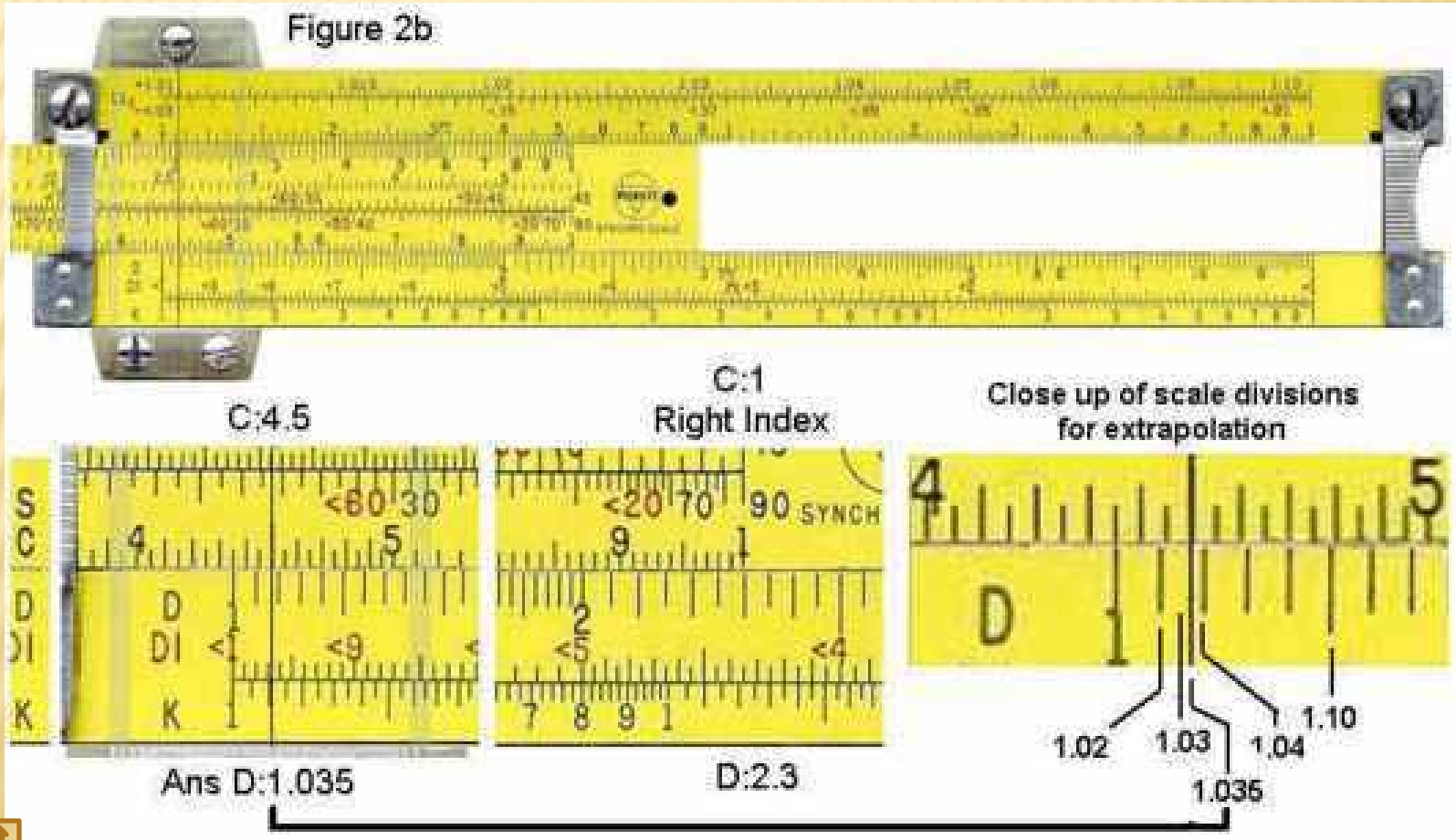
Figure 2a





WRAP-AROUND MULTIPLICATION (2)

- Move the right Index on C (C:1) to over 2.3 on the D scale (D:2.3)
- Slide the cursor to 4.5 on the C scale (C:4.5)
- On the D scale, you'll see that the hairline is between divisions. Extrapolate the answer to 1.035.
- We mentally calculate $2 \times 5 = 10$, so we adjust the decimal place to get 10.35 or 10.4.





HOW TO FIND THE DECIMAL POINT

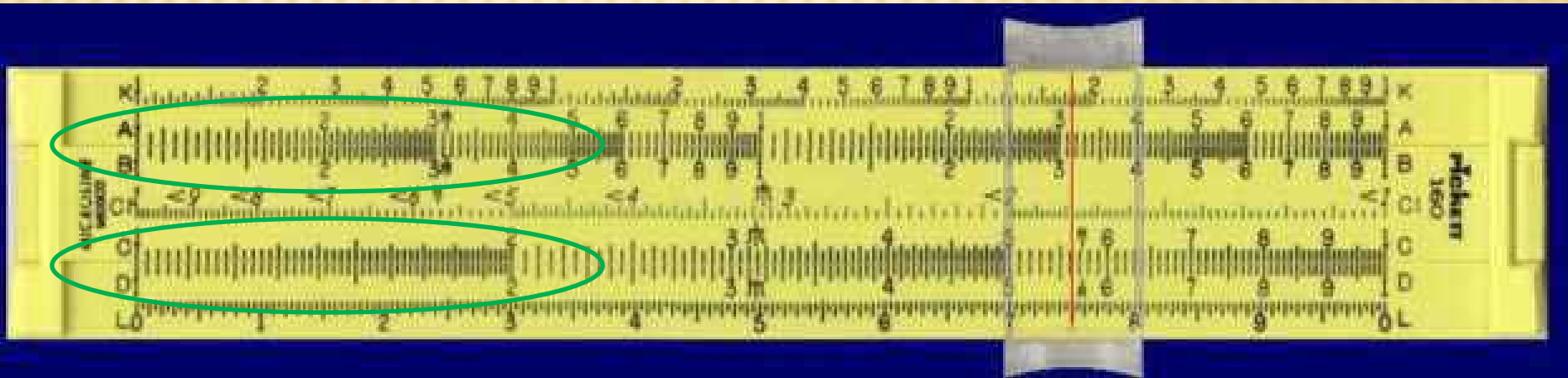
- ∅ Method I: Common Sense
- ∅ Method II: Estimation
- ∅ Method III: Standard Format
 1. Reduce all numbers to scientific notation
 2. If multiplying add the exponents, subtract if dividing
 3. If you had to use the right index add 1 to the exponent.
(If dividing and you had to use the right index subtract 1)
 4. The correct answer is now in scientific format.
- ✓ $2.3 \times 10^0 \cdot 4.5 \times 10^0 = 1.035 \times 10^{0+1} = 10.35$

Estimate, Calculate and Evaluate



DIVISION

- ∅ Division normally uses the C & D scales.
- ∅ The A & B scales, used together, can also be used for Division.
- ∅ The CF/DF Pair, as well as CI/D or C/DI can also be used.



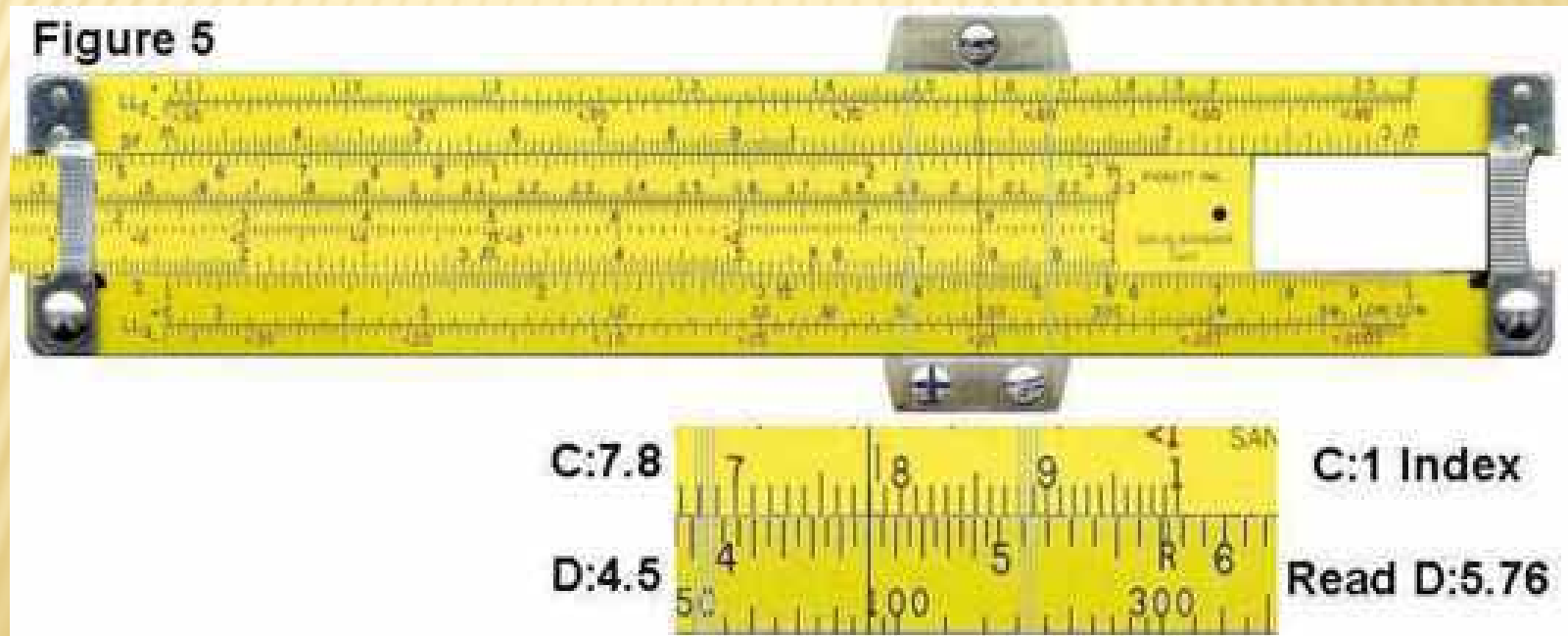


DIVISION

Calculate $4.5 / 7.8$

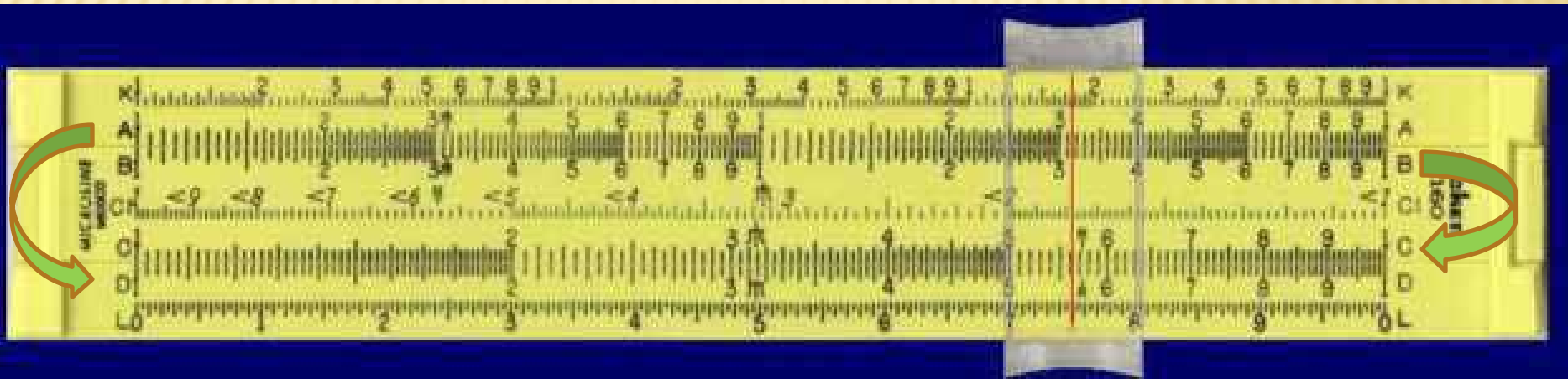
- Move the cursor to 4.5 on the D scale.
- Slide 7.8 on the C scale to the cursor.
- Move the cursor to either the leftmost or rightmost '1' on the C scale, whichever is in range. In this case, you would move it to the rightmost '1'.
- The cursor is now at 5.76 on the D scale.
- We know that the correct answer is near $4/8 = 0.5$, so move decimal to get 0.576.

Figure 5



SQUARES AND SQUARE ROOTS

- ∅ The Square and Square Root functions use 'A' in conjunction with 'D' or 'B' with 'C'.

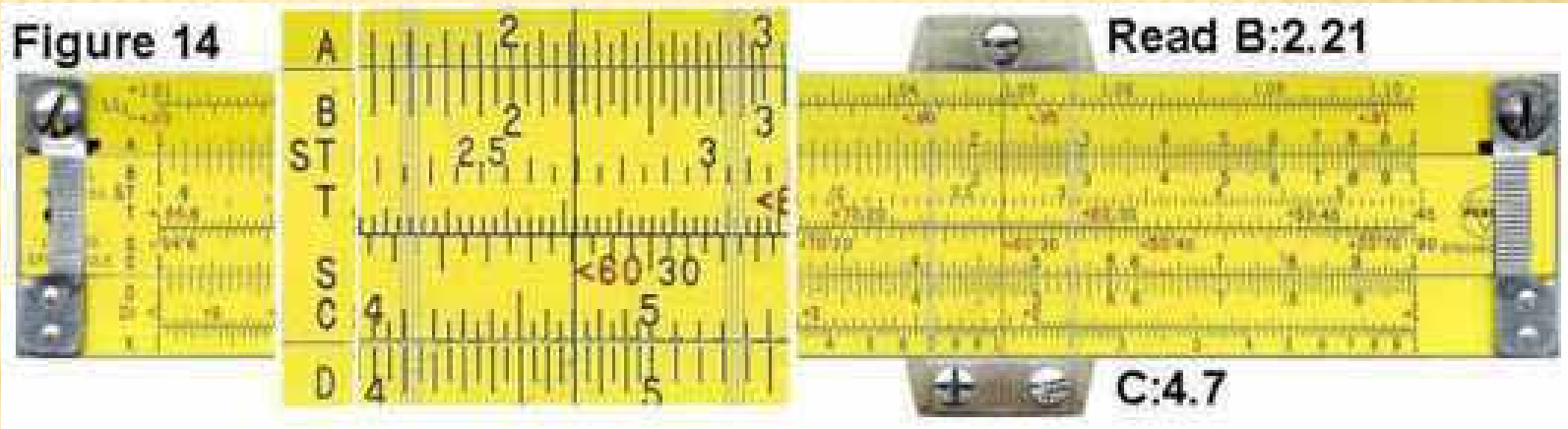


- ∅ Some slide rules also have double length scales for squares and square roots used with the 'D' scale. These are designated as R_1, R_2 ; Sq_1, Sq_2 ; W_1, W_2 .

SQUARES

Calculate 4.7^2

- Move the cursor to 4.7 on the C scale.
- The cursor is now at 2.2 on the B scale.



Decimal Point Location

- I. We know that the correct answer is near $5^2 = 25$, so adjust the decimal to get 22.1
- II. Scientific Notation, multiply exponent by 2, if on RH side of 'B' scale add 1.
 $(4.7 \times 10^0) = (4.7)^2 \times (10^{0 \times 2}) = 2.21 \times 10^{(0+1)}$

SQUARE ROOTS

Calculate $\sqrt{4500}$

- You will notice that the B scale has two similar halves. Which half to use?
- Use “scientific notation” with an even exponent (e.g. $4500 = 45 \times 10^2$)
- The left half is used to find the square root of numbers with an odd number of digits, the right half for even digits. Since 45×10^2 has an even digits, use the right half.
- Move the cursor to 4.5 on the right half of the B scale.

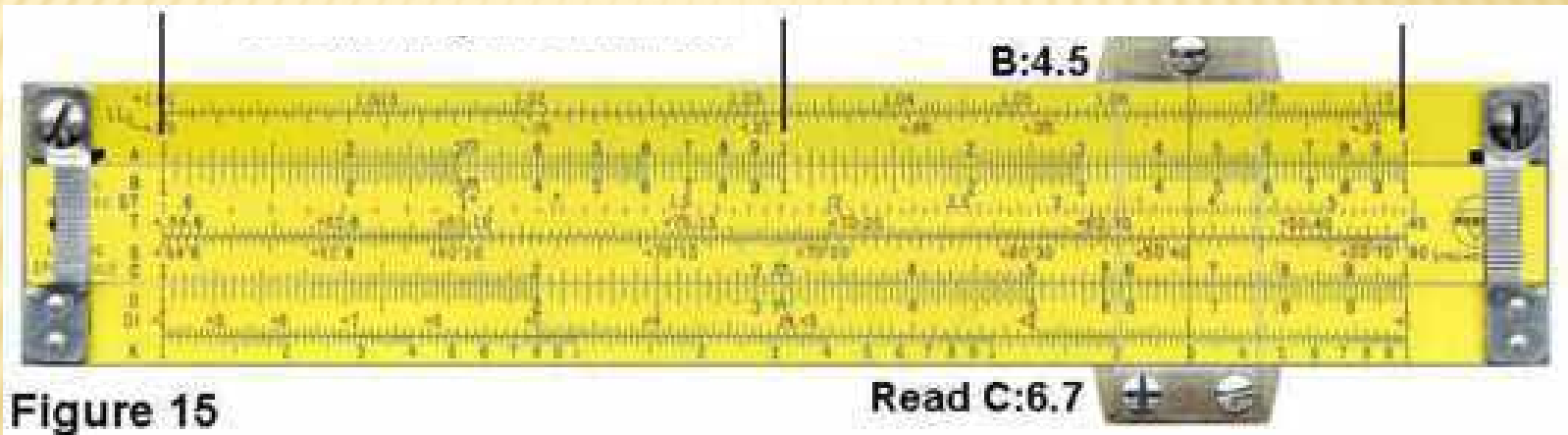


Figure 15

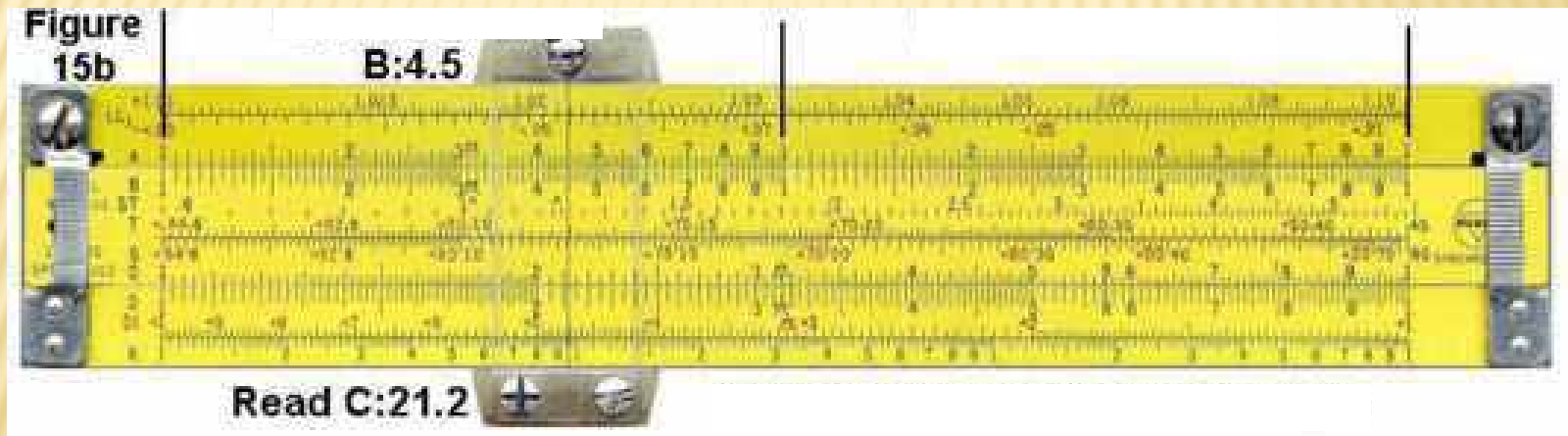
Decimal Point Location

- I. The cursor is now at 6.7 on the C scale. We know that $70^2 = 4900$, which is in the ballpark of 4500. Therefore we adjust the decimal point to get a result of 67.
- II. Scientific Notation: Place the decimal point to give an even exponent and divide the exponent by 2, $(45 \times 10^2) = \sqrt{45 \times 10^{(2)/2}} = 6.7 \times 10^{(2/2)} = 6.7 \times 10^1 = 67$

SQUARE ROOTS

Calculate $\sqrt{450}$

- Try it again with a 3 digit number Which half to use?
- Scientific notation with an even exponent. (e.g. 4.5×10^2)
- The left half is used to find the square root of numbers with an odd number of digits, the right half for even digits. Since 4.5×10^2 has odd digits, use the left half.
- Move the cursor to 4.5 on the left half of the B scale.



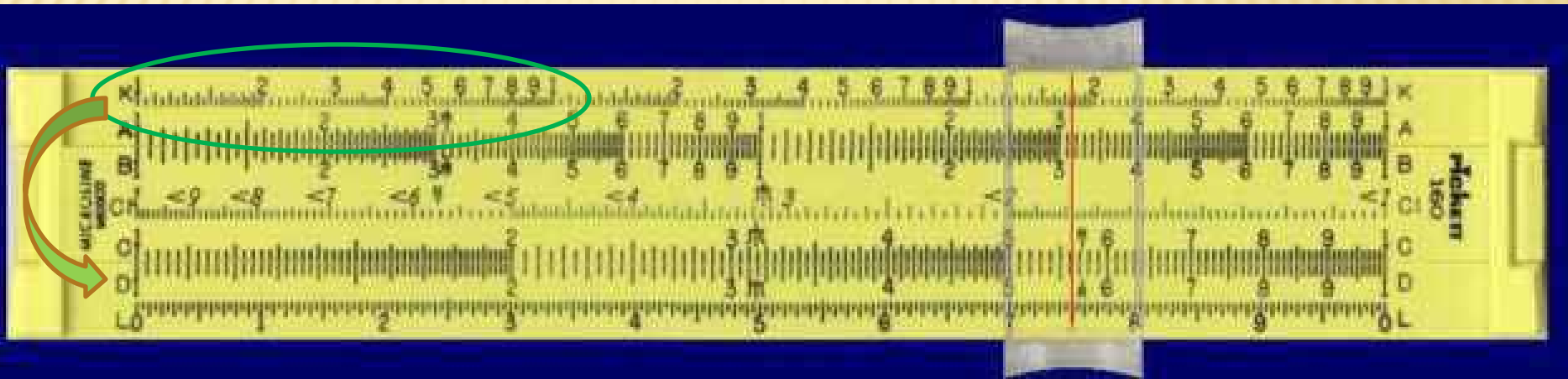
Decimal Point Location

- I. The cursor is now at 2.12 on the C scale. We know that $20^2 = 400$, which is in the ballpark of 450. Therefore we adjust the decimal point to get a result of 21.2.
- II. Scientific Notation: Place the decimal to give an even exponent and divide the exponent by 2, $(4.5 \times 10^2) = \sqrt{4.5 \times 10^{(2)/2}} = 2.12 \times 10^{(2/2)} = 2.12 \times 10^1 = 21.2$



CUBES AND CUBE ROOTS

- ∅ The cube and cube root functions use the 'K' scale in conjunction with the 'D' scale.

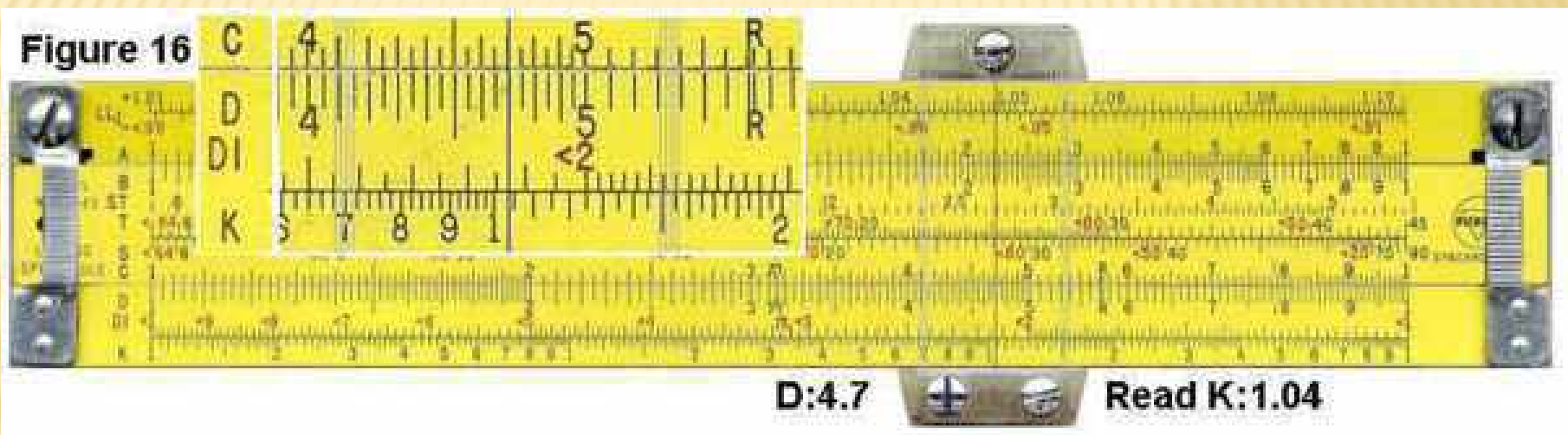


CUBES

Calculate 4.7^3

Move the cursor to 4.7 on the D scale.

The cursor is now at 1.04 on the K scale.



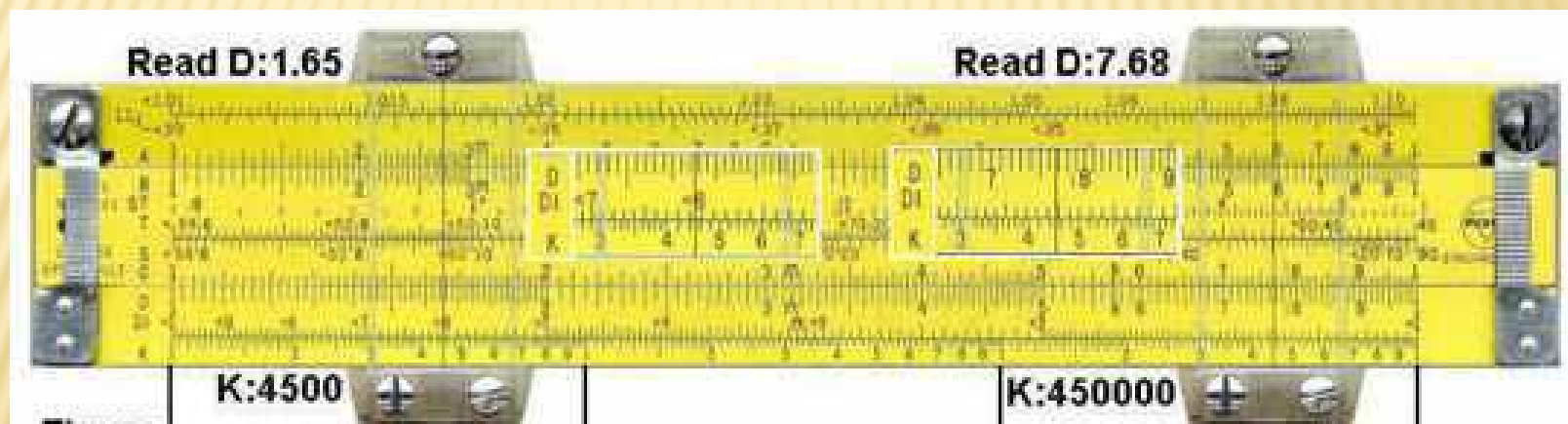
Decimal Point Location

- I. We know that the correct answer is near $5 \times 5 \times 5$, which, to further approximate, is near $5 \times 5 \times 4 = 5 \times 20 = 100$. Therefore adjust the decimal to get a result of 104.
- II. Scientific Notation. multiply exponent by 3, in left sector add 0, center +1, right +2
 $(4.7 \times 10^0) = (4.7)^3 \times (10^{0 \times 3}) = 1.04 \times 10^{(0+2)} = 1.04 \times 10^{(2)} = 104$

CUBE ROOTS

Calculate cube root of 4500

- You will notice that the K scale has three similar sectors. Which sector to use?
- “Scientific notation” with 1, 2 or 3 leading digits and exp. divisible by 3 (e.g. 4.5×10^3)
- The left sector is used to find the cube root of numbers with 1 digit, the 2nd 2 digits and the 3rd sector 3 digits, then cycling back around.
- Move the cursor to 4.5 on the left sector of the K scale and read 1.65 on ‘D’.



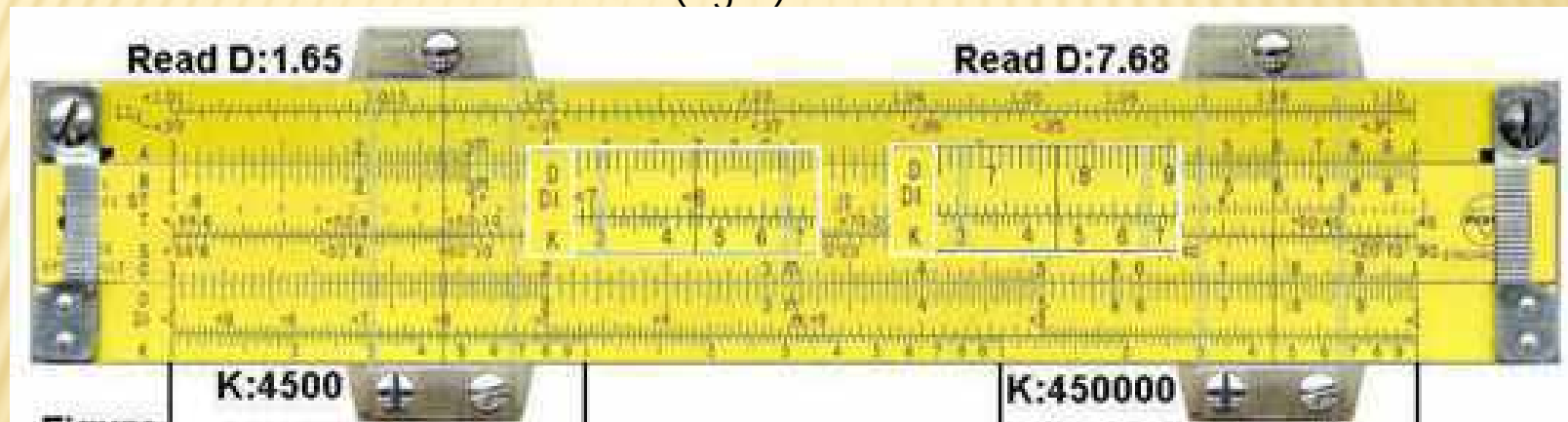
Decimal Point Location

- I. We guess that the answer is ~ 10 . 10^3 is 1000 and 20^3 is 8000. We are between 10 & 20 so move the decimal place and get the correct result of 16.5. .
- II. Scientific Notation: Place the decimal to get an exponent divisible by 3 and divide by 3. $(4.5 \times 10^3) = \sqrt[3]{4.5 \times 10^{(3/3)}} = 1.65 \times 10^{(1)} = 1.65 \times 10^1 = 16.5$

CUBE ROOTS

Calculate cube root of 450000

- You will notice that the K scale has three similar sectors. Which sector to use?
- Scientific notation with 1, 2 or 3 leading digits and exp. divisible by 3 (e.g. 450×10^3)
- The 1st sector is used to find the cube root of numbers with 1 digit, the 2nd 2 digits and the 3rd sector 3 digits, then cycling back around.
- Move the cursor to 450 on the 3rd (right) sector of the K scale and read 7.68 on 'D'.



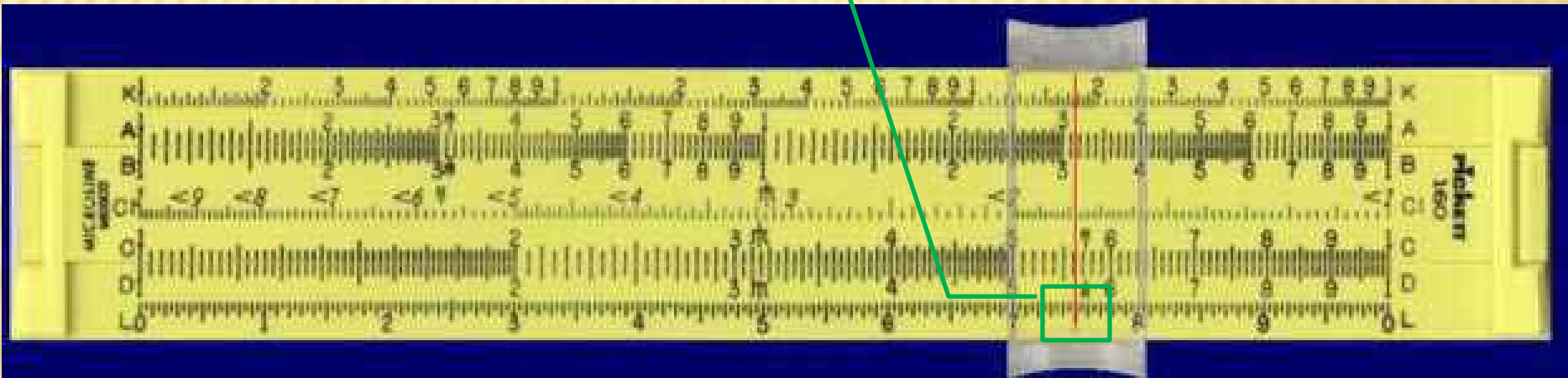
Decimal Point Location

- I. We guess that the answer is ~ 10 . 10^3 is 1000 and 100^3 is 1000000. We are between 10 & 100 so move the decimal place and get the correct result of 76.8.
- II. Scientific Notation: Place the decimal to get an exponent divisible by 3 and divide the exponent by 3. $\sqrt[3]{(450 \times 10^3)} = \sqrt[3]{450} \times 10^{(3/3)} = 7.68 \times 10^{(1)} = 76.8$



LOGARITHMS

- § The 'L' or Logarithmic scale computes the fractional or *mantissa* of the common Logarithm of the number on the 'C' or 'D' scale opposite it.
- § The integer portion or *characteristic* is taken directly from the exponent of the number as expressed in scientific notations.
- § $562 = 5.62 \times 10^2$ $\text{Log}_{10}(562) = 0.75 + 2 = 2.75$



LOGARITHMS

- § If your S, L & T Scales are on the back, **DON'T** pull out the slide and flip it over.
- § The 'L' scale on the back is used with the 'C' & 'D' scales on the front.
- § To find the logarithm of a number set the index of the 'C' scale over the number on 'D'. Flip over the slide rule and read the mantissa at the index line on the right end.

EXAMPLE: Find the log of 1.35. (See Figs. 31 and 32).

(1st) Set Left "C" Index
Over 135 on "D"

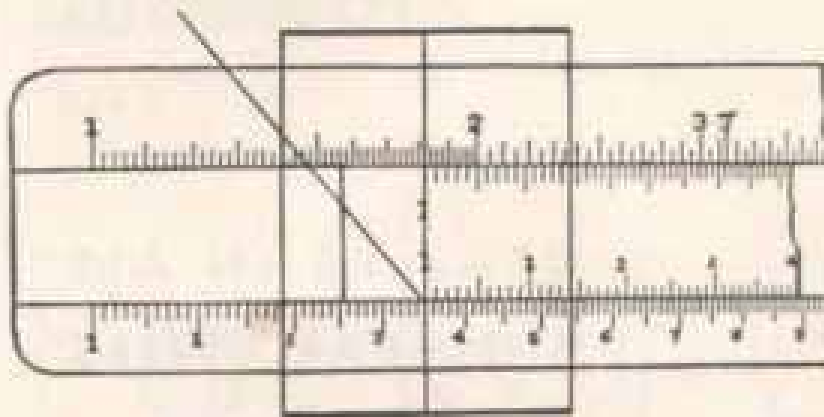


Fig. 31

(2nd) Read 1303 on Scale "L"
Over the Lower Index Line
of Right Hand Slot on
Reverse Side of Rule.

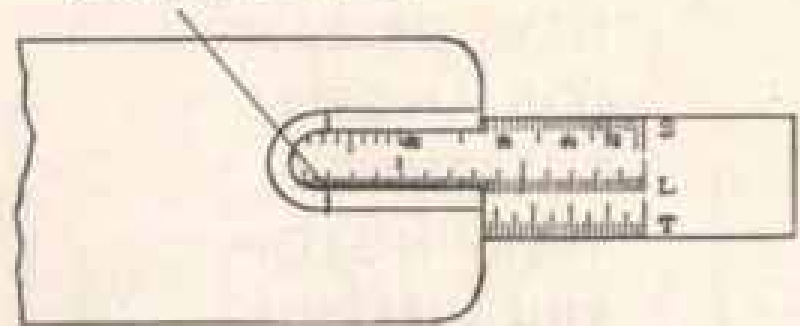


Fig. 32



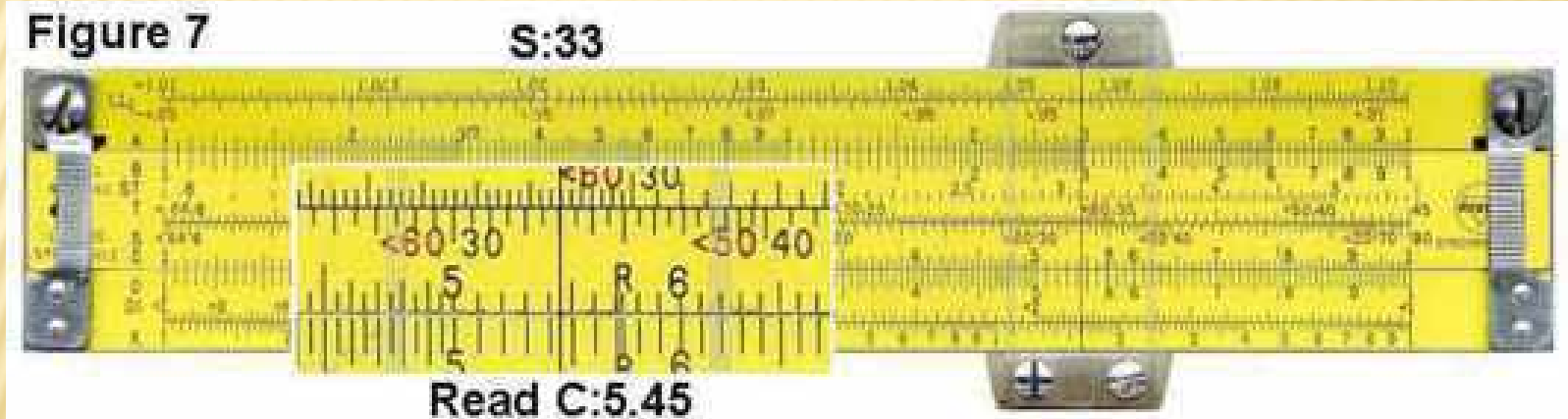
TRIGONOMETRY

$\sin(x)$

Calculate $\sin(33^\circ)$

- Move the cursor to 33 on the S scale.
- The cursor is at 5.45 on the C scale.
- We know that the correct answer for $\sin(x)$ in this range is between 0.1 and 1, so we adjust the decimal place to get 0.545.

Figure 7



- ✓ This is for $\sin(x)$ between 5.7° and 90°
- ✓ N.B. If you have a classic Mannheim design, S,T & L on the rear of the slide, these instructions will NOT work. We'll cover those soon.

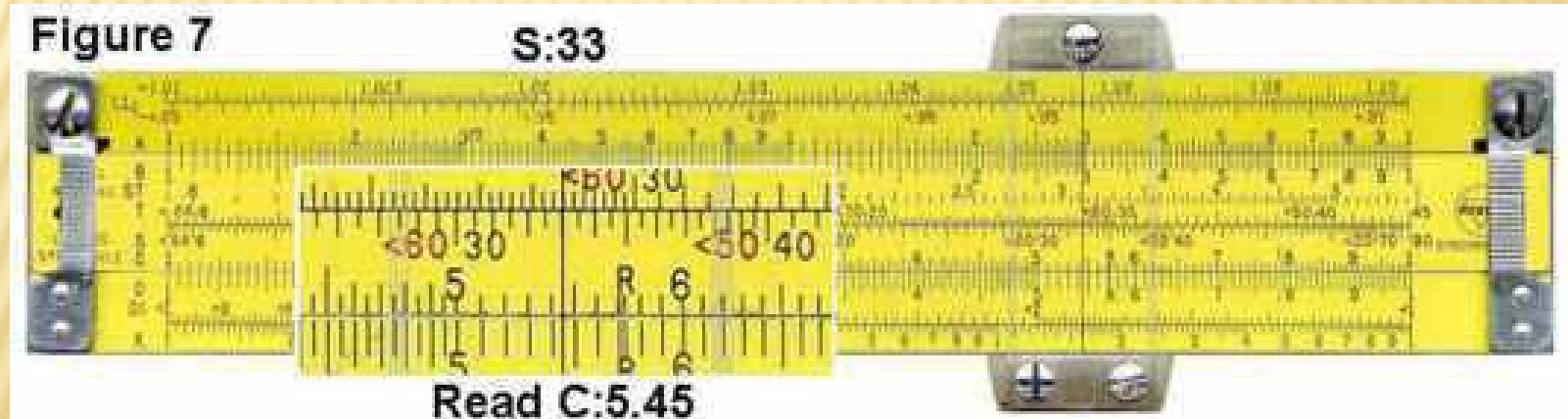


TRIGONOMETRY

cos(x)

Calculate $\cos(33^\circ)$

- The cos scale shares the sin 'S' scale. Instead of increasing from left to right like the sin scale, cos increases from right to left. Sometimes shown as <## or ##
- Move the cursor to <33 on the S scale.
- The cursor is now at 8.4 on the C scale.
- We know that the correct answer for $\cos(x)$ in this range is between 0.1 and 1, so we adjust the decimal place to get 0.84.



✓ This is for $\cos(x)$ between 5.7° and 90°

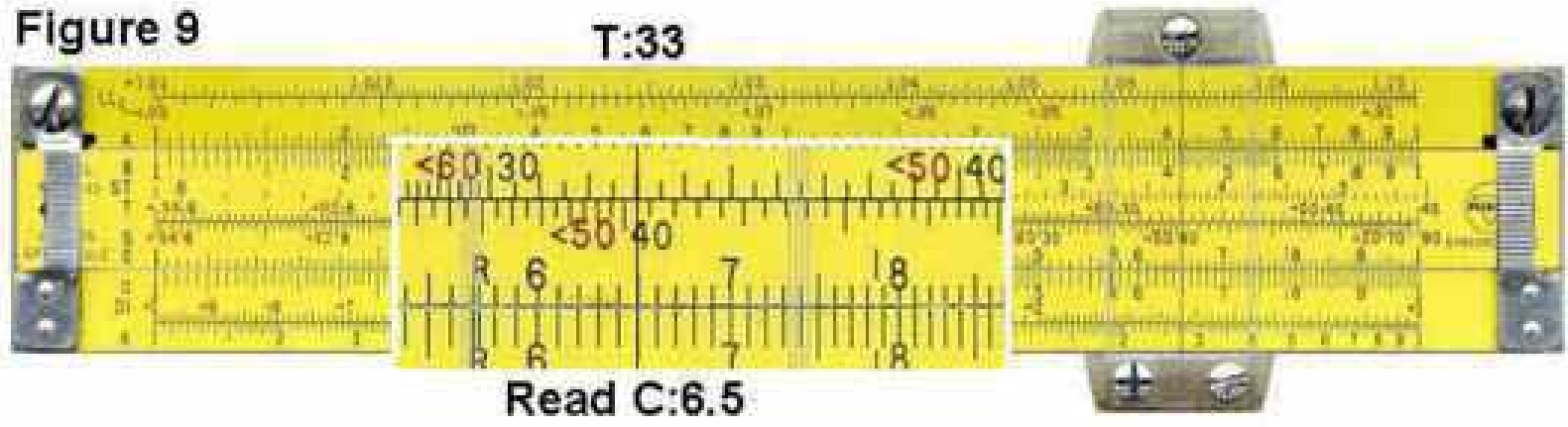
TRIGONOMETRY

$$\tan(x) \quad x \leq 45^\circ$$

Calculate $\tan(33^\circ)$

- Move the cursor to 33 on the T scale.
- The cursor is now at 6.5 on the C scale.
- We know that the correct answer for a tan in this range is between 0.1 and 1, so we adjust the decimal place to get 0.65.

Figure 9



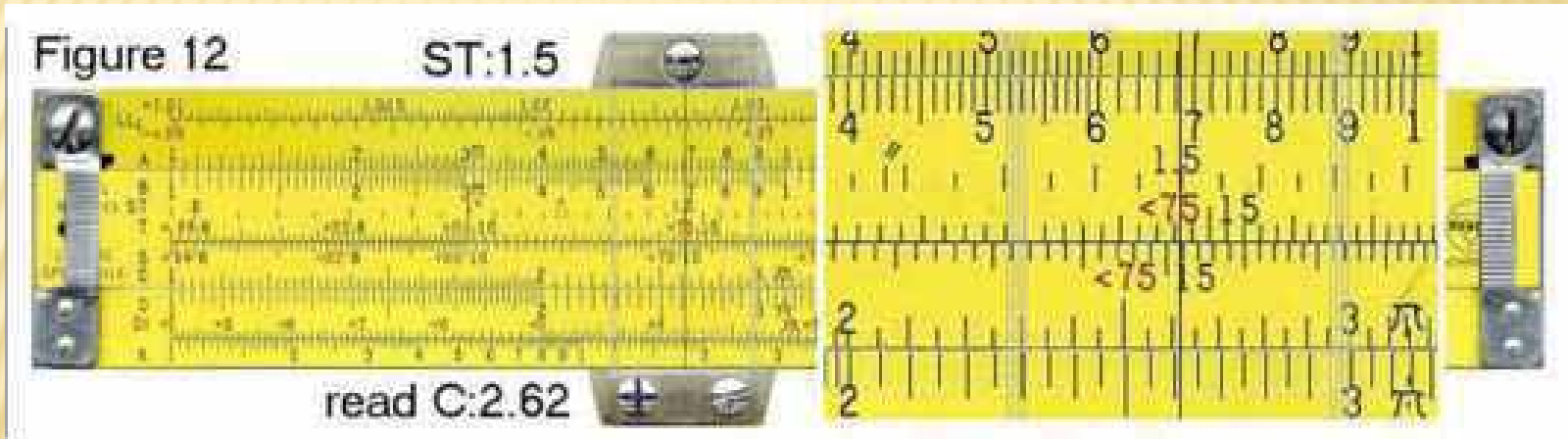
- ✓ This is for $\tan(x)$ between 5.7° and 45°
- ✓ For angles between 45° and 84° use backwards 'T' and CI (if your slide is aligned use DI, otherwise flip rule over for CI)
- ✓ $\tan(x)$ in this range are from 1 to 10. Scale appropriately.

TRIGONOMETRY

small angles $0.6 \leq x \leq 5.7^\circ$

Calculate $\sin(1.5^\circ)$

- Move the cursor to 1.5 on the ST scale.
- The cursor is now at 2.62 on the C scale.
- We know that the correct answer for a sin in this range is between 0.01 and 0.1, so we adjust the decimal place to get 0.0262.



- ✓ In this range $\sin(x)$ and $\tan(x)$ are nearly equal, so use same scale for both.
- ✓ For very small angles, the sin or tan function can be approximated closely by the equation: $\sin(x) = \tan(x) = x / (180/\pi) = x / 57.3$.
- ✓ Knowing this, the calculation becomes a simple division.
This technique can also be used on rules without an ST scale.



TRIGONOMETRY

For the 'classics'

- § If your S, L & T Scales are on the back, **DON'T** pull out the slide and flip it over.
- § The Sine 'S' scale is used with the 'A' & 'B' scales.
- § Set the upper right hand index line on the rear with any value of degrees on the 'S' scale. The right index of 'A' on the reverse will indicate the sine on 'B'.

EXAMPLE: Find the natural sine of 32 degrees. (See Figs. 25 and 26)

(1st) Set 32° on "S" to
Upper Index Line in
Right Slot on Reverse Side

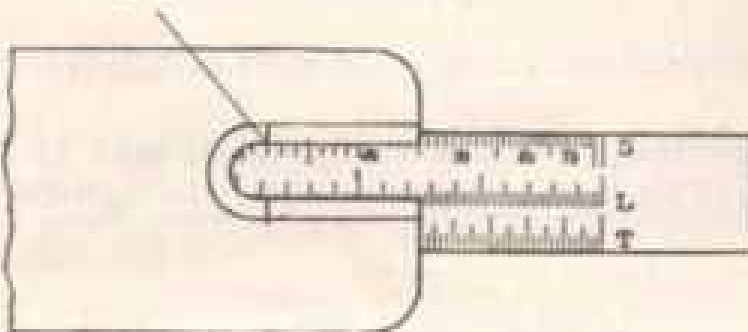


Fig. 25

(2nd) Under Right "A" Index
Read 53 on "B"

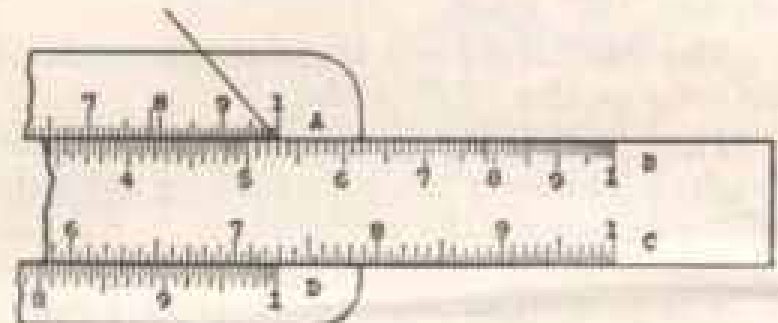


Fig. 26

NOTE: The natural sines read on the right half of scale "B" have a decimal point before the first significant figure. Therefore, the correct result of above example is 0.53.



TRIGONOMETRY

For the 'classics'

- § For small angles, the procedure is the same!
- § The Sine 'S' scale is used with the 'A' & 'B' scales.
- § Set the upper right hand index line on the rear with any value of degrees on the 'S' scale. The right index of 'A' on the reverse will indicate the sine on 'B'.

EXAMPLE: Find the natural sine of 2 degrees. (See Figs. 27 and 28).

(1st) Set 2° on "S" to
Upper Index Line in
Right Slot on Reverse Side

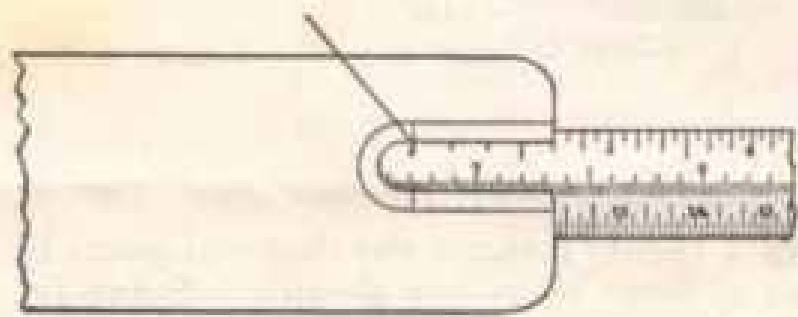


Fig. 27

(2nd) Under Right "A" Index
Read 349 on "B"

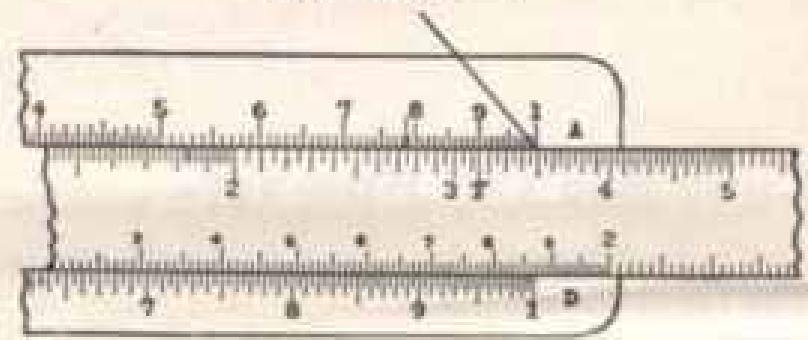


Fig. 28

NOTE: The natural sines read on the left half of the "B" scales have a zero between the decimal point and the first significant figure. Therefore, the correct result of the above example is 0.0349.





TRIGONOMETRY

For the 'classics'

- § If your S, L & T Scales are on the back, **DON'T** pull out the slide and flip it over.
- § The tangent 'T' scale is used with the 'C' & 'D' scales.
- § Set the lower left hand index line on the rear with any value of degrees on the 'T' scale. The left index of 'D' on the reverse will indicate the tangent on 'C'.

EXAMPLE: Find tangent of angle 7° 40'. (See Figs. 29 and 30).

(1st) Set 7° 40' on 'T' to Lower Index Line in Left Slot on Reverse Side

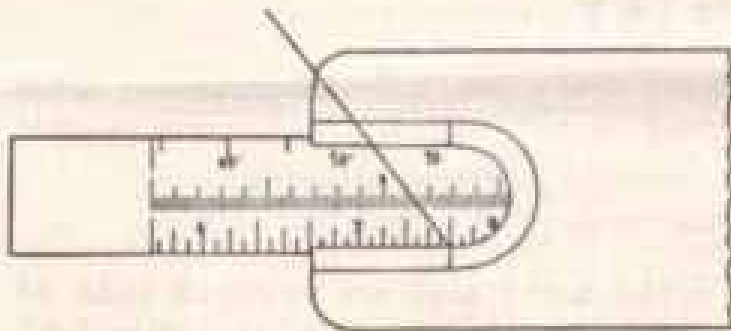


Fig. 29

(2nd) Over Left "D" Index Read 1346 on "C"

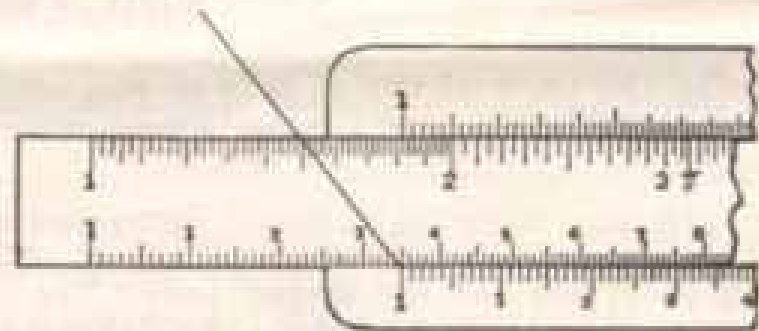


Fig. 30

- § The 'T' scale only goes to 45°, for angles > 45° use $\cot(x) = \tan(90^\circ - x)$
- § The co-tangent of 'T' will be on the 'D' scale under the right 'C' index.



TIME SAVERS AND TRICKS

- ∅ Folded Scale Multiplication
- ∅ Reciprocals
- ∅ Using CI & DI
- ∅ For extra precision reciprocals close to 1 use the values of $LLn/LL0n$
- ∅ Slide rules are ideal for 'Solve for the Unknown' problems by pretending you already know it. (covered a little later)





FOLDED SCALE MULTIPLICATION

Remember multiply 2.3×4.5 ?

- Slide the leftmost Index, '1', on C over the 2.3 on the D scale (D:2.3).
- We can't move the cursor to 4.5 on the C scale; it's out of range.
- **We can use the folded scales to get this answer. 1 on CF is already at 2.3 on DF!**
- Move the cursor to 4.5 on the CF scale (CF:4.5).
- The cursor is now at 1.04 on the DF scale (DF:1.04).

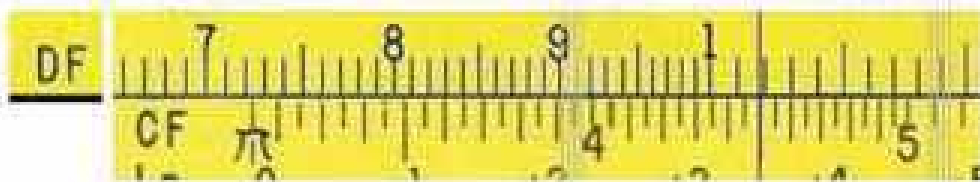
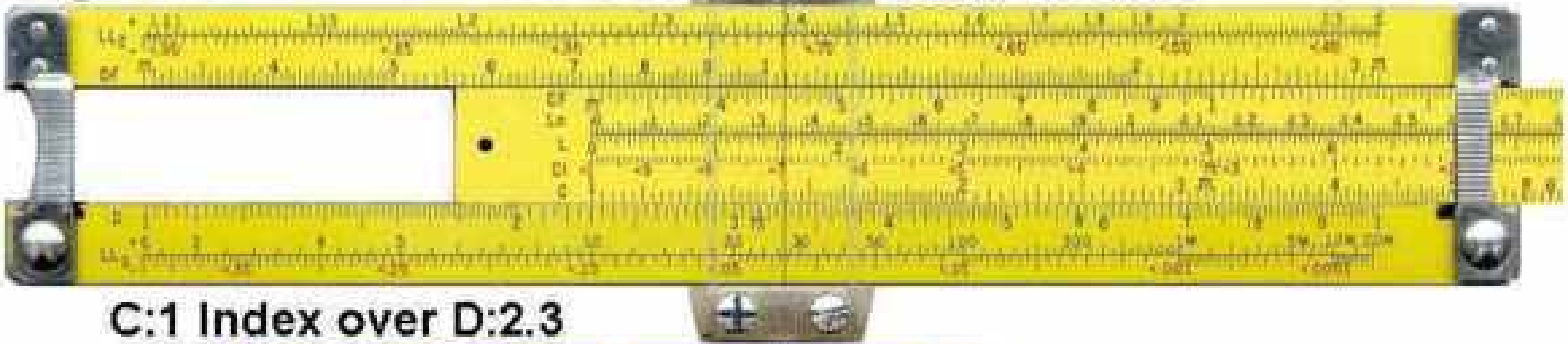


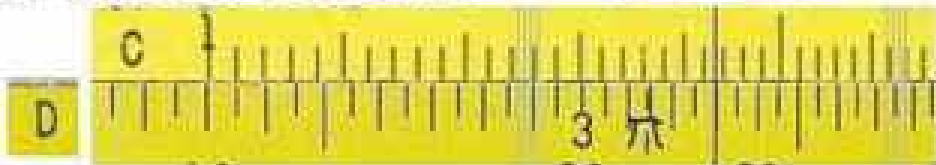
Figure 3

CF:4.5

Read DF:1.04



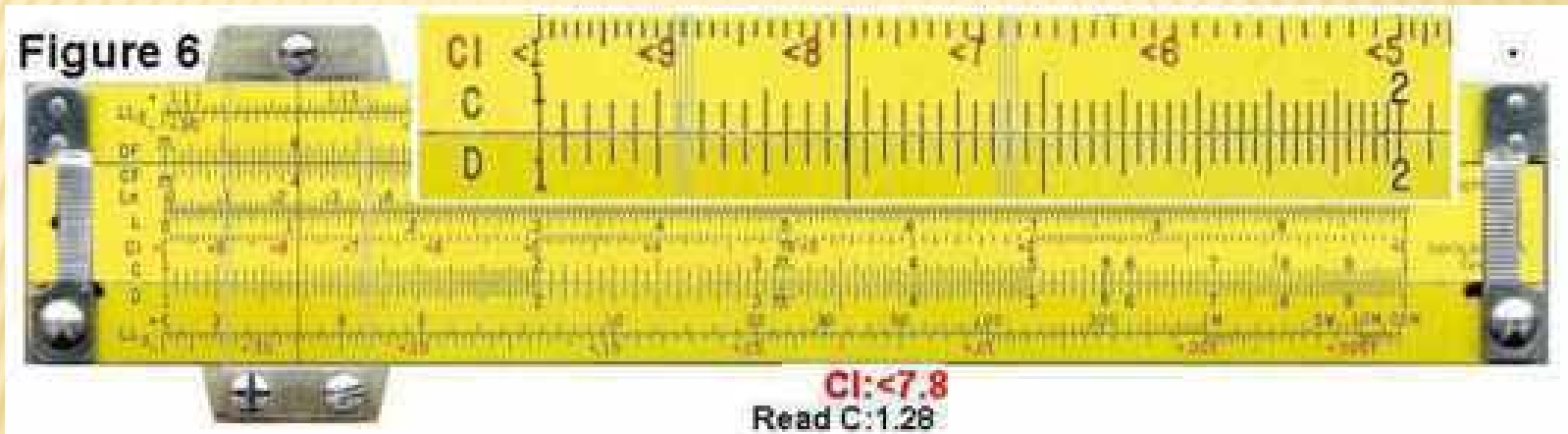
C:1 Index over D:2.3



RECIPROCAL

Calculate the reciprocal of 7.8, or $1/7.8$

- Move the cursor to 7.8 on the CI scale. Note that the CI scale increases from right to left, as indicated by the '<' symbols before the numbers.
- The cursor is now at 1.28 on the C scale.



Decimal Point Location

- I. We know that the correct answer is near $1/10 = 0.1$, so move decimal to get 0.128.
- II. Scientific Notation, multiply exponent by -1, then always subtract 1.
 $(7.8 \times 10^0) = 1/7.8 \times (10^{-1 \times 0 - 1}) = 1.28 \times 10^{(0-1)} = 1.28 \times 10^{(-1)} = .128$



USING RECIPROCALLS FOR SPEED

- ∅ We saw earlier that in division you set the divisor over the dividend to 'subtract' the values.
- ∅ Multiplying is the same as dividing by the reciprocal.
(e.g. $2.3 \times 4.5 = 2.3 \div 1/4.5$)
- ∅ In this manner you never have to reverse the slide!
- ∅ So, always multiply by setting the CI (of CIF) over the multiplicand on the D or (DF) scale and reading the result on D (or DF) against '1' on C (or CF).
- ∅ This also sets you up for compound or 'chain' operations by multiplying [D x C in the old fashioned way] or dividing by 'multiplying' the result by CI [D x CI].



COSINE, SECANT,
TANGENT, SINE 3.14159
INTEGRAL, RADICAL,
UDV, SLIPSTICK,
SLIDE RULE, MIT! --
MASSACHUSETTS INSTITUTE OF
TECHNOLOGY FOOTBALL CHEER





ADVANCED OPERATIONS

- ∅ Powers (A log-log slide rule is required)
 - ∅ Proportions and Conversions
 - ∅ Solving Quadratic Equations
-
- ✓ Most examples and the explanatory text is courtesy the self-guided slide rule course on the International Slide Rule Museum website, images created from emulators designed by [Derek Ross](#). Initial examples based on Derek's Self Guided demo and expanded by Mike Konshak





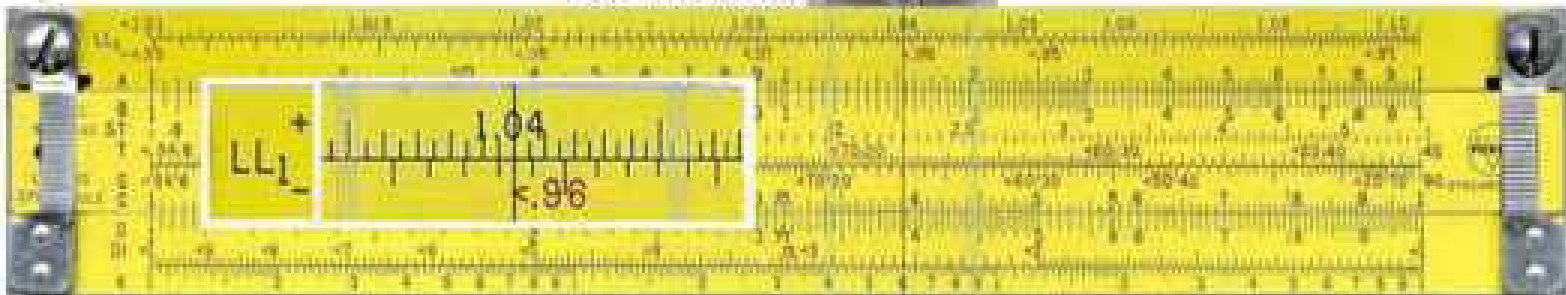
RAISING A NUMBER TO A POWER OF 10 ($N > 1$)

Calculate 1.04^{100}

- Move the cursor to 1.04 on the LL1 scale.
- On this model slide rule it is necessary to flip the slide rule over to get to LL3
- The cursor is at 50.5 on the LL3 scale. This is the correct answer.

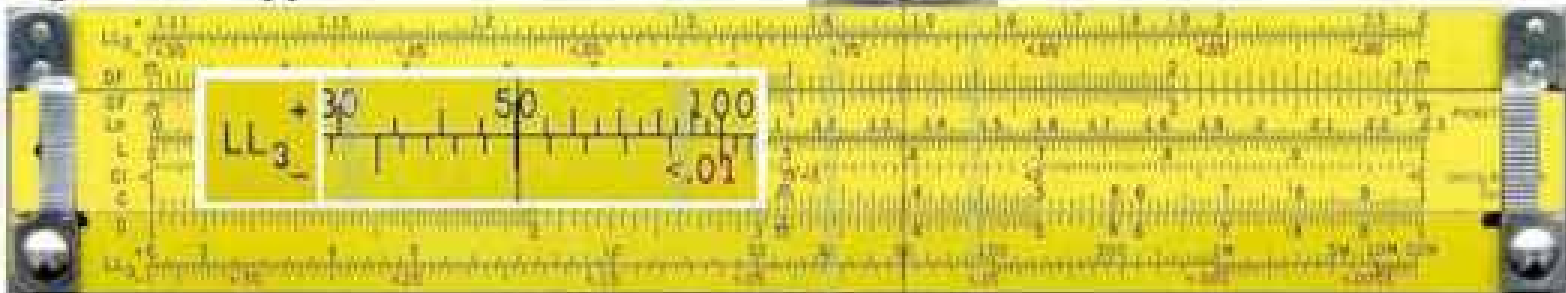
Figure 18b

Set LL1:1.04



Do not move the cursor while flipping the SR over!

Figure 18b Flipped



Read LL3:50.5

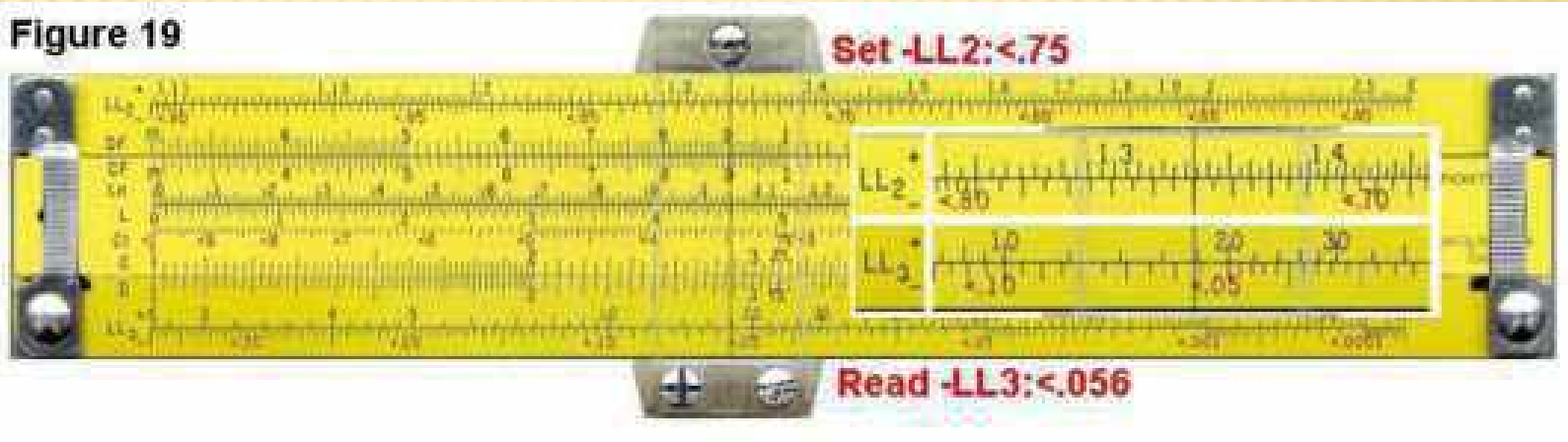
RAISING A NUMBER TO A POWER OF 10 ($N < 1$)

- ∅ The reciprocals of the LL scales are the -LL scales. They work the same way, but you have to make sure that you look for the answer on a -LL scale. They are **RED** in color and increase to the <LEFT

Calculate 0.75^{10}

- Move the cursor to <0.75 on the -LL2 scale.
- The cursor is at <0.056 on the -LL3 scale. This is the correct answer.

Figure 19



- ∅ Note: The positions of the LL scales vary greatly from one model slide rule to another.

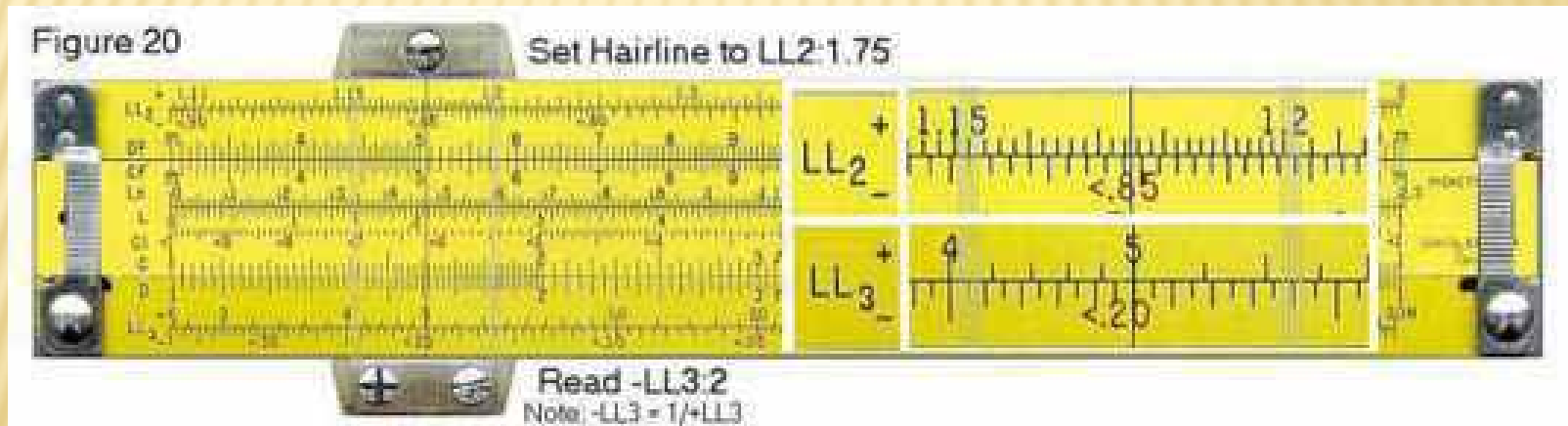


RAISING A NUMBER TO A POWER OF -10 ($N > 1$)

- ∅ The reciprocals of the LL scales are the -LL scales. They work the same way, but you have to make sure that you look for the answer on a -LL scale. They are **RED** in color and increase to the <LEFT

Calculate 1.175^{-10}

- Move the cursor to 1.175 on the LL2 scale.
- The hairline is over 5 on the LL3 scale. This is 1.175^{10}
- The cursor is also over .2 on the -LL3 scale. This is the reciprocal of 1.175^{-10} or $1/1.175^{10}$ and is the correct answer.

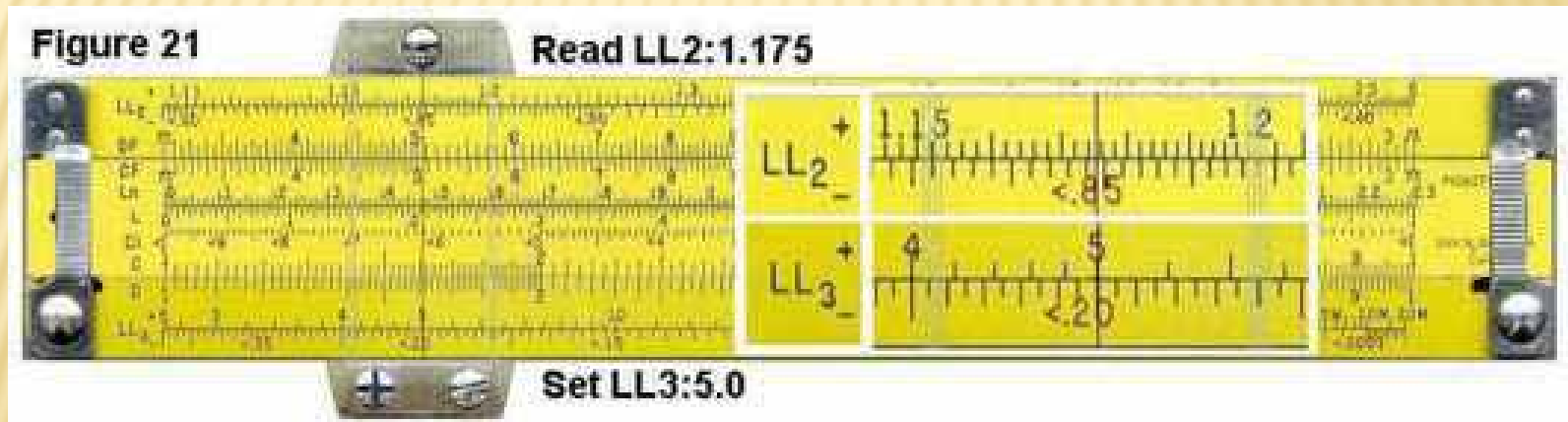


FINDING THE 10TH ROOT ($N > 1$)

- ∅ To raise a number to the 10th power, use the next highest LL scale.
- ∅ To find a tenth root, use the next lowest LL scale.
- ∅ Finding the tenth root is the same as raising a number to the power of 0.1.

Calculate $10\sqrt{5}$, or $5^{0.1}$

- Set the cursor over 5 on the LL3 scale.
- The cursor is now at 1.175 on the LL2 scale. This is the correct answer

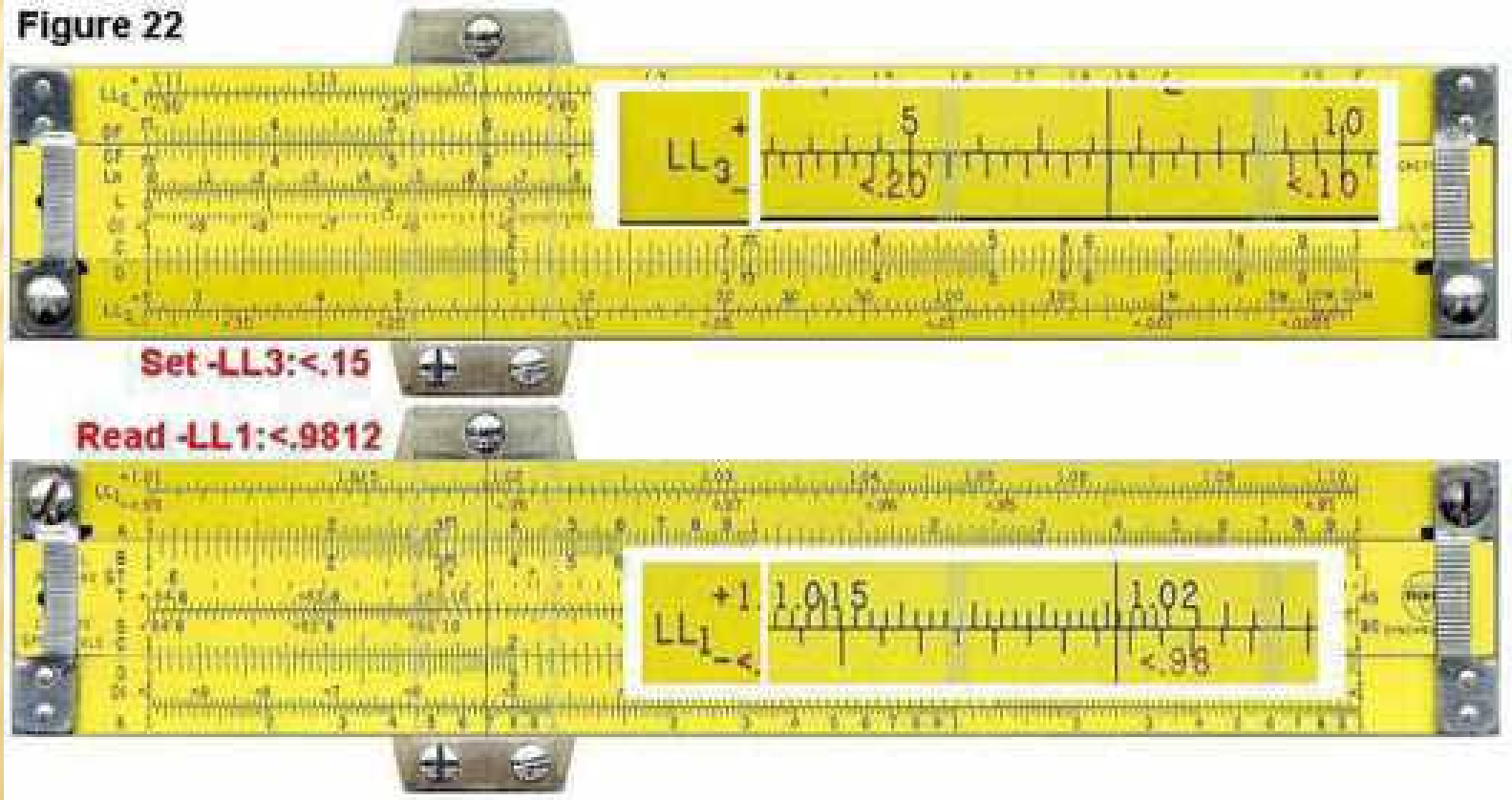


FINDING THE 10TH ROOT ($N < 1$)

Calculate $^{100}\sqrt{0.15}$, or $0.15^{0.01}$

- Set the cursor to 0.15 on the -LL3 scale.
- Flip the SR to the other side to read the LL1 scale.
- The cursor reads 0.9812 on the -LL1 scale. This is the correct answer.

Figure 22



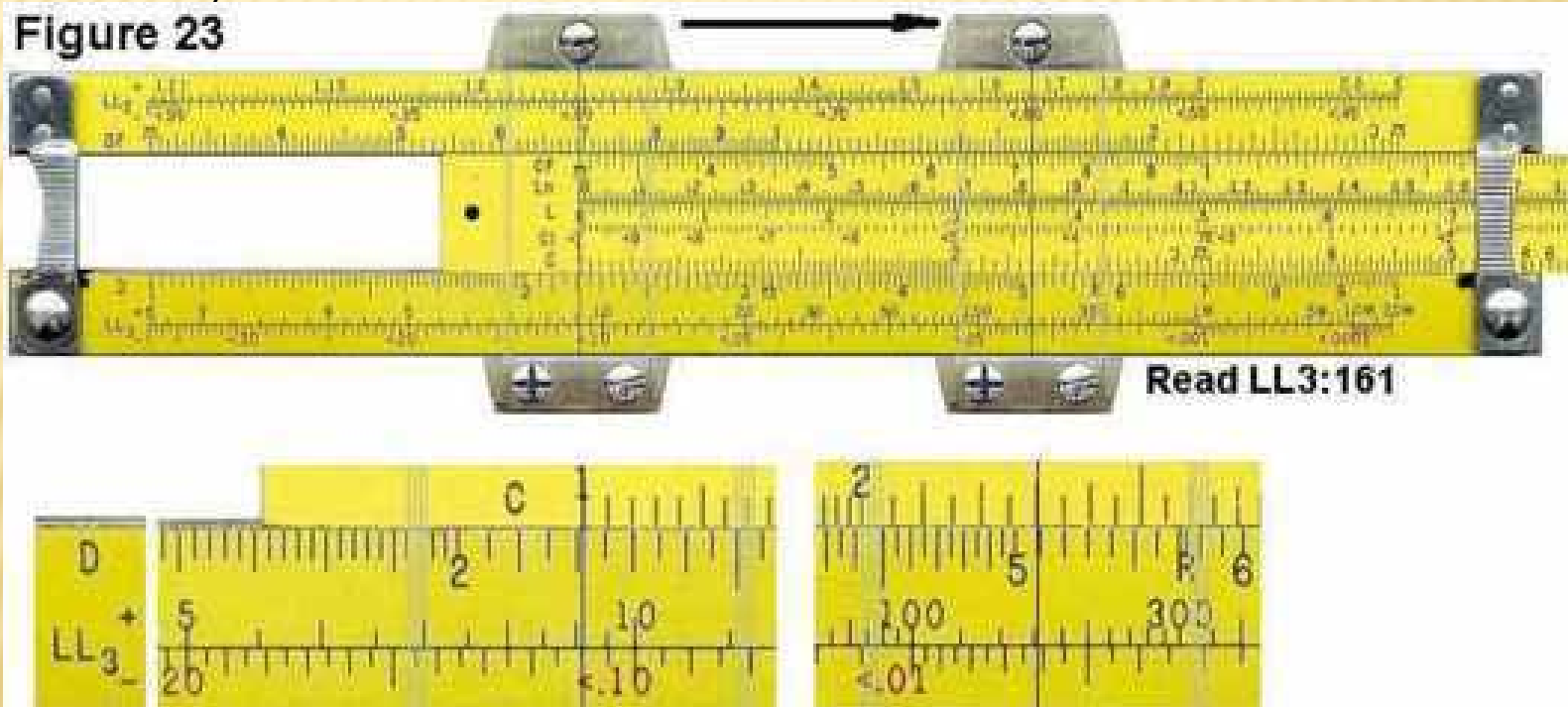


RAISING A NUMBER TO AN ARBITRARY POWER

Calculate $9.1^{2.3}$

- Set the cursor hairline to 9.1 on the LL3 scale.
- Slide the leftmost Index '1' on C to the hairline.
- Move the hairline over 2.3 on the C scale.
- The cursor is now at about 161 on the LL3.
- This is very close to the correct answer of 160.6.

Figure 23

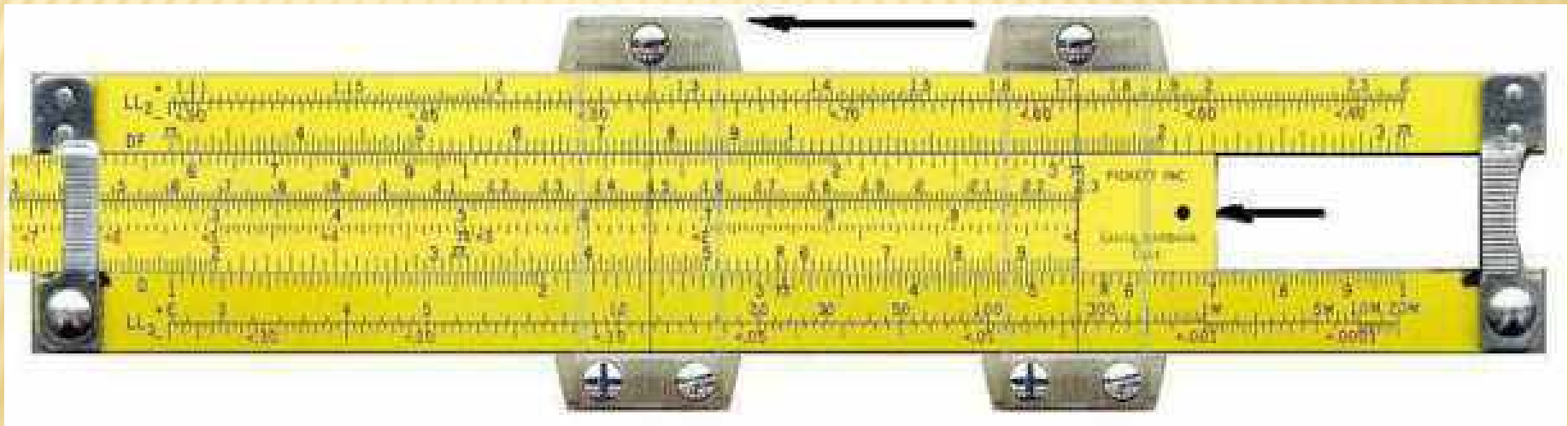




RAISING A NUMBER TO AN ARBITRARY POWER

Calculate $230^{0.45}$

- Set the cursor to 230 on the LL3 scale.
- Since we're raising to a power that's less than 1, we have to go left on the LL scale.
- Slide the rightmost Index '1' on the C scale to the cursor hairline.
- Move the cursor to 4.5 on the C scale.
- The cursor is at ~ 11.6 on the LL3 scale. This is close to the correct answer of 11.56





RAISING A NUMBER RULES

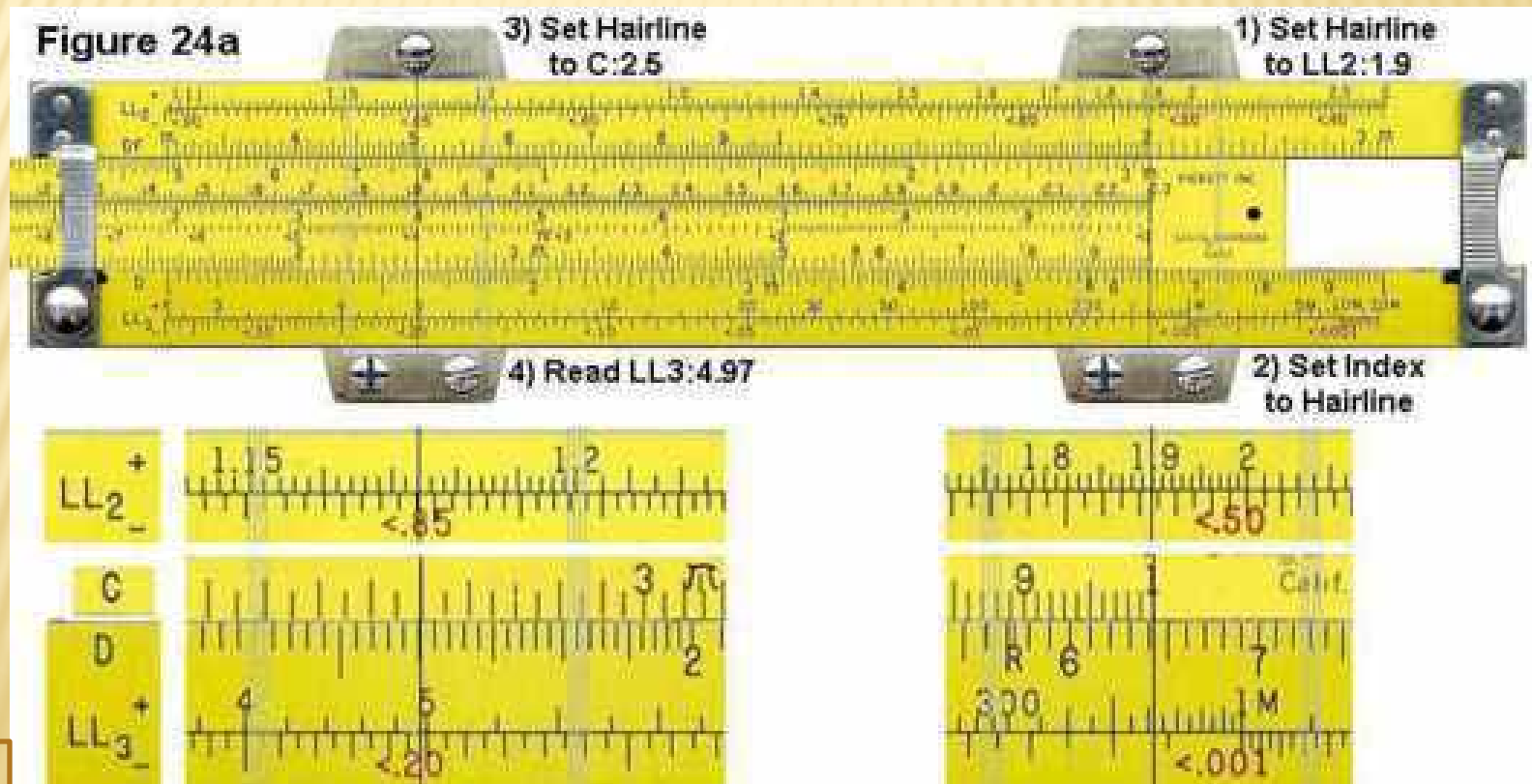
- ∅ If you raise a number to a power >10 you jump up a scale.
- ∅ If you raise a number to a power >1 and you have to reverse the slide (right index) you jump up a scale. (This could be two jumps if $>10!$)
- ∅ Same rules work for jumping down if power <1 .
- ∅ In the earlier example you jumped down for $0.45 <10$... but back up for the right index.



RAISING NUMBER TO ARBITRARY POWER

Calculate $1.9^{2.5}$

- If we try to calculate this the easy way, the power 2.5 is out of range for the scale.
- Set the cursor hairline to 1.9 on the LL2 scale.
- Slide the **rightmost** Index '1' on the C scale to the hairline and cursor over 2.5 on C
- The hairline is now at $1.9^{0.25}$ on the LL2 scale. But jump "one scale" to the LL3 scale.
- The cursor is at 4.97 on the LL3 scale. This is the correct answer.

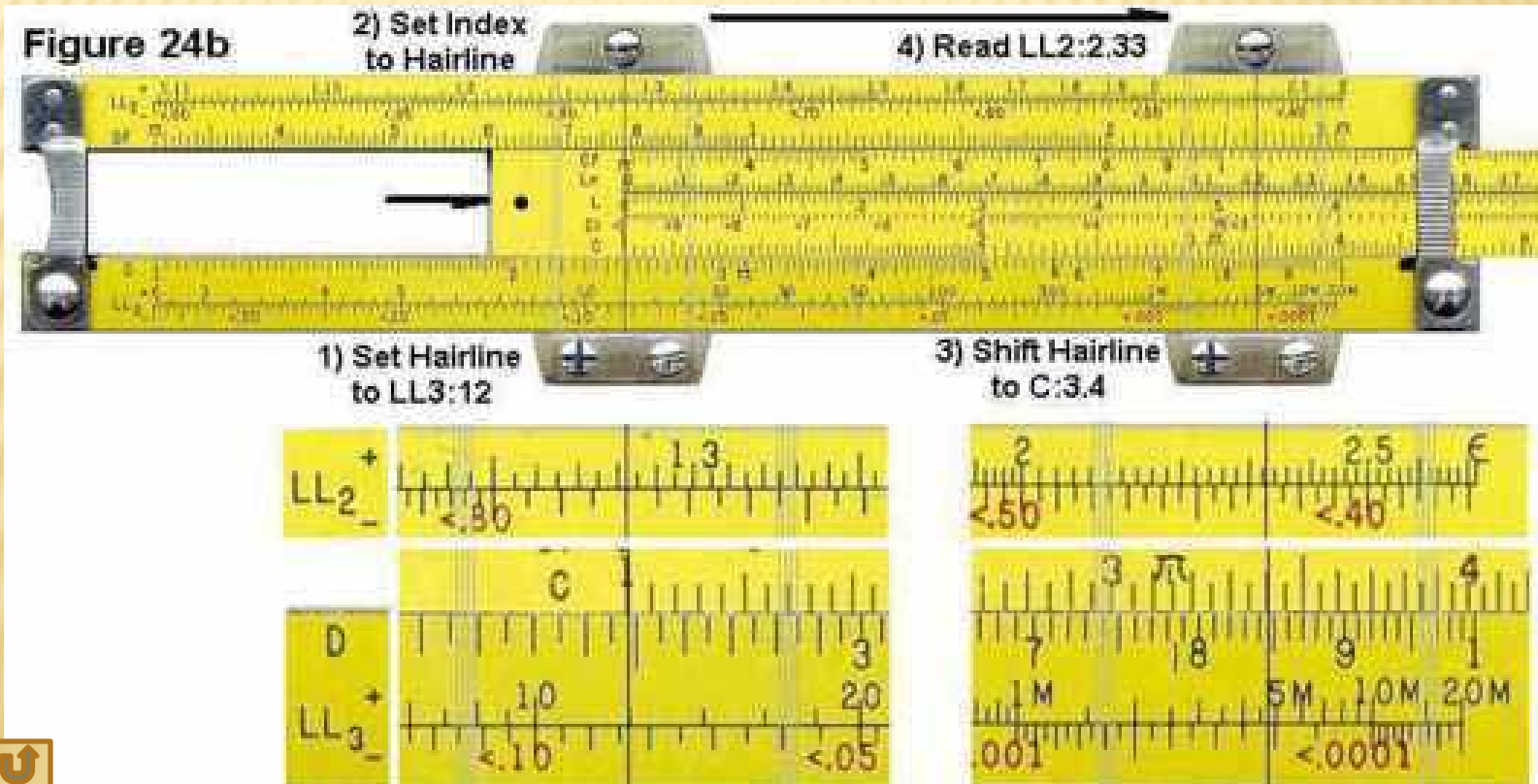




RAISING NUMBER TO ARBITRARY POWER

Calculate $12^{0.34}$

- Move the cursor to 12 on the LL3 scale.
- Slide the leftmost '1' on the C scale to the cursor.
- Move the cursor to 3.4 on the C scale.
- The cursor is now at $12^{3.4}$ on the LL3 scale, but since $.34 < 1$ jump down to LL2 scale.
- The cursor is now at 2.33 on the LL2 scale. This is the correct answer.





PROPORTIONS AND CONVERSIONS

- ∅ On the back of many Mannheim and single sided slide rules are handy conversion factors.
- ∅ Usually in the form of $X:Y$ $C:D$ To change from 'X' to 'Y' set C over D as indicated and for any X on C, read Y on D.
- ∅ The factors may not make sense, for example:
Yards:Meters 187:171, but really 1:0.914. (However that is too hard to set.)

If we know that 231 cubic inches equals 4 quarts and we wish to ascertain the number of quarts contained in 410 cubic inches, we set 231 on "C" over 4 on "D" and under 410 on "C", we read 7.1 quarts on "D" (See Fig. 17).

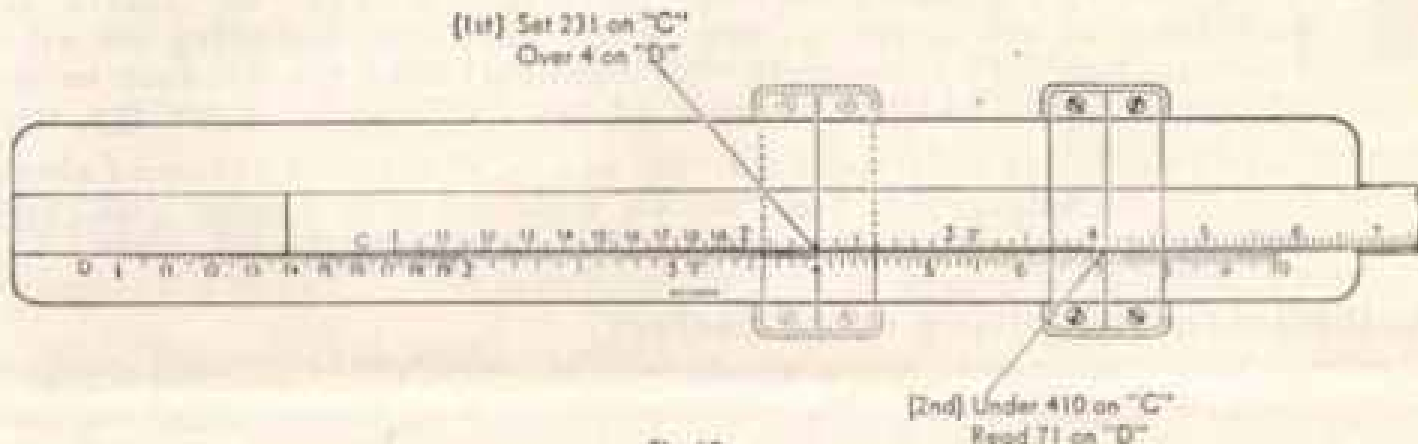


Fig. 17



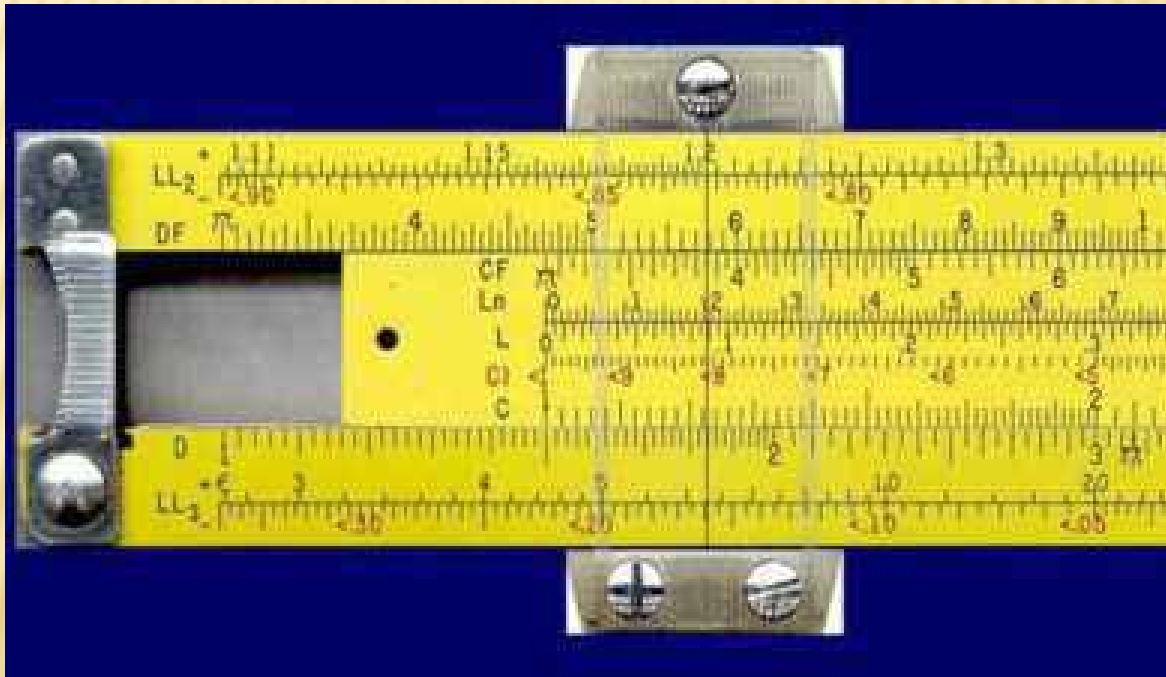
SOLVING QUADRATIC EQUATIONS

- ∅ Even if you've used a slide rule, probably never used it for this!
- ∅ Find the roots of $x^2 + bx + c = 0$; If u and v are the roots of the equation then:
$$(x-u)(x-v) = x^2 - (u+v)x + uv = x^2 + bx + c$$
- ∅ So we want to find two numbers (u,v) that add to b and multiply to c .
 1. Move the hairline over c on the 'D' scale, and place the closest index there.
 2. Now, anywhere you put the hairline, the product of 'D' & 'CI' (or 'DF' & 'CIF') equals c , the product of uv .
 3. All that you need to do is look up and down the slide until you find two numbers on 'D' and 'CI' that add to b . Once you have the numbers u, v then compute the signs from the signs of b and c .



SOLVING QUADRATIC EQUATIONS

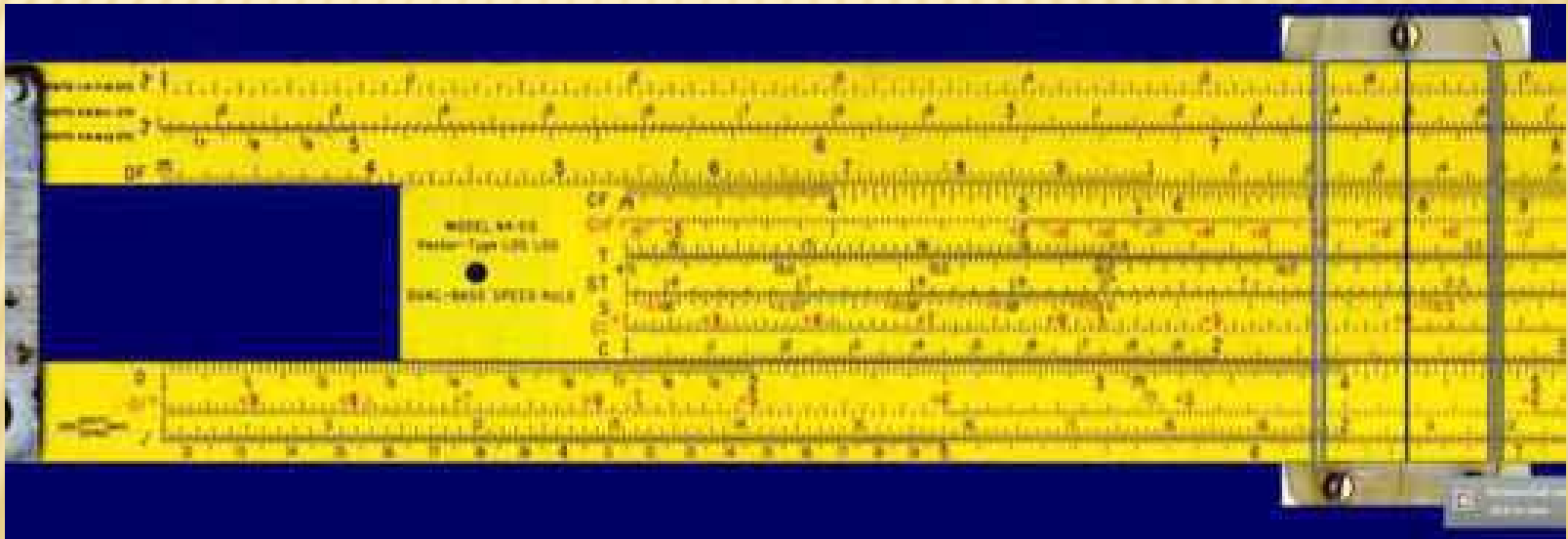
- ∅ Find the roots of $X^2+10x+15=0$
 - Set the left index of the C scale over 15 on the D scale.
 - Move the cursor until the sum of the numbers on CI and D equal 10.
 - This occurs when $D=1.84$ and $CI=8.15$ (=9.99 but close enough)





SOLVING QUADRATIC EQUATIONS

- ∅ Find the roots of $X^2 - 12.2x - 17.2 = 0$
 - Set the left index of the C scale over 17.2 on the D scale.
 - Since this is actually negative, -17.2 and it is the product of u and v, one must be positive the other negative.
 - Also since the sum of these numbers = -12.2. we are really looking for a difference between D and CI that totals 12.2.
 - This occurs when D=13.5 and CI=1.275 (= -12.225 but close enough)

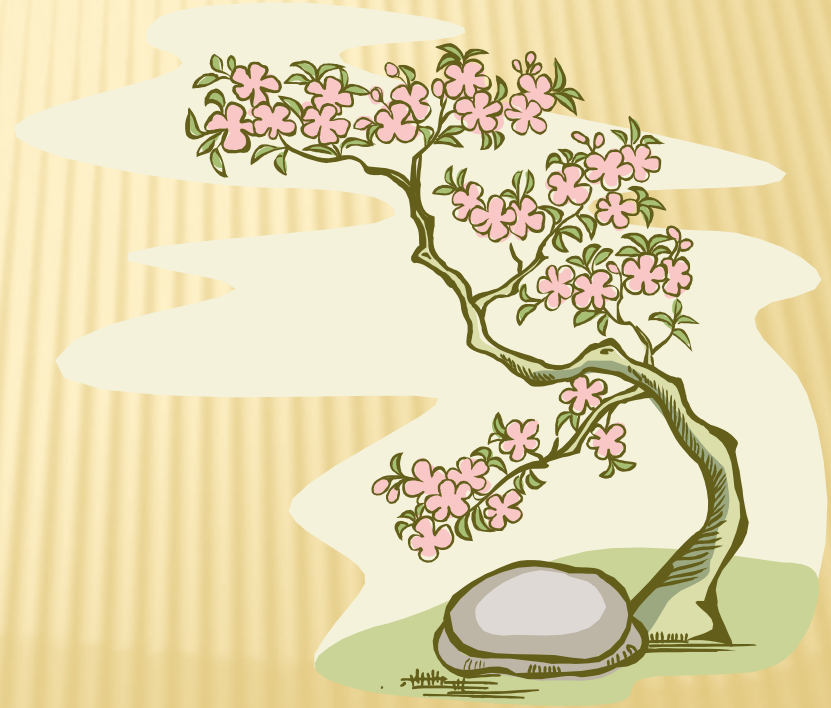


SLIDE RULE HAIKU

The old engineer

keeps drafting tools and slide rule

near the computer.





APPLICATIONS

- ∅ X/R to τ
- ∅ % DC
- ∅ Total Current
- ∅ Vectors
- ∅ Finance
- ∅ Probability

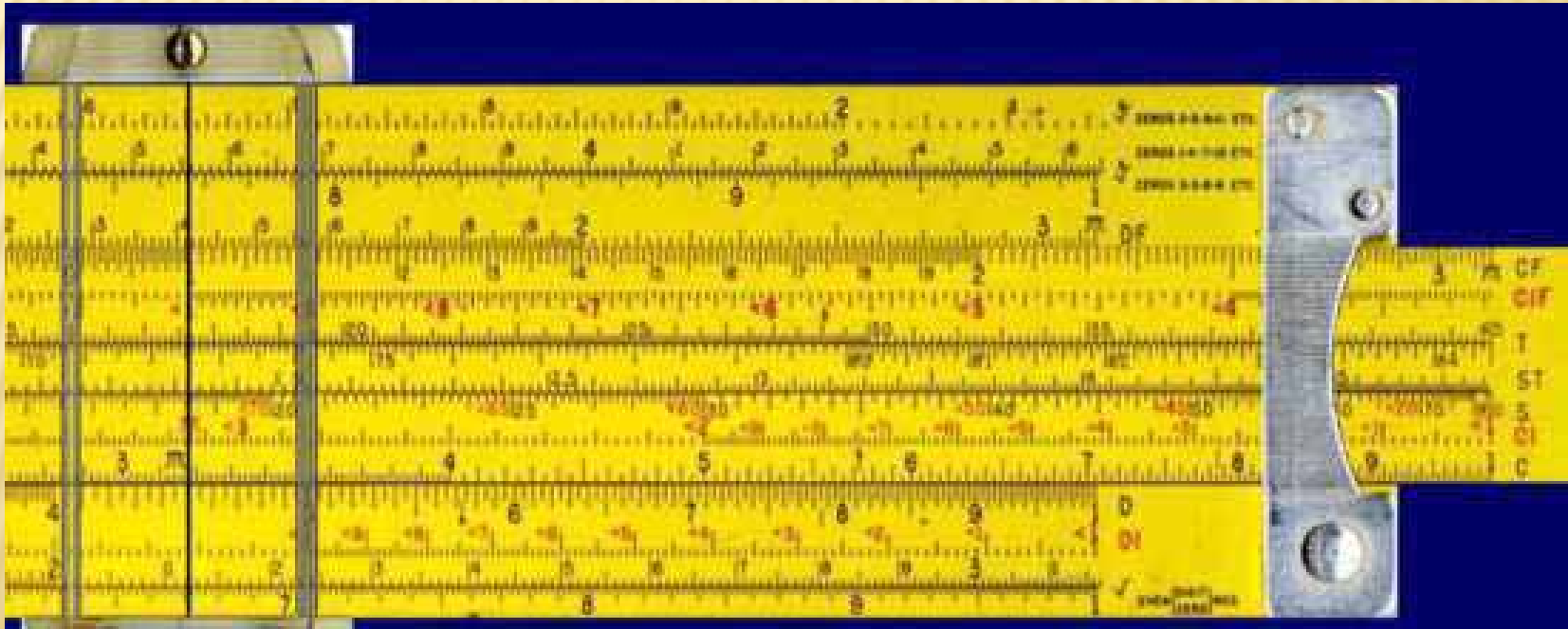


TIME CONSTANT

Calculate τ for $X/R = 17$

- Set the cursor to 17 on the 'DF' Scale.
 - Align 120 (2×60) on CF under the hairline.
 - Slide the cursor to the index ('1') and CF and read .045 on 'D'.
- (Moving from CF/DF to C/D divides by π)

$$t = \frac{X/R}{2pf}$$



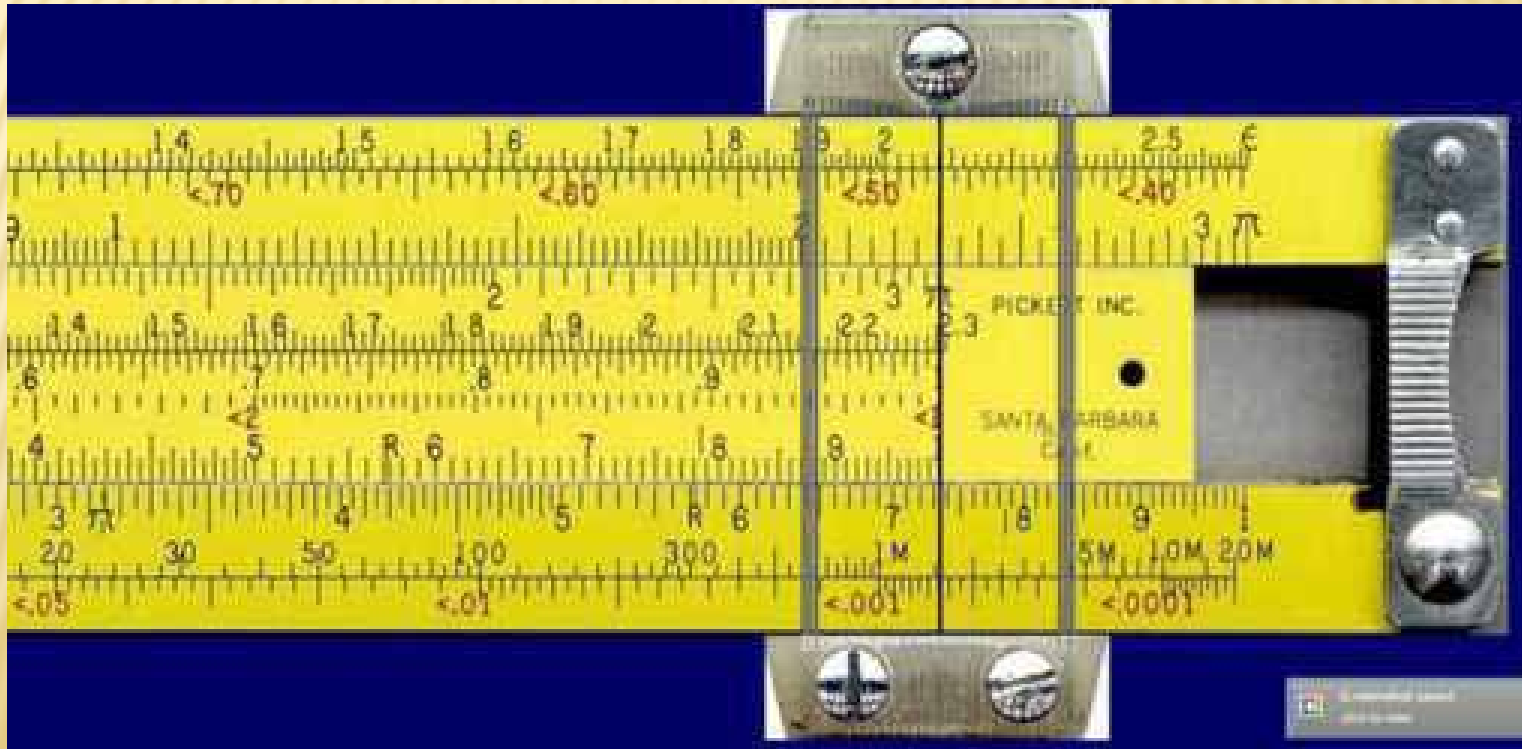


% DC FOR OPENING TIME t

$$\% DC = 100e^{-t/\tau}$$

Calculate %DC for $t=24.7$ msec, $\tau =45$ msec .

- Mentally add 8.3 msec to $t = 24.7 + 8.3 = 33$ msec.
- Put the hairline on 33 on 'D' and align it with 45 on 'C' to divide.
- Slide the cursor to the right index ('1') on 'C' and examine **LL02**
- Find the answer 0.48 (48%)





TOTAL CURRENT FOR %DC

$$I_t = I_{sym} \sqrt{1 + 2 \frac{(\%DC)^2}{100}}$$

Calculate Total Current for 48% DC and $I_{sym}=40$ kA

- Place the hairline on 0.48 on D and obtain the square on 'A' (0.23 but doesn't matter)
- Place the right index of B under the hairline and shift the cursor to 2 on the B scale.
- Read $2 * (\%DC/100)^2$ on 'A' above 2 on 'B' mentally note the value (0.46) and add 1.



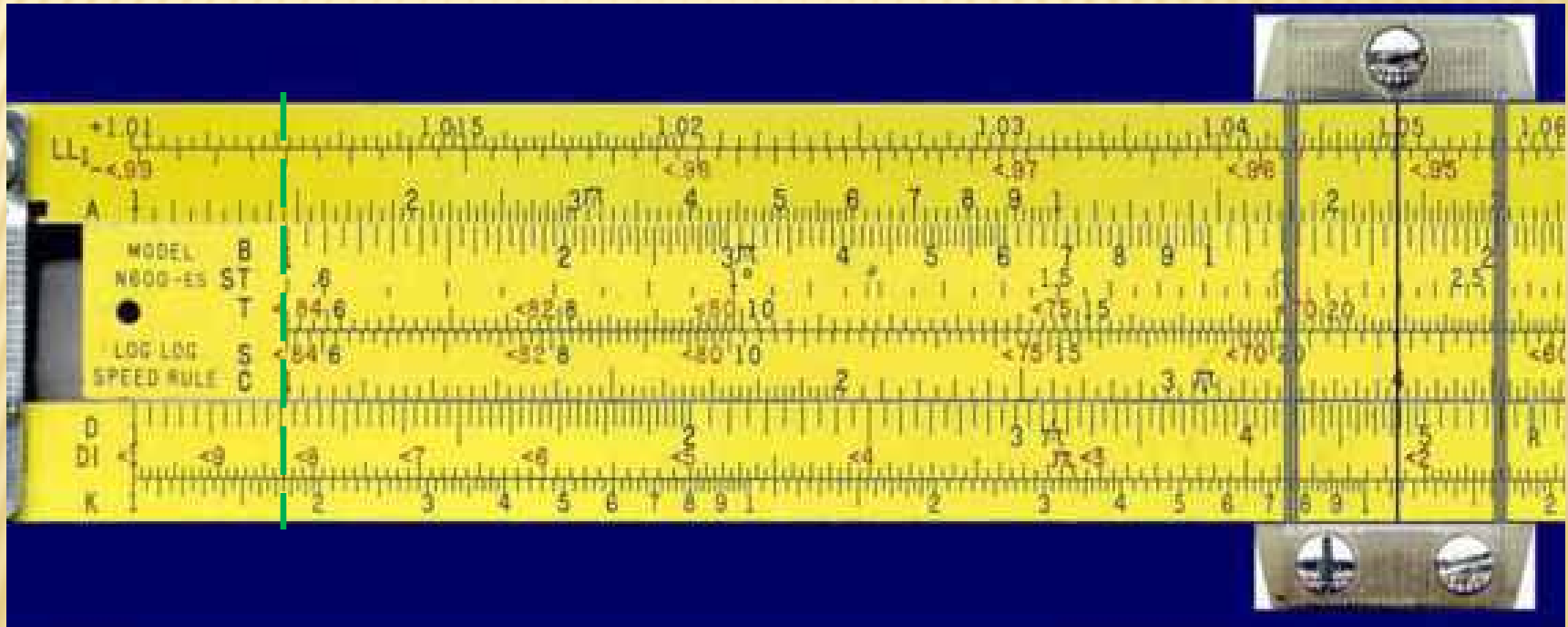


TOTAL CURRENT FOR %DC

Calculate Total Current for 48% DC and $I_{sym}=40$ kA (Part 2)

$$I_t = I_{sym} \sqrt{1 + 2 \frac{\%DC}{100}}$$

- Adding 1 to the previous value of 0.46 gives 1.46.
- Place the cursor at 1.46 on the left half of the 'A' scale, and read the $\sqrt{\quad}$ on 'D' scale
- Place 1 on C over the 'S' factor on D scale (1.21 but doesn't matter)
- Read the total current under the I_{sym} of 40 on the C scale = 48.35.



VECTORS

∅ $R \cdot e^{jq} = R \cdot \cos(q) + j \cdot R \cdot \sin(q)$

∅ $R \cdot e^{jq} = x + jy$ where $x = R \cdot \cos(q)$

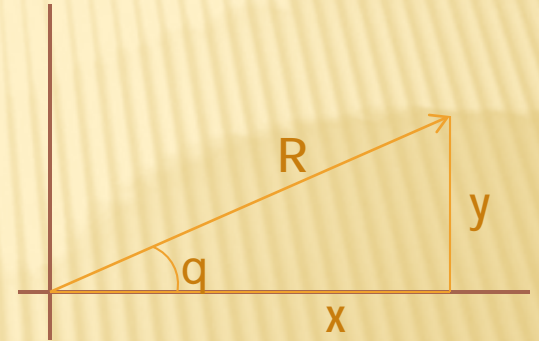
and $y = R \cdot \sin(q)$

∅ $y/x = \tan(q)$ or $y = \tan(q) \cdot x$ **and** $R = y/\sin(q)$

∅ Note: In the following example a powerful feature of slide rules is employed: Solve for the Unknown or "Reverse Division"

∅ In solving $\tan(q) \cdot x = y$, if you knew both x & $\tan(q)$, then in solving the equation, x and y would be on the D scale, and $\tan(q)$ on the slide.

∅ 'Pretend' you know $\tan(q)$ and set up the slide rule. [next]

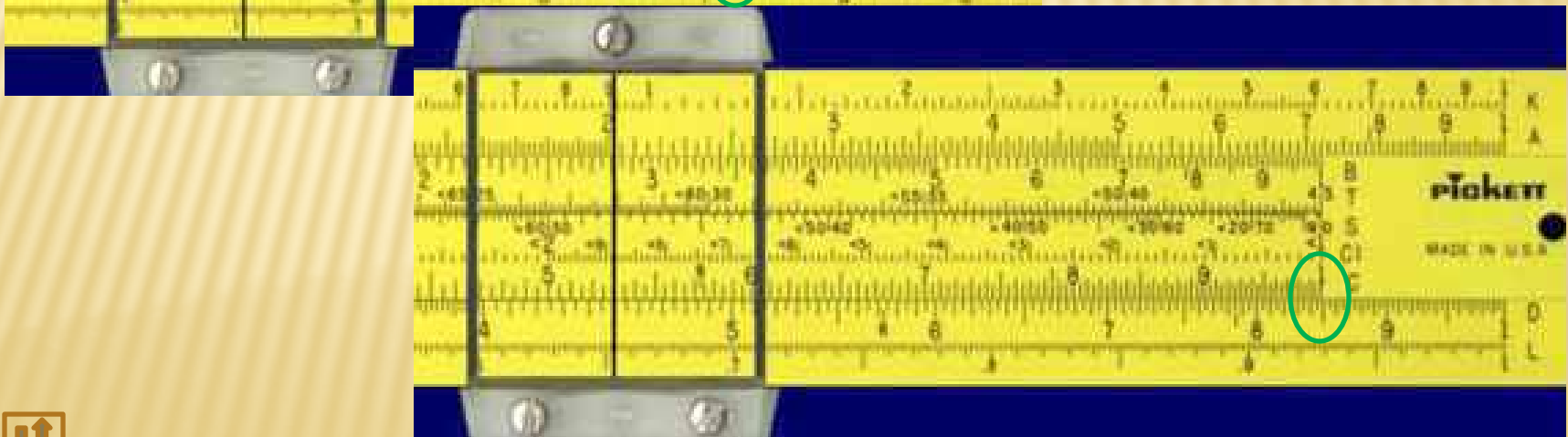
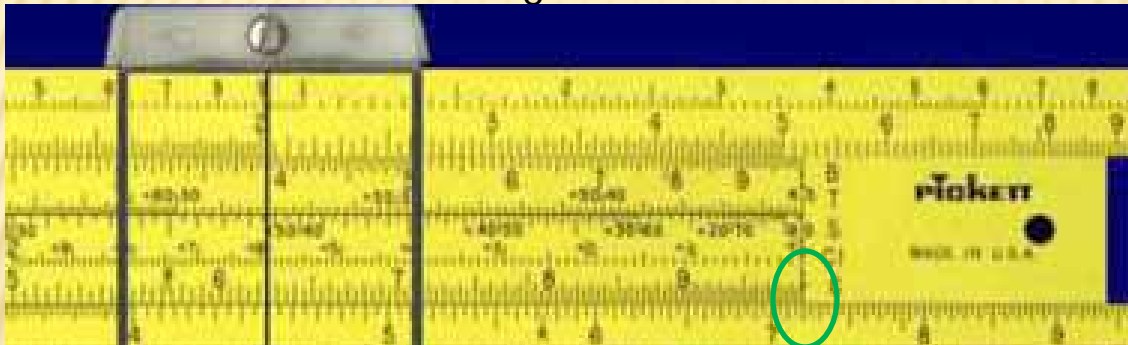


VECTORS

Solve for the
Unknown Method

Calculate $R\mathbf{e}^{j\theta}$ for $7.2+j4.5$

- Set right index of C to 7.2 on D
- Move cursor to 4.5 on D, read $\theta=32^\circ$ on T at the hairline
- Leaving the cursor at 4.5 on D, Move the slide to place 32° on S at hairline
- Read $R=8.49$ on D at right index of C the answer is $8.49\mathbf{e}^{j32^\circ}$





PROBABILITY

- ∅ The probability of r successes in a random sample of N from a population P for a binomial distribution is:

$$\text{Pr} = C_N^r P^r (1 - P)^{N-r}$$

N = sample size

r = number of defects in population

P = proportion of defects in the population

- ∅ What is the probability of finding exactly 2 defects in a random sample of 10 parts from a lot that contains 5% defective parts?

$$\text{Pr} = \frac{10!}{8! 2!} (0.05)^2 (0.95)^8$$

PROBABILITY

$$\text{Pr} = \frac{10!}{8!2!} (0.05)^2 (0.95)^8$$

- Ø Set the hairline at 0.95 on LL/1 slide the right index of C to the hairline & move the hairline to 8 on C, and read 0.664 on LL/2. (N < 1, being raised to 1 < p < 10, but right index so a jump up.)
- Ø Set the hairline to 0.05 on LL/3, slide 2 on CI to the hairline and read 0.0025 on LL/3. (N < 1, being raised to 1 < p < 10, no reversals, so stay on LL/3)
- Ø Can't do factorials on a slide rule (easily) but $10!/8! = 10 \times 9$. Mentally, $90 / 2 = 45$.
- Ø Problem is now reduced to chain multiplication.
- Ø Place the cursor on 45 on D and line .0025 on CI over it.
- Ø The left index of C is over the product on D, but doesn't matter.
- Ø Move the hairline to .0664 on C and read .0746 on D.
- Ø Probability of finding exactly two defects is ~ 7.5%

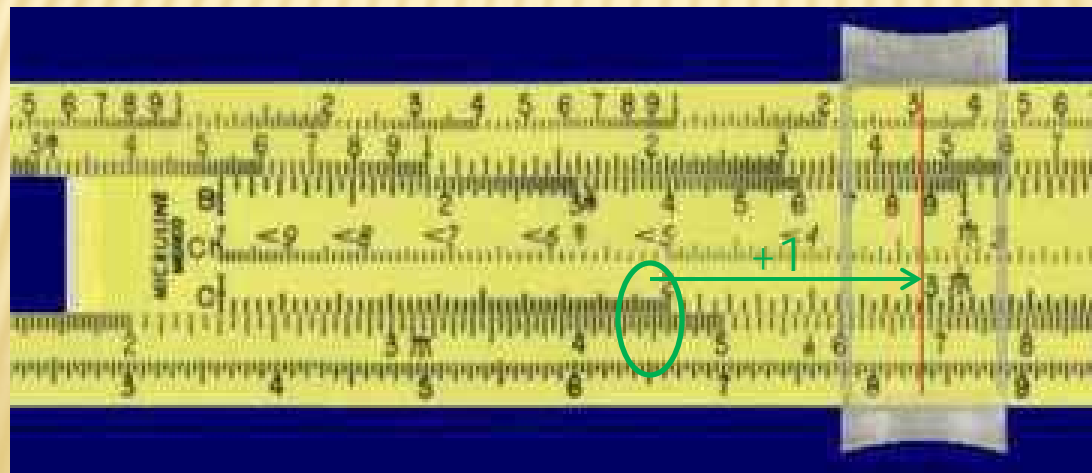
ADDITION & SUBTRACTION

- ∅ Addition and subtraction
- ∅ The C and D scales are logarithmic scales designed for multiplication. We want $x + y$, we need to find a multiplier (let's call it m) such that $x * m = x + y$.

Solving for m :

$$x * m = x + y$$
$$m = \frac{1}{x}(x + y) = 1 + \frac{y}{x}$$

- ✓ Move left index (1) on C to 2.3 on D Scale
- ✓ Move cursor to 4.5 on D Scale, Observe cursor is on 1.956 on C.
- ✓ Mentally add 1 to 1.956 to get 2.956. Move the cursor to 2.956 on C Scale.
- ✓ Now, you have 'conventional' multiplication of $2.3 \times (4.5/2.3+1)$, cursor is on 6.8 on D Scale.



"REVERSE DIVISION"

$$\frac{4.5}{2.3} = c$$

$$2.3 * c = 4.5$$

∅ A pipe gives a wise man time to think;
and a fool something to stick in his mouth.

-- *Anonymous*

∅ A slide rule gives a wise man time to think;
and a fool something to keep his hands busy.

-- *John Webb*



TOP 10 REASONS TO USE A SLIDE RULE

10. For many problems, after you do it once, repeat calculations faster than on a calculator. As are reverse (what number gives me?) calculations.
9. Empire State Building, Hoover Dam, Golden Gate Bridge, Nuclear Power, Apollo Moon Rocket, ...
8. You think about solving the problem, not just getting an answer.
7. Calculators are Nerdy, Slide Rules are Cool!
6. Significant figures: Just because you *can* calculate to 12 digits doesn't mean you should.





TOP 10 REASONS TO USE A SLIDE RULE

5. Your boss will never say “Hell, I could do that!”
4. Closer to the numbers: You know what makes up the answer, which parts are sensitive to change and which aren’t.
3. Even when you’re wrong, people are still impressed.
2. Less likely to make a mistake, because you think!
You estimate, calculate & evaluate.
1. Two Words: ~~No Batteries~~ **Zombie Apocalypse**





HOW TO GET YOUR OWN SLIDE RULE

- ∅ Ebay (*used slide rules, wide price swings*)
 - ✓ Rod Lovett's Slide Rule price history
<http://sliderules.lovett.com/srsearch.html>
- ∅ Sphere Research Corporation (*new & used*)
 - ✓ Slide Rule Universe
<http://www.sphere.bc.ca/test/sruniverse.html>
- ∅ Rose Vintage Instruments (*new & Collector specials*)
 - ✓ <http://www.rose-vintage-instruments.com/>





REFERENCES, RESOURCES & CREDITS

- ∅ International Slide Rule Museum www.sliderulemuseum.com
- ∅ The Oughtred Society www.oughtred.org
- ∅ Eric's Slide Rule Page www.sliderule.ca
- ∅ Derek's Virtual SR Gallery <http://solo.dc3.com/VirtRule/index.html>
- ∅ HP Museum www.hpmuseum.org/sliderul.htm
- ∅ *When Slide Rules Ruled* by Clifford Stohl, Scientific American, May '06
- ∅ History of the Logarithmic Slide Rule
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- ∅ Wolfram Mathworld <http://mathworld.wolfram.com/>
- ∅ VersaLog Slide Rule Instruction Manual
E.I. Fisenheiser, Frederick Post Co.; Chicago, IL © 1951
- ∅ The Log Log Duplex Trig Slide Rule No. 4080, A Manual
L.M. Kellis, W.F. Kern, J. R. Bland, K&E NY, NY © 1943
- ∅ Dietzgen Redi-Rule Pocket Slide Rule, Eugene Dietzgen Co.





THIS WAS DONE WITH A
SLIDE RULE.

Your eighth grader has more computing power in his cell phone,
and can't do arithmetic. Think there's a connection?